CCGPS Frameworks Teacher Edition

Mathematics

Fourth Grade

Grade Level Overview

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"Making Education Work for All Georgians"

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Grade Level Overview

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Common Core Georgia Performance Standards
Fourth Grade

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These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain.							

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 3-5 Key: G= Geometry, MD=Measurement and Data, NBT= Number and Operations in Base Ten, NF = Number and Operations, Fractions, OA = Operations and Algebraic Thinking.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Students are expected to:

1. Make sense of problems and persevere in solving them.

In fourth grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. Reason abstractly and quantitatively.

Fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others.

In fourth grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

4. Model with mathematics.

Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

5. Use appropriate tools strategically.
Fourth graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.

6. Attend to precision.
As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.

7. Look for and make use of structure.
In fourth grade, students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.

8. Look for and express regularity in repeated reasoning.
Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

******Mathematical Practices 1 and 6 should be evident in EVERY lesson******

CONTENT STANDARDS

Operations and Algebraic Thinking

CLUSTER #1: USE THE FOUR OPERATIONS WITH WHOLE NUMBERS TO SOLVE PROBLEMS.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***multiplication/multiply, division/divide, addition/add, subtraction/subtract, equations, unknown, remainders, reasonableness, mental computation, estimation, rounding.***

MCC.4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "a is n times as much as b"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.
Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

Examples:
5 x 8 = 40: Sally is five years old. Her mom is eight times older. How old is Sally's Mom?
5 x 5 = 25: Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

MCC.4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems. Refer Table 2, included at the end of this document, for more examples.
Examples:

- **Unknown Product:** A blue scarf costs \$3. A red scarf costs 6 times as much. How much does the red scarf cost? $(3 \times 6 = p)$
- **Group Size Unknown:** A book costs \$18. That is 3 times more than a DVD. How much does a DVD cost? $(18 \div p = 3 \text{ or } 3 \times p = 18)$

- **Number of Groups Unknown:** A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? *($18 \div 6 = p$ or $6 \times p = 18$)*

  When distinguishing multiplicative comparison from additive comparison, students should note the following.
- Additive comparisons focus on the difference between two quantities.
  - For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?
  - A simple way to remember this is, "How many more?"
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other.
  - For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?

  A simple way to remember this is "How many times as much?" or "How many times as many?"

MCC.4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.
Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?
Some typical estimation strategies for this problem are shown below.

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	Student 2
First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\ ^{2}/_{6}$

Possible solutions:

- Problem **A: 7.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

- Problem **B: 7 r 2.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

- Problem **C: 8.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

- Problem **D: 7 or 8.**
  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E:** $\mathbf{7^2/_6}$**.**
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $7^2/_6$
  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).*
  Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)

- **Clustering around an average** (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)

- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)

- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)

- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

<u>CLUSTER #2:</u> GAIN FAMILIARITY WITH FACTORS AND MULTIPLES.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***multiplication/multiply, division/divide, factor pairs, factor, multiple, prime, composite.***

MCC.4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

Common Misconceptions

A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.

Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

Prime vs. Composite:

- A prime number is a number greater than 1 that has only 2 factors, 1 and itself.
- Composite numbers have more than 2 factors.

  Students investigate whether numbers are prime or composite by
- Building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g., 7 can be made into only 2 rectangles, 1 × 7 and 7 × 1, therefore it is a prime number).
- Finding factors of the number.

  Students should understand the process of finding factor pairs so they can do this for any number 1-100.

  Example:

  Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.

Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:

Factors of 24: 1, 2, 3, 4, 6,8, 12, 24

Multiples: 1, 2, 3, 4, 5, … , <u>24</u>

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, <u>24</u>

3, 6, 9, 12, 15, 15, 21, <u>24</u>

4, 8, 12, 16, 20, <u>24</u>

8, 16, <u>24</u>

12, <u>24</u>
<u>24</u>

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- All even numbers are multiples of 2.
- All even numbers that can be halved twice (with a whole number result) are multiples of 4.
- All numbers ending in 0 or 5 are multiples of 5.

<u>CLUSTER #3</u>: GENERATE AND ANALYZE PATTERNS.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***pattern (number or shape), pattern rule.***

MCC.4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. ***For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.***

Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example:

Pattern	**Rule**	**Feature(s)**
3, 8, 13, 18, 23,28, .	Start with 3; add 5	The numbers alternately end with a 3 or an 8
5, 10, 15, 20, …	Start with 5; add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that 3nd with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:

Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 8, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 – 1 = 2, 9 – 3 = 6, 27 – 9 = 18, etc.).

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

Day	Operation	Beans
0	$3 \times 0 + 4$	4
1	$3 \times 1 + 4$	7
2	$3 \times 2 + 4$	10
3	$3 \times 3 + 4$	13
4	$3 \times 4 + 4$	16
5	$3 \times 5 + 4$	19

Number and Operation in Base Ten

CLUSTER #1: GENERALIZE PLACE VALUE UNDERSTANDING FOR MULTI-DIGIT WHOLE NUMBERS.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***place value, greater than, less than, equal to, ‹, ›, =, comparisons/compare, round.***

MCC.4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. ***For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.***

This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

Example:
How is the 2 in the number 582 similar to and different from the 2 in the number 528?

MCC.4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones.

Students should also be able to compare two multi-digit whole numbers using appropriate symbols.

MCC.4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.

This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First, I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 2040, so we need about 240 more bottles.

Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did your family travel?
Some typical estimation strategies for this problem:

Student 1	**Student 2**	**Student 3**
I first thought about 276 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be

200 together, I get 500.	have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	about 530.

Example:
Round 368 to the nearest hundred.
This will either be 300 or 400, since those are the two hundreds before and after 368.
Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400.

Common Misconceptions

There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand do not cause a problem; however a number like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but then instead of placing the 2 in the ones place, students will write the numbers as they hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition method.
Students often assume that the first digit of a multi-digit number indicates the "greatness" of a number. The assumption is made that 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.
Students need to be aware of the greatest place value. In this example, there is one number with the lead digit in the thousands and another number with its lead digit in the hundreds.

<u>CLUSTER #2</u>: USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC.

Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they *develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are:* ***partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction.***

MCC.4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

Example:

$$\begin{array}{r} 3892 \\ \underline{+\ 1567} \end{array}$$

Student explanation for this problem:

1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the10 tens as one more hundred.(*Denotes with a 1 above the hundreds column*)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.
5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (*Denotes with a 1 above the thousands column*)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

Example:

$$\begin{array}{r} 3546 \\ \underline{-\ \ 928} \end{array}$$

Student explanations for this problem:

1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (*Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.*)
2. Sixteen ones minus 8 ones is 8 ones. (*Writes an 8 in the ones column of answer.*)
3. Three tens minus 2 tens is one ten. (*Writes a 1 in the tens column of answer.*)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (*Marks through the 3 and notates with a 2 above it. Writes down a 1 above the hundreds column.*) Now I have 2 thousand and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (*Writes a 6 in the hundreds column of the answer.*)
6. I have 2 thousands left since I did not have to take away any thousands. (*Writes 2 in the thousands place of answer.*)

   Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

MCC.4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

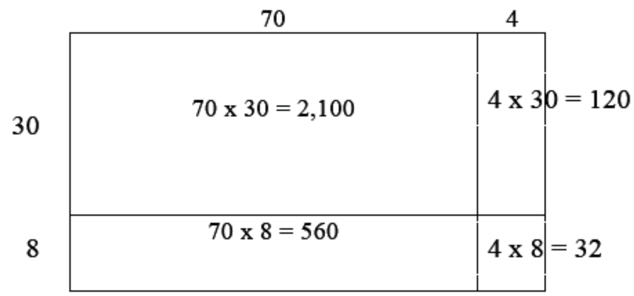
Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. **Use of the standard algorithm for multiplication is an expectation in the 5th grade.**

This standard calls for students to multiply numbers using a variety of strategies.

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

Student 1	**Student 2**	**Student 3**
25 × 12 I broke 12 up into 10 and 2. 25 × 10 = 250 25 × 2 = 50 250 + 50 = 300	25 × 12 I broke 25 into 5 groups of 5. 5 × 12 = 60 I have 5 groups of 5 in 25. 60 × 5 = 300	25 × 12 I doubled 25 and cut 12 in half to get 50 × 6. 50 × 6 = 300

Example:
What would an array area model of 74 x 38 look like?



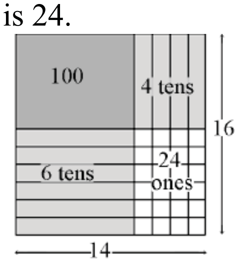
2,100 = 560 + 1,200 + 32 = 2,812

Examples:
To illustrate 154 × 6, students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property,

154 × 6 = (100 + 50 + 4) × 6
= (100 × 6) + (50 × 6) + (4 × 6)
=600 + 300 + 24 = 924.

The area model below shows the partial products for 14 x 16 = 224.
Using the area model, students first verbalize their understanding:

- 10 × 10 is 100
- 4 × 10 is 40
- 10 × 6 is 60, and
- 4 × 6 is 24.



100 + 40 + 60 + 24 = 224 Students use different strategies to record this type of thinking.

Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

```
   25
×  24
      400 (20 × 20)
      100 (20 × 5)
 80 (4 × 20)
 20 (4 × 5)
600
```

MCC.4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.
Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

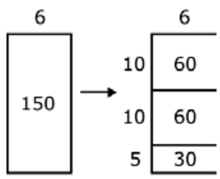
- **Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** $260 \div 4 = (200 \div 4) + (60 \div 4)$
- **Using Multiplication:** $4 \times 50 = 200$, $4 \times 10 = 40$, $4 \times 5 = 20$; $50 + 10 + 5 = 65$; so $260 \div 4 = 65$

This standard calls for students to explore division through various strategies.
Example:
There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

Student 1	**Student 2**	**Student 3**
592 divided by 8 There are 70 eights in 560. 592 – 560 = 32 There are 4 eights in 32. 70 + 4 = 74	592 divided by 8 I know that 10 eights is 80. If I take out 50 eights that is 400. 592 – 400 = 192 I can take out 20 more eights which is 160. 192 – 160 = 32 8 goes into 32 four times. I have none left. I took out 50, then 20 more, then 4 more. That's 74.	I want to get to 592. 8 × 25 = 200 8 × 25 = 200 8 × 25 = 200 200 + 200 + 200 = 600 600 – 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams.

Example:
Using an Open Array or Area Model
After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

1. **150 ÷ 6**



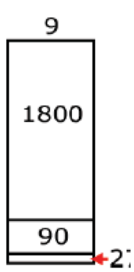
Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, "6 times what number is a number close to 150?" They recognize that 6 × 10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left, they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6 × 5 is 30, they write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Student express their calculations in various ways:

   a. 150
   <u>–60</u> (6 × 10)   150 ÷ 6 = 10 + 10 + 5 = 25
   90
   <u>–60</u> (6 × 10)
   30
   <u>–30</u> (6 × 5)
   0

   b. 150 ÷ 6 = (60 ÷ 6) + (60 ÷ 6) + (30 ÷ 6) = 10 + 10 + 5 = 25

2. 1917 ÷ 9



A student's description of his or her thinking may be:
I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. 1917 ÷ 9 = 213.

Common Misconceptions

Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

Number and Operations- Fractions

<u>CLUSTER #1</u>: EXTEND UNDERSTANDING OF FRACTION EQUIVALENCE AND ORDERING.

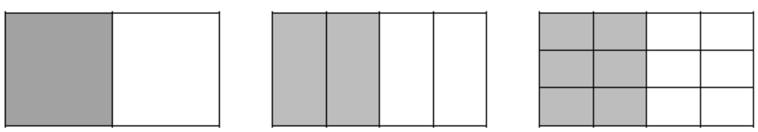
Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***partition(ed), fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, comparison/compare, ‹, ›, =, benchmark fraction.***

MCC.4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

Example:



1/2 = 2/4 = 6/12

Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80

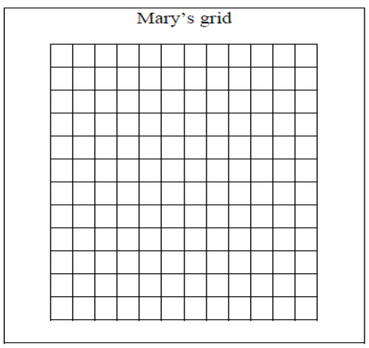
MCC.4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

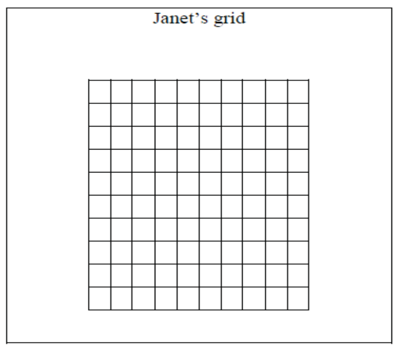
This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. When tested, models may or may not be included. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., 1/2 and 1/8 of two medium pizzas is very different from 1/2 of one medium and 1/8 of one large).

Example:
Use patterns blocks.

1. If a red trapezoid is one whole, which block shows 1/3?
2. If the blue rhombus is 1/3, which block shows one whole?
3. If the red trapezoid is one whole, which block shows 2/3?

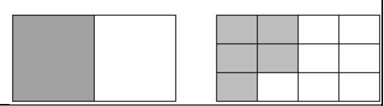
Example:
Mary used a 12 × 12 grid to represent 1 and Janet used a 10 × 10 grid to represent 1. Each girl shaded grid squares to show ¼. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?
Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so ¼ of each total number is different.



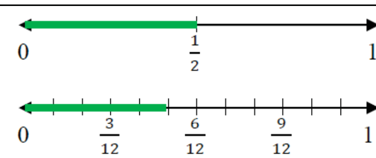


Example:
There are two cakes on the counter that are the same size. The first cake has 1/2 of it left. The second cake has 5/12 left. Which cake has more left?

Student 1: ***Area Model***
The first cake has more left over. The second cake has 5/12 left which is smaller than 1/2.

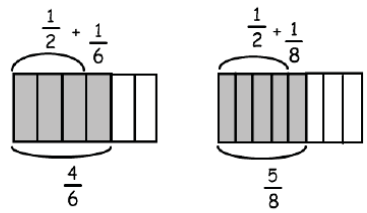


Student 2: ***Number Line Model***
The first cake has more left over: 1/2 is bigger than 5/12.



Student 3: ***Verbal Explanation***
I know that 6/12 equals 1/2, and 5/12 is less than 1/2. Therefore, the second cake has less left over than the first cake. The first cake has more left over.

Example:
When using the benchmark of $\frac{1}{2}$ to compare to $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:



$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

Common Misconceptions

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing 12 to sixths. They would multiply the denominator by 3 to get 16, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.
Students need to use a fraction in the form of one such as 33 so that the numerator and denominator do not contain the original numerator or denominator.

<u>CLUSTER #2</u>: BUILD FRACTIONS FROM UNIT FRACTIONS BY APPLYING AND EXTENDING PREVIOUS UNDERSTANDINGS OF OPERATIONS ON WHOLE NUMBERS.

Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***operations, addition/joining, subtraction/separating, fraction, unit fraction, equivalent, multiple, reason, denominator, numerator, decomposing, mixed number, rules about how numbers work (properties), multiply, multiple.***

MCC.4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.

Example: $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$

Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example: $1\frac{1}{4} - \frac{3}{4} = ?$ → $\frac{4}{4} + \frac{1}{4} = \frac{5}{4}$ → $\frac{5}{4} - \frac{3}{4} = \frac{2}{4}$ or $\frac{1}{2}$

Example of word problem:

Mary and Lacey decide to share a pizza. Mary ate $\frac{3}{6}$ and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat together?

Possible solution: The amount of pizza Mary ate can be thought of a $\frac{3}{6}$ or $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. The amount of pizza Lacey ate can be thought of a $\frac{1}{6} + \frac{1}{6}$. The total amount of pizza they ate is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or $\frac{5}{6}$ of the pizza. A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.

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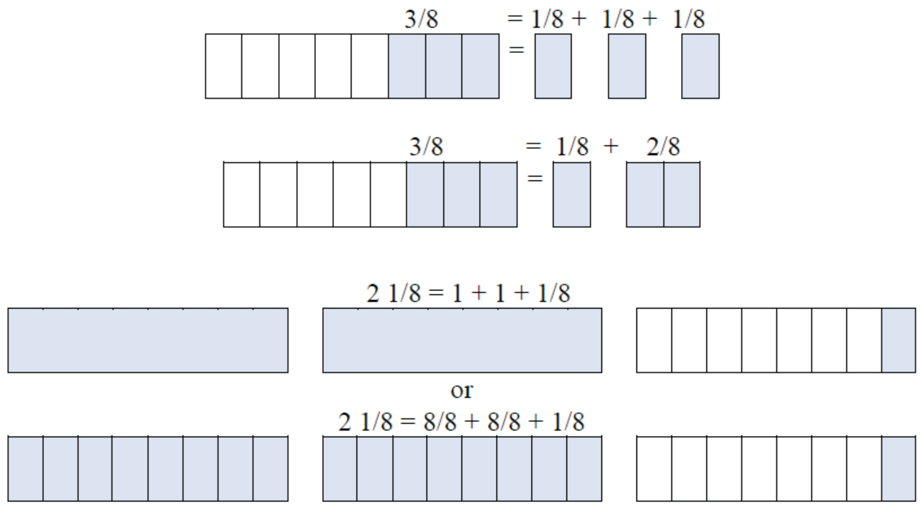
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. ***Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.***

Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

Example:



c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:

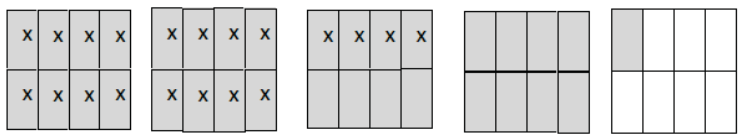
Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. I can write this as $3\frac{1}{8} + 5\frac{3}{8}$. I know they have 8 feet of ribbon by adding the 3 and 5. They also have $\frac{1}{8}$ and $\frac{3}{8}$ which makes a total of $\frac{4}{8}$ more. Altogether they have $8\frac{4}{8}$ feet of ribbon. $8\frac{4}{8}8$ is larger than $8\frac{3}{8}$ so they will have enough ribbon to complete the project. They will even have a little extra ribbon left: $\frac{1}{8}$ foot.

Example:

Trevor has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?

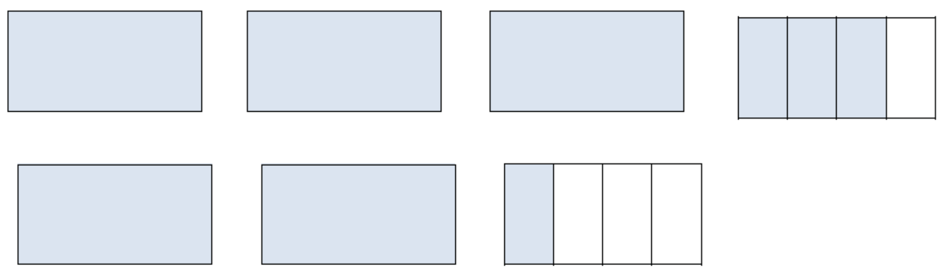
Possible solution: Trevor had $4\frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x's show the pizza he has left which is $2\frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is $\frac{13}{8}$ or $1\frac{5}{8}$ pizzas.



Mixed numbers are introduced for the first time in 4th Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions.

Example:

While solving the problem, $3\frac{3}{4} + 2\frac{1}{4}$, students could do the following:



Student 1: $3 + 2 = 5$ and $\frac{3}{4} + \frac{1}{4} = 1$, so $5 + 1 = 6$.

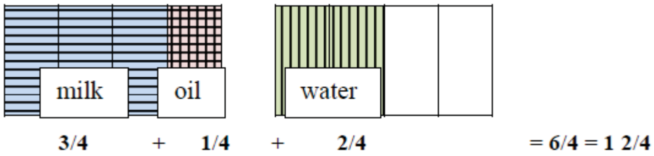
Student 2: $3\frac{3}{4} + 2 = 5\frac{3}{4}$, so $5\frac{3}{4} + \frac{1}{4} = 6$.

Student 3: $3\frac{3}{4} = \frac{15}{4}$ and $2\frac{1}{4} = \frac{9}{4}$, so $\frac{15}{4} + \frac{9}{4} = \frac{24}{4} = 6$.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Example:

A cake recipe calls for you to use $\frac{3}{4}$ cup of milk, $\frac{1}{4}$ cup of oil, and $\frac{2}{4}$ cup of water. How much liquid was needed to make the cake?

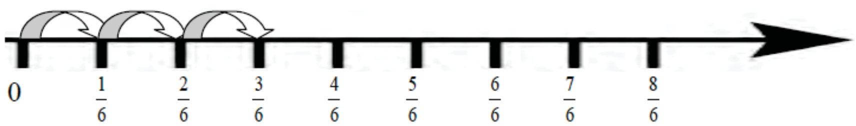


MCC.4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

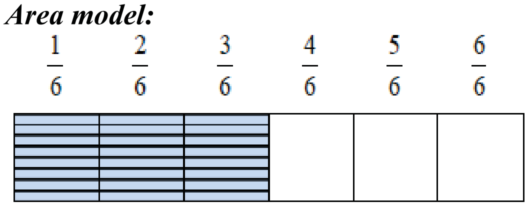
a. Understand a fraction *a*/*b* as a multiple of 1/*b*. ***For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).***

This standard builds on students' work of adding fractions and extending that work into multiplication.
Example: $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3 \times \frac{1}{6}$

Number line:

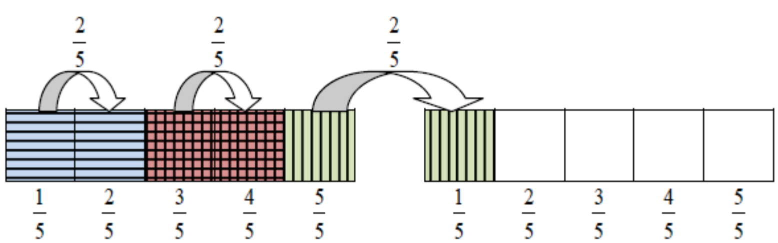


Area model:



b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. ***For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)***

This standard extended the idea of multiplication as repeated addition. For example, $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$.
Students are expected to use and create visual fraction models to multiply a whole number by a fraction.

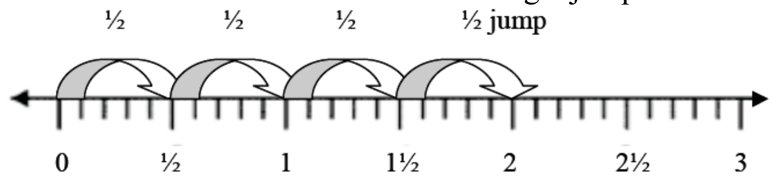


c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ***For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party,***

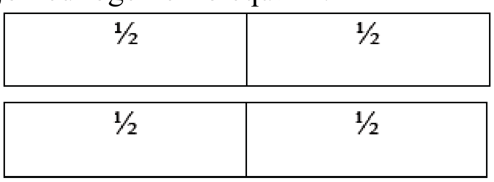
how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.
Example:
In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

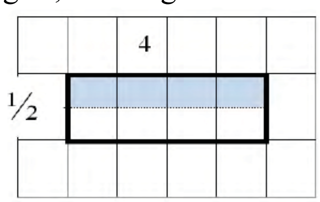
Student 1 – Draws a number line showing 4 jumps of ½:



Student 2 – Draws an area model showing 4 pieces of ½ joined together to equal 2:



Student 3 – Draws an area model representing 4 × ½ on a grid, dividing one row into ½ to represent the multiplier:

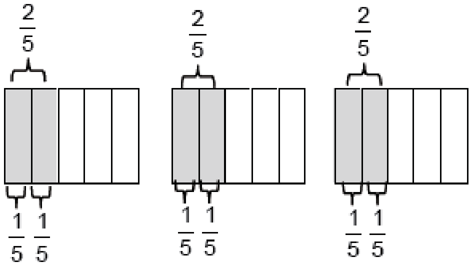


Example:
Heather bought 12 plums and ate $\frac{1}{3}$ of them. Paul bought 12 plums and ate $\frac{1}{4}$ of them. Which statement is true? Draw a model to explain your reasoning.

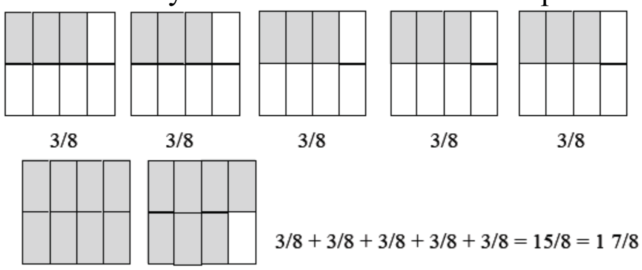
a. Heather and Paul ate the same number of plums.
b. Heather ate 4 plums and Paul ate 3 plums.
c. Heather ate 3 plums and Paul ate 4 plums.
d. Heather had 9 plums remaining.

Examples:
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

1. $3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$



2. If each person at a party eats $\frac{3}{8}$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie? A student may build a fraction model to represent this problem:



Common Misconceptions

Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

CLUSTER #4: UNDERSTAND DECIMAL NOTATION FOR FRACTIONS, AND COMPARE DECIMAL FRACTIONS.

*Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: fraction, numerator, denominator, equivalent, reasoning, decimals, tenths, hundreds, multiplication, comparisons/compare, ‹, ›, =.***

MCC.4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[1] ***For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.***

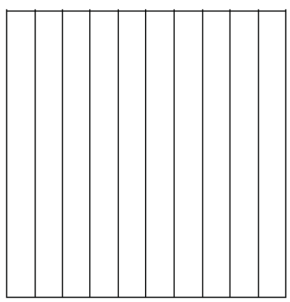
[1] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement for this grade.

This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (CCGPS.4.NF.6 and CCGPS.4.NF.7), experiences that allow students to shade decimal grids (10×10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

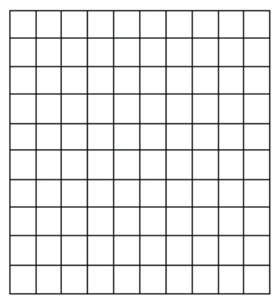
This work in 4th grade lays the foundation for performing operations with decimal numbers in 5th grade.

Ones	.	Tenths	Hundredths

Tenths Grid



Hundredths Grid

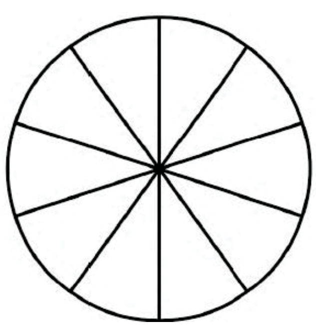


Example: **.3 = 3 tenths = 3/10** **.30 = 30 hundredths = 30/100**

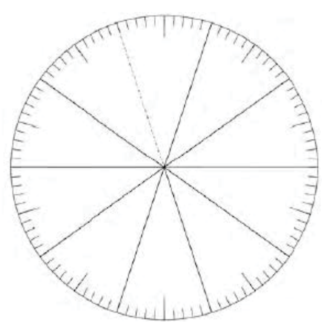
Example:

Represent 3 tenths and 30 hundredths on the models below.

Tenths circle



Hundredths circle



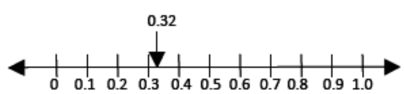
MCC.4.NF.6 Use decimal notation for fractions with denominators 10 or 100. ***For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.***

Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown on the following page.

Hundreds	Tens	Ones	•	Tenths	Hundredths
			•	3	2

Students use the representations explored in MCC.4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$. Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.



MCC.4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

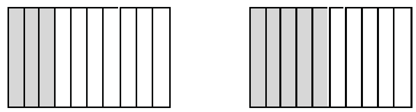
Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.

When the wholes are the same, the decimals or fractions can be compared.

Example:

Draw a model to show that 0.3 < 0.5. (*Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.*)



Common Misconceptions

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that).03 is greater than 0.3.

Measurement and Data

CLUSTER #1: SOLVE PROBLEMS INVOLVING MEASUREMENT AND CONVERSION OF MEASUREMENTS FROM A LARGER UNIT TO A SMALLER UNIT.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***measure, metric, customary, convert/conversion, relative size, liquid volume, mass, length, distance, kilometer (km), meter (m), centimeter (cm), kilogram (kg), gram (g), liter (L), milliliter (mL), inch (in), foot (ft), yard (yd), mile (mi), ounce (oz), pound (lb), cup (c), pint (pt), quart (qt), gallon (gal), time, hour, minute, second, equivalent, operations, add, subtract, multiply, divide, fractions, decimals, area, perimeter.***

MCC.4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. ***For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...***

The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create.
Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.

Example:
Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	$n \times 3$

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).

Understand the relationship between the size of a unit and the number of units needed (compensatory principle[2]).

MCC.4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

Example:
Charlie and 10 friends are planning for a pizza party. They purchased 3 quarts of milk. If each glass holds 8oz will everyone get at least one glass of milk?
Possible solution: Charlie plus 10 friends = 11 total people
11 people × 8 ounces (glass of milk) = 88 total ounces
1 quart = 2 pints = 4 cups = 32 ounces
Therefore 1 quart = 2 pints = 4 cups = 32 ounces
2 quarts = 4 pints = 8 cups = 64 ounces
3 quarts = 6 pints = 12 cups = 96 ounces

If Charlie purchased 3 quarts (6 pints) of milk there would be enough for everyone at his party to have at least one glass of milk. If each person drank 1 glass then he would have 1- 8 oz glass or 1 cup of milk left over.

Additional examples with various operations:

[4] The compensatory principle states that the smaller the unit used to measure the distance, the more of those units that will be needed. For example, measuring a distance in centimeters will result in a larger number of that unit than measuring the distance in meters.

- Division/fractions: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get?

Students may record their solutions using fractions or inches. (The answer would be 2/3 of a foot or 8 inches. Students are able to express the answer in inches because they understand that 1/3 of a foot is 4 inches and 2/3 of a foot is 2 groups of 1/3.)

- Addition: Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?
- Subtraction: A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?
- Multiplication: Mario and his 2 brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a volume measure on the side of a container.

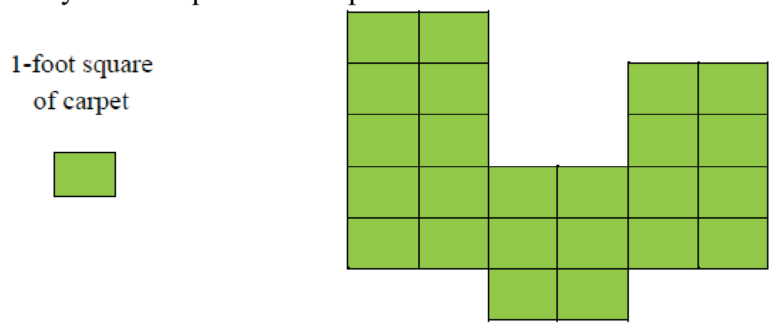
Example:
At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.

MCC.4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. ***For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.***

Students developed understanding of area and perimeter in 3rd grade by using visual models.

While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is I x w and the answer will always be in square units. The formula for perimeter can be 2 l + 2 w or 2 (l + w) and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:
Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?



Common Misconceptions

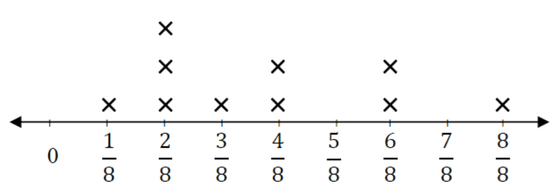
Students believe that larger units will give the larger measure. Students should be given multiple opportunities to measure the same object with different measuring units. For example, have the students measure the length of a room with one-inch tiles, with one-foot rulers, and with yard sticks. Students should notice that it takes fewer yard sticks to measure the room than rulers or tiles.

CLUSTER #2: REPRESENT AND INTERPRET DATA.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***data, line plot, length, fractions.***

MCC.4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. ***For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.***

This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.
Example:
Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 inch. They displayed their data collected on a line plot. How many objects measured 1/4 inch? 1/2 inch? If you put all the objects together end to end what would be the total length of **all** the objects.



Common Misconceptions

Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

CLUSTER #3: GEOMETRIC MEASUREMENT – UNDERSTAND CONCEPTS OF ANGLE AND MEASURE ANGLES.

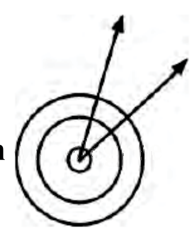
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***measure, point, end point, geometric shapes, ray, angle, circle, fraction, intersect, one-degree angle, protractor, decomposed, addition, subtraction, unknown.***

MCC.4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

This standard brings up a connection between angles and circular measurement (360 degrees).

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.



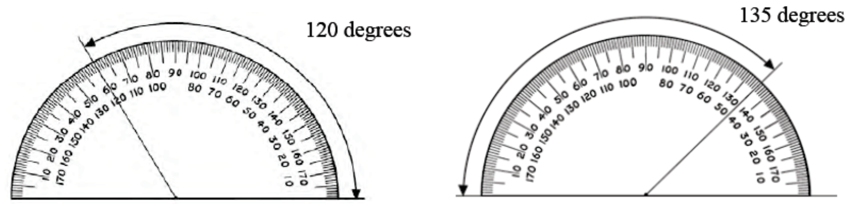
b. An angle that turns through n ngles is said to have an angle measure of n degrees.

This standard calls for students to explore an angle as a series of "one-degree turns." A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?

MCC.4.MD.6 Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.

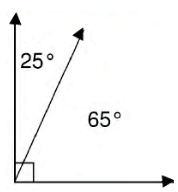
Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

Students should measure angles and sketch angles.



MCC.4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.



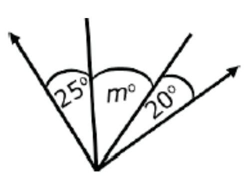
Example:

A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses, how many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of *m*?



Example:

Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

Common Misconceptions

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

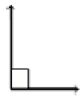
Geometry

CLUSTER #1: DRAW AND IDENTIFY LINES AND ANGLES, AND CLASSIFY SHAPES BY PROPERTIES OF THEIR LINES AND ANGLES.

*Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are**: classify shapes/figures, (properties)-rules about how numbers work, point, line, line segment, ray, angle, vertex/vertices, right angle, acute, obtuse, perpendicular, parallel, right triangle, isosceles triangle, equilateral triangle, scalene triangle, line of symmetry, symmetric figures, two dimensional. From previous grades: polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere.***

MCC.4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily.
Students do not easily identify lines and rays because they are more abstract.

right angle 

acute angle 

obtuse angle 

straight angle 

segment

line 

ray 

parallel lines 

perpendicular lines 

Example:
Draw two different types of quadrilaterals that have two pairs of parallel sides?
Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.

Examples:
How many acute, obtuse and right angles are in this shape?

Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

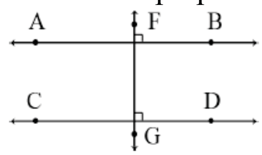
MCC.4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

Parallel or Perpendicular Lines:
Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they

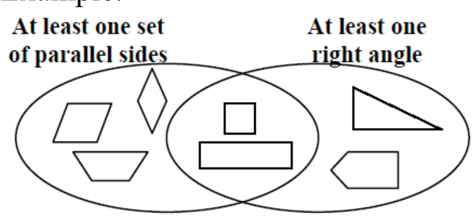
intersect in right angles (90º). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

Parallel and perpendicular lines are shown below:



This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.

Example:



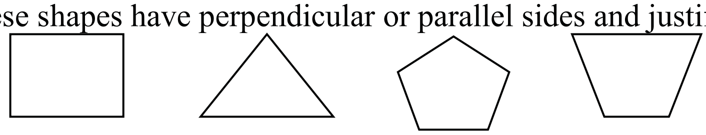
Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counterexample.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. *(impossible)*
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram.

Example:
Identify which of these shapes have perpendicular or parallel sides and justify your selection.

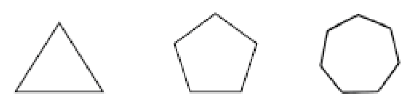


A possible justification that students might give is: “The square has perpendicular lines because the sides meet at a corner, forming right angles.”

Angle Measurement:
This expectation is closely connected to CCGPS.4.MD.5, CCGPS.4.MD.6, and CCGPS.4.G.1. Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.
Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

MCC.4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry.
This standard only includes line symmetry, not rotational symmetry.
Example:
For each figure at the right, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions.



Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.

<u>Common Misconceptions</u>
Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

"When classrooms are workshops-when learners are inquiring, investigating, and constructing- there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

Opening: Set the stage
Get students mentally ready to work on the task
Clarify expectations for products/behavior
How?

- Begin with a simpler version of the task to be presented
- Solve problem strings related to the mathematical idea/s being investigated
- Leap headlong into the task and begin by brainstorming strategies for approaching the task
- Estimate the size of the solution and reason about the estimate

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

Work session: Give 'em a chance
Students- grapple with the mathematics through sense-making, discussion, concretizing their mathematical ideas and the situation, record thinking in journals
Teacher- Let go. Listen. Respect student thinking. Encourage testing of ideas. Ask questions to clarify or provoke thinking. Provide gentle hints. Observe and assess.

Closing: Best Learning Happens Here
Students- share answers, justify thinking, clarify understanding, explain thinking, question each other
Teacher- Listen attentively to all ideas, ask for explanations, offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"
Anchor charts
Read Van de Walle 3-5, Chapter 1

BREAKDOWN OF A TASK (UNPACKING TASKS)

How do I go about tackling a task or a unit?

1. Read **the unit** in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units/tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school-wide. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

2. Read **the first task** your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

3. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks/centers/math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=
   K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html
   (this is a for-profit site with several free resources)
   Winnepeg resources- http://www.wsd1.org/iwb/math.htm
   Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

4. Points to remember:
   - Each task begins with a list of the standards specifically addressed in that task, however, that does not mean that these are the only standards addressed in the task. Remember, standards build on one another, and mathematical ideas are connected.
   - Tasks are made to be modified to match your learner's needs. If the names need changing, change them. If the materials are not available, use what is available. If a task doesn't go where the students need to go, modify the task or use a different resource.
   - The units are not intended to be all encompassing. Each teacher and team will make the units their own, and add to them to meet the needs of the learners.

ROUTINES AND RITUALS

Teaching Math in Context and Through Problems

"By the time they begin school; most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practices (SMP) should be at the forefront of every mathematics lessons and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMPs is to construct lessons built on context or through story problems. It is important for you to understand the difference between story problems and context problems. "Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around many problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult however; there are excellent reasons for making the effort.

- Problem solving focuses students' attention on ideas and sense making
- Problem solving develops the belief in students that they are capable of doing the mathematics and that mathematics makes sense
- Problem solving provides on going assessment data
- Problem solving is an excellent method for attending to a breadth of abilities
- Problem solving engages students so that there are few discipline problems
- Problem solving develops "mathematical power"
  (Van de Walle 3-5 pg. 15 and 16)

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are, which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 11 and 12

Use of Manipulatives

Used correctly manipulatives can be a positive factor in children's learning. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 6

When a new model of new use of a familiar model is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use.

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty. (Van de Walle and Lovin, Teaching Student-Centered Mathematics3-5 pg. 9

Use of Strategies and Effective Questioning

Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose;
- asking too many closed questions;
- asking several questions all at once;
- poor sequencing of questions;
- asking rhetorical questions;
- asking 'Guess what is in my head' questions;
- focusing on just a small number of learners;

- ignoring incorrect answers;
- not taking answers seriously.

In contrast, the research shows that effective questioning has the following characteristics:

- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A 'no hands' approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient 'wait time' between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

Mathematize the World through Daily Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance, doing a lunch count, determining how many items are needed for snack, lining up in a variety of ways (by height, age, type of shoe, hair color, eye color, etc.), and daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, and have productive discourse about the mathematics in which students are engaged. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of the routines are important to the development of students' number sense, flexibility, and fluency, which will support students' performances on the tasks in this unit.

Number Talks

Though the current understanding of mathematics may have been appropriate years ago, it is no longer sufficient to succeed in today's society. "Our students must have the ability to reason about quantitative information, possess number sense, and check for the reasonableness of solutions and answers (Parrish, 2010 – Number Talks*: Helping Children Build Mental Math and Computation Strategies K-5,* p. 4-5)." Students need to be encouraged and given plenty of opportunities to mentally compute and explain their strategy.

For example, if you are focusing on friendly numbers, you may include a computation problem such as 50-28. Students may look in a number of ways and given the opportunity to share.

Student 1 Strategy:	Student 2 Strategy:	Student 3 Strategy:	Student 4 Strategy:
I see that 28 is two away from 30. Then, I can just add 20 more to get to 50, so my answer is 22.	I pretended that 28 was 25 and I know that 25 + 25 = 50. But if I added 28 + 25 that would be 53 so took 3 away from 25 and that equals 22.	I jumped back 2 from 50 to 48 and jumped back another 20 to 28 to find the difference. I know that 2 and 20 more is 22.	I know that 28 + 30 is 58 and that is too much so I know I need to remove 8 from 30 and that is 22.

When providing a solution, students should always be required to justify, even if it is not correct. Designating as little as 10-15 minutes a day for mental computation and talking about numbers will help students look and think about numbers flexibly.

In a classroom number talk, students begin to share the authority of determining whether answers are accurate, and are expected to think through all solutions and strategies carefully (Parrish, 2010). During the number talk, the teacher is not the definitive authority. The teacher maintains the role of the facilitator, and is listening and learning for and from the students' natural mathematical thinking. The discussions should maintain a focus, assist students in learning appropriate ways to structure comments and misunderstandings, and the conversation should flow in a meaningful and natural way (Parrish, 2010).

Workstations and Learning Centers

When thinking about developing work stations and learning centers you want to base them on student readiness, interest, or learning profile such as learning style or multiple intelligence. This will allow different students to work on different tasks. Students should be able to complete the tasks within the stations or centers independently, with a partner or in a group.

It is important for students to be engaged in purposeful activities within the stations and centers. Therefore, you must careful consider the activities selected to be a part of the stations and centers. When selecting an activity, you may want to consider the following questions:

- Will the activity reinforce or extend a concept that's already been introduced?
- Are the directions clear and easy to follow?
- Are materials easy to locate and accessible?
- Can students complete this activity independently or with minimal help from the teacher?
- How will students keep a record of what they've completed?
- How will students be held accountable for their work?

(Laura Candler, *Teaching Resources*)

When implementing work stations and learning centers within your classroom, it is important to consider when the stations and centers will be used. Will you assign students to specific stations or centers to complete each week or will they be able to select a station or center of their choice? Will this opportunity be presented to all students during particular times of your math block or to students who finish their work early?

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned.

Games

"A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience.

Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 28

Journaling

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.

When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student's thinking is made visible!

GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem? assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use?

Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

Representing

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK cont'd…

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others."

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students. Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from "doing it myself!"

Problem Solving Rubric (3-5)

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, and selected and applied an appropriate problem solving strategy that lead to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, and selected an applied and appropriate problem solving strategy that lead to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem solving strategy that lead to a thorough and accurate solution. In addition, student will check answer using another method.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words. Student is able to make connections between models and equations.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words. The student connects quantities to written symbols and create a logical representation with precision.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected an appropriate tools or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition this students was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

SUGGESTED LITERATURE

One Hundred Hungry Ants, Elinor J Pinczes
Clean Sweep Campers, Lucille Recht Penner
Two Ways to Count to Ten, Ruby Dee and Susan Meddaugh
A Million Fish More or Less, Fred McKissack
Jump Kangaroo Jump, Stuart J. Murphy and Kevin O'Malley
Fraction Action, Loreen Leedy
Hershey Fraction Book, Jerry Pallotta
If You Hopped Like a Frog, David M. Schwartz
When a line bends a shape begins, Rhonda Gowler Greene
Grandfather Tang, Ann Tompert
Greedy Triangle, Marilyn Burns
Sir Cumference and the Knights of Angleland, Cindy Neuschwander
Sam Johnson and the Blue Ribbon Quilt, Lisa Campbell Ernst
House for Birdie, Stuart J. Murphy
Pastry School in Paris, Cindy Neuschwander
Hamster Champs, Stuart J. Murphy
Racing Around, Stuart J. Murphy
Bigger, Better, Best, Stuart J. Murphy

TECHNOLOGY LINKS

Unit 1

- http:// www.internet4classrooms.com Additional standard base activities and interactive games.
- http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes A brief definition and an animation of a Sieve being completed
- http://highered.mcgraw-hill.com/sites/dl/free/0072533072/78543/CentimeterGrid.pdf Printable centimeter grid paper
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=12
- http://lwf.ncdc.noaa.gov/oa/climate/online/ccd/nrmlprcp.html Monthly Precipitation table (in inches) provides data on several cities in the states of the United States
- http://money.cnn.com/magazines/moneymag/bplive/2007/ Top ten places to live in the United States according to Money Magazine
- http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html An electronic sieve that will either eliminate multiples or shade them in different colors.
- http://nlvm.usu.edu/en/nav/frames_asid_202_g_3_t_2.html
- http://s22318.tsbvi.edu/mathproject/ch5-sec5.asp Focuses on teaching mental math strategies to students who are visually impaired. Contains a variety of ideas all teachers can use to assist their students in developing sound number sense and an ability to think more deeply about multiplication and division.\

- http://s22318.tsbvi.edu/mathproject/ch5-sec5.asp Focuses on teaching mental math strategies to students who are visually impaired. Contains a variety of ideas all teachers can use to assist their students in developing sound number sense and an ability to think more deeply about multiplication and division.\
- http://ulm.edu/~esmith/nctmregional/blocks.htm Short videos demonstrating multiplication using different strategies – useful for teacher/parent background knowledge
- http://www.aaamath.com/ Students compare large numbers and determine their placement in a series of numbers.
- http://www.allcountries.org/uscensus/411_normal_monthly_and_annual_precipitation_selected.html Precipitation rates for cities in the United States
- http://www.arcytech.org/java/b10blocks/b10blocks.html Interactive base-ten blocks
- http://www.arcytech.org/java/b10blocks/b10blocks.html Virtual base 10 blocks for student use
- http://www.census.gov/schools/facts/ Student friendly site provides statistics about the states in the United States
- http://www.cut-the-knot.org/Curriculum/Arithmetic/Eratosthenes.shtml Another sieve, this one shows the numbers 1-100 and crosses out the multiples (contains advertising).
- http://www.ers.usda.gov/statefacts/ State facts regarding population, income, education, employment, farming, and exports provided
- http://www.etacuisenaire.com/pdf/gridpaper.pdf Printable centimeter grid paper
- http://www.ezschool.com/Games/Order.html Interactive opportunities to compare large numbers and determine the value of given digits in various-sized numbers
- http://www.faust.fr.bw.schule.de/mhb/eratosiv.htm An electronic sieve for the numbers 1-400. You can select any number and its multiples will be eliminated.
- http://www.gameclassroom.com Additional standard base activities and interactive games.
- http://www.internet4kids.com Additional standard base activities and interactive games
- http://www.justriddlesandmore.com/math2.html - Math riddles, patterns, multiplication problems, problem solving, and more (contains advertising)
- http://www.k-5mathteachingresources.com/4th-grade-number-activities.html Additional activities and read aloud activities.
- http://www.learner.org/courses/learningmath/number/session4/part_b/index.html Detailed information and activities for teachers regarding multiplication and division using base ten blocks.
- http://www.mathcats.com/grownupcats/ideabankmultiplication.html Additional ideas for approaching multiplication instruction
- http://www.mathwire.com/standards/standards.html Additional standard base activities
- Click on 7 "Multiplying by 2-Digit Numbers" then click on 7.3 "Multiplication: Arrays and an Expanded Algorithm" to find an example of multiplying using an array.

- http://www.phttp://www.pblunit10.com/envis4math/envisionmath.html blunit10.com /envis4math/envisionmath.html Click on 7 "Multiplying by 2-Digit Numbers" then click on 7.3 "Multiplication: Arrays and an Expanded Algorithm" to find an example of multiplying using an array.
- http://www.sheppardsoftware.com/mathgames/numbers/fruit_shoot_prime.htm
- http://www.statemaster.com/index.php Site provides a drop-down list of features to research regarding states in the United States, advertising is used.
- http://www.stfx.ca/special/mathproblems/grade5.html Problem solving activities and math riddles
- http://www2.edc.org/mathpartners/pdfs/3-5%20Patterns%20and%20Functions.pdf A variety of lessons for helping students to understand different types of patterns
- This website offers additional strategies and explanations for how to teach students both the conceptual information behind division as well as mental math strategies: http://www.nzmaths.co.nz/Number/Operating%20Units/MultDivStrategies.aspx

Unit 2

- For Fraction Kit Template - http://www.eastsideliteracy.org/tutorsupport/documents/HO_Fractions.pdf
- http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes A brief definition and an animation of a Sieve being completed
- http://gingerbooth.com/flash/patblocks/patblocks.php. Students can manipulate pattern blocks online and easily print and then label their work from this web site.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=12
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=18 This applet allows students to individually practice working with relationships among fractions and ways of combining fractions.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=18 This applet allows students to individually practice working with relationships among fractions and ways of combining fractions.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=80 Creating Equivalent Fractions by Shading Squares and Circles.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=80 Students are able to use a square or circle model to show equivalent fractions.
- http://nlvm.usu.edu/en/NAV/frames_asid_158_g_3_t_1.html An electronic sieve that will either eliminate multiples or shade them in different colors.
- http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html?open=activities&from=category_g_2_t_3.html Students can manipulate virtual pattern blocks easily on this site and record their work on the student sheet.
- http://visualfractions.com/CompareC/compareC.html Comparing fractions using circles.
- http://visualfractions.com/CompareL/comparel.html Compare fractions on a number line.

- http://www.cut-the-knot.org/Curriculum/Arithmetic/Eratosthenes.shtml Another sieve, this one shows the numbers 1-100 and crosses out the multiples (contains advertising).
- http://www.faust.fr.bw.schule.de/mhb/eratosiv.htm An electronic sieve for the numbers 1-400. You can select any number and its multiples will be eliminated.
- http://www.learningplanet.com/sam/ff/index.asp Students practice identifying equivalent fractions.
- http://www.sheppardsoftware.com/mathgames/fractions/memory_equivalent1.htm Students play a matching game of equivalent fractions with picture representations provided. Levels 1 and 2 are appropriate for fourth grade.
- http://www.uen.org/Lessonplan/preview.cgi?LPid=21526 Click on "Pattern Block Equivalent Fractions" under "Materials." Pattern Block problems that could be used as an introduction to this task or as extension for this task. This activity provides interactive practice enabling students to work more with factoring. They can work alone or with a partner.

Unit 3

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=11 Students can manipulate fraction sets easily on this site and record their work on the student sheet.
- http://jeopardylabs.com/ - This site allows you to create a Jeopardy game on a web-based template.
- http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=grade_g_2.html Students can manipulate fraction tiles easily on this site and record their work on the student sheet.
- http://www.jmu.edu/madison/teacher/jeopardy/Create_Jeopardy.pdf - Directions for creating a Jeopardy game using PowerPoint.
- http://www.learningwave.com/lwonline/fractions/add_sub_frac.html Gives a problem solving situation that requires adding fractions with like denominators and elapsed.
- http://www.primarygames.com/fractions/question1.htm In this game, students are asked to identify remaining fraction of a pizza. Note this site contains advertising
- http://www.visualfractions.com/AddEasyCircle.html Provides students with practice adding mixed numbers with like denominators using circle models.
- http://www.youtube.com/watch?v=14-KD4J3yLo A short YouTube video of Dwight Phillips, the 2009 USA long jump champion. He jumped 28' $1\frac{1}{2}$".
- http://www.visualfractions.com/AddEasyCircle.html Provides students with practice adding mixed numbers with like denominators using circle models.

Unit 4

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=11 can use the "set" application to produce similar models to those in this task.
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=205 Students can manipulate fraction tiles easily on this site and record their work on the student sheet.
- http://nlvm.usu.edu/en/nav/frames_asid_203_g_2_t_1.html?from=grade_g_2.html Students can manipulate fraction tiles easily on this site and record their work on the student sheet.
- http://www.visualfractions.com/Identify_sets.html This site would be best utilized with a lot of guidance from the teacher. The website asks students to determine what fraction of the set are square cookies or round cookies, something students should easily be able to accomplish. The teacher would then have to guide the students into making a multiplication sentence. For instance, the example below asks how many cookies are round. The answer is ⅙. The teacher should then ask what is the whole (6) and then ask what type of multiplication problem could you create from this scenario (i.e. 6 x ⅙ = 1)

Unit 5

- http://nces.ed.gov/nceskids/createagraph/default.aspx Allows students to create several types of graphs.
- http://nlvm.usu.edu/en/nav/frames_asid_334_g_2_t_1.html?from=category_g_2_t_1.html This is a interactive number line that students can "zoom" in on to show smaller and smaller units. Make sure you chose the correct place value on the bottom left, option range from billions down to decimals.
- http://www.decimalsquares.com/dsGames/games/beatclock.html Students get to race against time or a partner writing the correct decimal for a given model. (One whole is represented by a large square divided into 100 small squares.) For fourth grade, choose "beginner" when playing this game.
- http://www.decimalsquares.com/dsGames/games/concentration.html Students play a concentration game matching decimal numbers in the tenths (in red) with decimal numbers in the hundredths (in red).
- http://www.factmonster.com/ipka/A0772159.html Provides images of flags around the world.
- http://www.mathsonline.co.uk/freesite_tour/resource/whiteboard/decimals/dec_notes.html An interactive number line that can be divided into tenths and hundredths. Can be used for large group or small group work or students can print the number line with a certain point identified.
- http://www.widro.com/throwpaper.html - Students can play the waste basket basketball virtually. This is very challenging because a virtual fan is blowing. Note – web site contains advertisements.

Unit 6

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=34 Students can continue exploring sort shapes by characteristics using this online activity from NCTM Illuminations website.
- http://illuminations.nctm.org/LessonDetail.aspx?id=L720 The introduction to symmetry using the NCTM Illuminations web sites may be done as a whole group with a projector or in a computer lab individually or in pairs. If the students work on the tasks as individuals or in pairs, prepare a list of questions for them to answer while exploring the web site. At the end of this session, whether the activity is done as a whole group, individually, or in pairs, students should report to the whole class what they have learned or have found interesting about symmetry. The purpose of these activities is to provoke class discussion. Students should also be challenged to look for symmetries in other places during the next week and a half and report these findings to the class as well.
- http://mathforum.org/geometry/rugs/symmetry/ Be sure to check out the rug gallery to see different rug patterns and their symmetries!
- http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities&from=category_g_2_t_3.htmls Students can use a virtual geoboard rather than actual geoboards to create and compare their triangle
- http://www.basic-mathematics.com/types-of-triangles.html Gives basic definitions for the types of triangles. Note: this web site contains advertising.

RESOURCES CONSULTED

Content:
Ohio DOE
http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704
Arizona DOE
http://www.azed.gov/standards-practices/mathematics-standards/
Nzmaths
http://nzmaths.co.nz/

Teacher/Student Sense-making:
http://www.youtube.com/user/mitcccnyorg?feature=watch
http://www.insidemathematics.org/index.php/video-tours-of-inside-mathematics/classroom-teachers/157-teachers-reflect-mathematics-teaching-practices
https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
or http://secc.sedl.org/common_core_videos/

Journaling:
http://www.mathsolutions.com/index.cfm?page=wp10&crid=3

Community of Learners:
http://www.edutopia.org/math-social-activity-cooperative-learning-video
http://www.edutopia.org/math-social-activity-sel
http://www.youtube.com/user/responsiveclassroom/videos
http://www.responsiveclassroom.org/category/category/first-weeks-school
http://www.stenhouse.com/shop/pc/viewprd.asp?idProduct=9282&r=n206w
Work stations
http://www.stenhouse.com/shop/pc/viewprd.asp?idProduct=9336
Number sense