CCGPS Frameworks Teacher Edition

Mathematics

Third Grade

Grade Level Overview

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"Making Education Work for All Georgians"

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Grade Level Overview

TABLE OF CONTENTS

Common Core Georgia Performance Standards
Elementary School Mathematics
Third Grade

Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Numbers and Operations in Base Ten	**Operations and Algebraic Thinking: the Relationship Between Multiplication and Division**	**Operations and Algebraic Thinking: Patterns in Addition and Multiplication**	**Geometry**	**Representing and Comparing Fractions**	**Measurement**	**Show What We Know**
MCC3.NBT.1 **MCC3.NBT.2** **MCC3.MD.3** **MCC3.MD.4**	**MCC3.OA.1** **MCC3.OA.2** **MCC3.OA.3** **MCC3.OA.4** **MCC3.OA.5** **MCC3.OA.6** **MCC3.OA.7** **MCC3.NBT.3** **MCC3.MD.3** **MCC3.MD.4**	**MCC3.OA.8** **MCC3.OA.9** **MCC3.MD.3** **MCC3.MD.4** **MCC3.MD.5** **MCC3.MD.6** **MCC3.MD.7**	**MCC3.G.1** **MCC3.G.2** **MCC3.MD.3** **MCC3.MD.4** **MCC3.MD.7** **MCC3.MD.8**	**MCC3.NF.1** **MCC3.NF.2** **MCC3.NF.3** **MCC3.MD.3** **MCC3.MD.4**	**MCC3.MD.1** **MCC3.MD.2** **MCC3.MD.3** **MCC3.MD.4**	**ALL**

STANDARDS FOR MATHEMATICAL PRACTICE

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction. The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Students are expected to:

1. Make sense of problems and persevere in solving them.
In third grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.

2. Reason abstractly and quantitatively.
Third graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities.

3. Construct viable arguments and critique the reasoning of others.
In third grade, students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.

4. Model with mathematics.
Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Third graders should evaluate their results in the context of the situation and reflect on whether the results make sense.

5. Use appropriate tools strategically.
Third graders consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles

6. Attend to precision.
As third graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle they record their answers in square units.

7. Look for and make use of structure.
In third grade, students look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to multiply and divide (commutative and distributive properties).

8. Look for and express regularity in repeated reasoning.
Students in third grade should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7 x 8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

******Mathematical Practices 1 and 6 should be evident in EVERY lesson******

CONTENT STANDARDS

OPERATIONS AND ALGEBRAIC THINKING (OA)

CLUSTER #1: REPRESENT AND SOLVE PROBLEMS INVOLVING MULTIPLICATION AND DIVISION.

Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. The terms students should learn to use with increasing precision with this cluster are: ***products, groups of, quotients, partitioned equally, multiplication**, **division, equal groups, arrays, equations**, **unknown.***

MCC.3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each.

For example, describe a context in which a total number of objects can be expressed as 5 × 7.

This standard interprets products of whole numbers. Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol '×' means "groups of" and problems such as 5 × 7 refer to 5 groups of 7.

Example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? *(5 groups of 3, 5 × 3 = 15)*

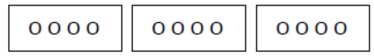
Describe another situation where there would be 5 groups of 3 or 5 × 3.

MCC.3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each.

For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

This standard focuses on two distinct models of division: partition models and measurement (repeated subtraction) models.

Partition models focus on the question, "How many in each group?" A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?



Measurement (repeated subtraction) models focus on the question, "How many groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?

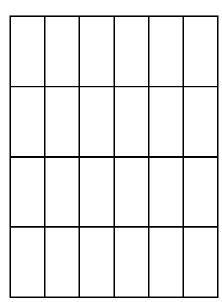
MCC.3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

This standard references various strategies that can be used to solve word problems involving multiplication and division. Students should apply their skills to solve word problems. Students should use a variety of representations for creating and solving one-step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many cookies does each person receive? *($4 \times 9 = 36$, $36 \div 6 = 6$).*

Table 2, located at the end of this document, gives examples of a variety of problem solving contexts, in which students need to find the product, the group size, or the number of groups. Students should be given ample experiences to explore all of the different problem structures.

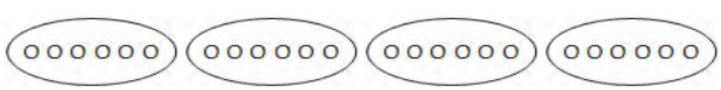
Examples of multiplication: There are 24 desks in the classroom. If the teacher puts 6 desks in each row, how many rows are there?

This task can be solved by drawing an array by putting 6 desks in each row. This is an array model:



This task can also be solved by drawing pictures of equal groups.

4 groups of 6 equals 24 objects

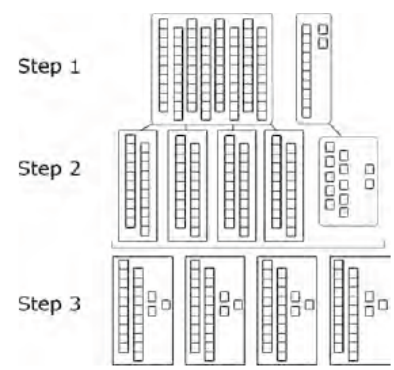


A student could also reason through the problem mentally or verbally, "I know 6 and 6 are 12. 12 and 12 are 24. Therefore, there are 4 groups of 6 giving a total of 24 desks in the classroom." A number line could also be used to show equal jumps. Third grade students should use a variety of pictures, such as stars, boxes, flowers to represent unknown numbers (variables).. Letters are also introduced to represent unknowns in third grade.

Examples of division: There are some students at recess. The teacher divides the class into 4 lines with 6 students in each line. Write a division equation for this story and determine how many students are in the class. *(□ ÷ 4 = 6. There are 24 students in the class).*

Determining the number of objects in each share (partitive division, where the size of the groups is unknown):

Example: The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



Determining the number of shares (measurement division, where the number of groups is unknown):

Example: Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max 4 bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	24 – 4 = 20	20 – 4 = 16	16 – 4 = 12	12 – 4 = 8	8 – 4 = 4	4 – 4 = 0

Solution: The bananas will last for 6 days.

MCC. 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?

This standard refers Table 2, included at the end of this document for your convenience, and equations for the different types of multiplication and division problem structures. The easiest problem structure includes Unknown Product (3 × 6 = ? or 18 ÷ 3 = 6). The more difficult problem structures include Group Size Unknown (3 × ? = 18 or 18 ÷ 3 = 6) or Number of Groups Unknown (? × 6 = 18, 18 ÷ 6 = 3). The focus of MCC.3.OA.4 goes beyond the traditional notion of *fact families*, by having students explore the inverse relationship of multiplication and division.

Students apply their understanding of the meaning of the equal sign as ”the same as” to interpret an equation with an unknown. When given 4 × ? = 40, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Example: Solve the equations below:

- 24 = ? × 6
- 72 ÷ ∆ = 9
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether?

3 × 4 = *m*

CLUSTER #2: MCC CLUSTER: UNDERSTAND PROPERTIES OF MULTIPLICATION AND THE RELATIONSHIP BETWEEN MULTIPLICATION AND DIVISION.

Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, strategies, (properties)-rules about how numbers work.

MCC.3.OA.5 Apply properties of operations as strategies to multiply and divide.

Examples:

If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.)
$3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.)
Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

This standard references properties (rules about how numbers work) of multiplication. While students DO NOT need to use the formal terms of these properties, students should understand that properties are rules about how numbers work, they need to be flexibly and fluently applying each of them. Students represent expressions using various objects, pictures, words and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication (but does make a difference in division). Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.

The associative property states that the sum or product stays the same when the grouping of addends or factors is changed. For example, when a student multiplies $7 \times 5 \times 2$, a student could rearrange the numbers to first multiply $5 \times 2 = 10$ and then multiply $10 \times 7 = 70$.

The commutative property (order property) states that the order of numbers does not matter when you are adding or multiplying numbers. For example, if a student knows that $5 \times 4 = 20$, then they also know that $4 \times 5 = 20$. The array below could be described as a 5×4 array for 5 columns and 4 rows, or a 4×5 array for 4 rows and 5 columns. There is no "fixed" way to write the dimensions of an array as rows × columns or columns × rows.

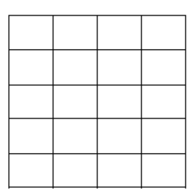
Students should have flexibility in being able to describe both dimensions of an array.

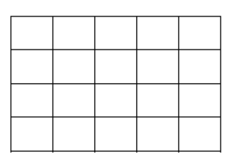
Example:

4×5

or

5×4





4×5

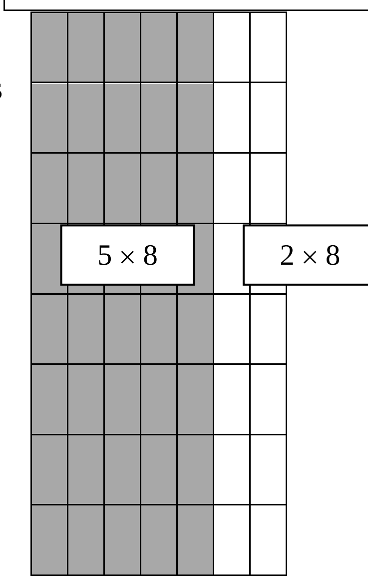
or

5×4

Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they don't know. Students would be using mental math to determine a product. Here are ways that students could use the distributive property to determine the product of 7×6. Again, students should use the distributive property, but can refer to this in informal language such as “breaking numbers apart”.

Student 1	**Student 2**	**Student 3**
7×6	7×6	7×6
$7 \times 5 = 35$	$7 \times 3 = 21$	$5 \times 6 = 30$
$7 \times 1 = 7$	$7 \times 3 = 21$	$2 \times 6 = 12$
$35 + 7 = 42$	$21 + 21 = 42$	$30 + 12 = 42$

Another example if the distributive property helps students determine the products and factors of problems by breaking numbers apart. For example, for the problem $7 \times 8 = ?$, students can decompose the 7 into a 5 and 2, and reach the answer by multiplying $5 \times 8 = 40$ and $2 \times 8 = 16$ and adding the two products ($40 + 16 = 56$).



To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- $0 \times 7 = 7 \times 0 = 0$ (Zero Property of Multiplication)
- $1 \times 9 = 9 \times 1 = 9$ (Multiplicative Identity Property of 1)
- $3 \times 6 = 6 \times 3$ (Commutative Property)
- $8 \div 2 = 2 \div 8$ (Students are only to determine that these are not equal)
- $2 \times 3 \times 5 = 6 \times 5$
- $10 \times 2 < 5 \times 2 \times 2$
- $2 \times 3 \times 5 = 10 \times 3$
- $0 \times 6 > 3 \times 0 \times 2$

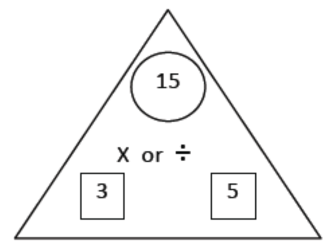
MCC.3.OA.6 Understand division as an unknown-factor problem.

For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

This standard refers to Table 2, included at the end of this document for your convenience, and the various problem structures. Since multiplication and division are inverse operations, students are expected to solve problems and explain their processes of solving division problems that can also be represented as unknown factor multiplication problems.

Example: A student knows that 2 x 9 = 18. How can they use that fact to determine the answer to the following question: 18 people are divided into pairs in P.E. class. How many pairs are there? Write a division equation and explain your reasoning.

Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient. Examples:



- 3 × 5 = 15 5 × 3 = 15
- 15 ÷ 3 = 5 15 ÷ 5 = 3

<u>MCC CLUSTER # 3</u>: MULTIPLY AND DIVIDE WITHIN 100.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, unknown, strategies, reasonableness, mental computation, property.

MCC.3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

This standard uses the word fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). “Know from memory” should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9 × 9).

By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
- Tens facts (relating to place value, 5 × 10 is 5 tens or 50)
- Five facts (half of tens)
- Skip counting (counting groups of __ and knowing how many groups have been counted)
- Square numbers (ex: 3 × 3)
- Nines (10 groups less one group, e.g., 9 × 3 is 10 groups of 3 minus one group of 3)
- Decomposing into known facts (6 × 7 is 6 x 6 plus one more group of 6)
- Turn-around facts (Commutative Property)
- Fact families (Ex: 6 × 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 × 6 = 24)
- Missing factors

General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

MCC CLUSTER #4: SOLVE PROBLEMS INVOLVING THE FOUR OPERATIONS, AND IDENTIFY AND EXPLAIN PATTERNS IN ARITHMETIC.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: operation, multiply, divide, factor, product, quotient, subtract, add, addend, sum, difference, equation, unknown, strategies, reasonableness, mental computation, estimation, rounding, patterns, (properties) – rules about how numbers work.

MCC.3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100.

This standard calls for students to represent problems using equations with a letter to represent unknown quantities.

Example:

> Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$).

This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving, and then revisit their estimate to check for reasonableness.

Example: Here are some typical estimation strategies for the problem:

> On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

MCC.3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. ***For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.***

This standard calls for students to examine arithmetic patterns involving both addition and multiplication. Arithmetic patterns are patterns that change by the same rate, such as adding the same number. For example, the series 2, 4, 6, 8, 10 is an arithmetic pattern that increases by 2 between each term.

This standard also mentions identifying patterns related to the properties of operations.

Examples:

- Even numbers are always divisible by 2. Even numbers can always be decomposed into 2 equal addends (14 = 7 + 7).

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

- Multiples of even numbers (2, 4, 6, and 8) are always even numbers.
- On a multiplication chart, the products in each row and column increase by the same amount (skip counting).
- On an addition chart, the sums in each row and column increase by the same amount.

What do you notice about the numbers highlighted in pink in the multiplication table? Explain a pattern using properties of operations. *When one changes the order of the factors (commutative property), they will still gets the same product; example 6 x 5 = 30 and 5 x 6 = 30.*

Teacher: What pattern do you notice when 2, 4, 6, 8, or 10 are multiplied by any number (even or odd)?

Student: The product will always be an even number.

Teacher: Why?

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

What patterns do you notice in this addition table? Explain why the pattern works this way?

Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically.

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	19	11	12	13	14	15	16	17	18	19	20

Example:

- Any sum of two even numbers is even.
- Any sum of two odd numbers is even.
- Any sum of an even number and an odd number is odd.
- The multiples of 4, 6, 8, and 10 are all even because they can all be decomposed into two equal groups.

- The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines.
- The multiples of any number fall on a horizontal and a vertical line due to the commutative property.
- All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

addend	addend	sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
□	□	□
□	□	□
□	□	□
20	0	20

NUMBERS AND OPERATIONS IN BASE TEN (NBT)

MCC CLUSTER: USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO PERFORM MULTI-DIGIT ARITHMETIC.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: place value, round, addition, add, addend, sum, subtraction, subtract, difference, strategies, (properties)-rules about how numbers work.

MCC.3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

MCC.3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

This standard refers to fluently, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using strategies such as the distributive property). The word algorithm refers to a procedure or a series of steps. There are other algorithms other than the standard algorithm. Third grade students should have experiences beyond the standard algorithm. A variety of algorithms will be assessed.

Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Students explain their thinking and show their work by using strategies and algorithms, and verify that their answer is reasonable.

Example: There are 178 fourth graders and 225 fifth graders on the playground. What is the total number of students on the playground?

Student 1

100 + 200 = 300

70 + 20 = 90

8 + 5 = 13

300+90+13 = 403 students

Student 2

I added 2 to 178 to get 180. I added 220 to get 400. I added the 3 left over to get 403.

Student 3

I know the 75 plus 25 equals 100. Then I added 1 hundred from 178 and 2 hundreds from 275. I had a total of 4 hundreds and I had 3 more left to add. So I have 4 hundreds plus 3 more which is 403.

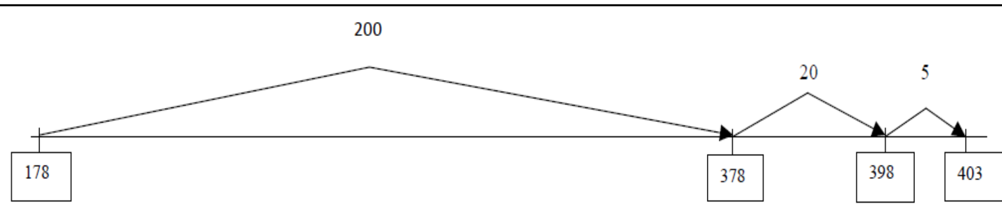
Student 4

178 + 225 = ?

178 + 200 = 378

378 + 20 = 398

398 + 5 = 403



MCC.3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

This standard extends students' work in multiplication by having them apply their understanding of place value.

This standard expects that students go beyond tricks that hinder understanding such as "just adding zeros" and explain and reason about their products. For example, for the problem 50 x 4, students should think of this as 4 groups of 5 tens or 20 tens. Twenty tens equals 200.

<u>Common Misconceptions</u>

The use of terms like "round up" and "round down" confuses many students. For example, the number 37 would round to 40 or they say it "rounds up". The digit in the tens place is changed from 3 to 4 (rounds up). This misconception is what causes the problem when applied to rounding down. The number 32 should be rounded (down) to 30, but using the logic mentioned for rounding up, some students may look at the digit in the tens place and take it to the previous number, resulting in the incorrect value of 20. To remedy this misconception, students need to use a number line to visualize the placement of the number and/or ask questions such as: "What tens are 32 between and which one is it closer to?" Developing the understanding of what the answer choices are before rounding can alleviate much of the misconception and confusion related to rounding.

NUMBER AND OPERATIONS IN FRACTIONS (NF)

MCC CLUSTER : DEVELOP UNDERSTANDING OF FRACTIONS AS NUMBERS.

Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

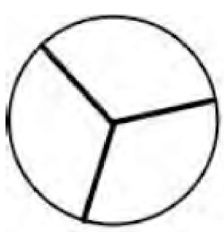
MCC.3.NF.1 Understand a fraction 1/*b* as the quantity formed by 1 part when *a* whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by a parts of size 1/*b*.

This standard refers to the sharing of a whole being partitioned or split. Fraction models in third grade include area (parts of a whole) models (circles, rectangles, squares) and number lines. Set models (parts of a group) are not introduced in Third Grade. In 3.NF.1 students should focus on the concept that a fraction is made up (composed) of many pieces of a unit fraction, which has a numerator of 1. For example, the fraction 3/5 is composed of 3 pieces that each have a size of 1/5.

Some important concepts related to developing understanding of fractions include:

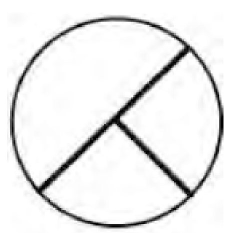
- Understand fractional parts must be equal-sized.

Example



These are thirds.

Non-Example



These are NOT thirds.

- The number of equal parts tells how many make a whole.
- As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.

- The size of the fractional part is relative to the whole.
- The number of children in one-half of a classroom is different than the number of children in one-half of a school. (The whole in each set is different; therefore, the half in each set will be different.)
- When a whole is cut into equal parts, the denominator represents the number of equal parts.
- The numerator of a fraction is the count of the number of equal parts.
  - ¾ means that there are 3 one-fourths.
  - Students can count *one fourth, two fourths, three fourths.*
    Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.

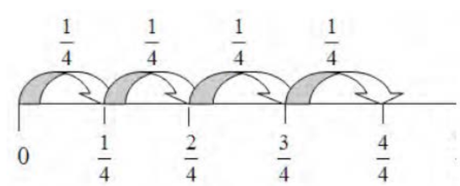
MCC.3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.

b. Represent a fraction *a*/*b* on a number line diagram by marking off a lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.

The number line diagram is the first time students work with a number line for numbers that are between whole numbers (e.g., that . is between 0 and 1).

In the number line diagram below, the space between 0 and 1 is divided (partitioned) into 4 equal regions. The distance from 0 to the first segment is 1 of the 4 segments from 0 to 1 or . (**MCC.3.NF.2a**). Similarly, the distance from 0 to the third segment is 3 segments that are each one-fourth long. Therefore, the distance of 3 segments from 0 is the fraction . (**MCC.3.NF.2b**).



MCC.3.NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, 1/8 is smaller than 1/2 because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

These standards call for students to use visual fraction models (area models) and number lines to explore the idea of equivalent fractions. Students should only explore equivalent fractions using models, rather than using algorithms or procedures.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. ***Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.***

This standard includes writing whole numbers as fractions. The concept relates to fractions as division problems, where the fraction 3/1 is 3 wholes divided into one group. This standard is the building block for later work where students divide a set of objects into a specific number of groups. Students must understand the meaning of a/1.

Example: If 6 brownies are shared between 2 people, how many brownies would each person get?

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

This standard involves comparing fractions with or without visual fraction models including number lines. Experiences should encourage students to reason about the size of pieces, the fact that 1/3 of a cake is larger than 1/4 of the same cake. Since the same cake (the whole) is split into equal pieces, thirds are larger than fourths.

In this standard, students should also reason that comparisons are only valid if the wholes are identical. For example, 1/2 of a large pizza is a different amount than 1/2 of a small pizza. Students should be given opportunities to discuss and reason about which 1/2 is larger.

<u>Common Misconceptions</u>

The idea that the smaller the denominator, the smaller the piece or part of the set, or the larger the denominator, the larger the piece or part of the set, is based on the comparison that in whole numbers, the smaller a number, the less it is, or the larger a number, the more it is. The use of

different models, such as fraction bars and number lines, allows students to compare unit fractions to reason about their sizes.
Students think all shapes can be divided the same way. Present shapes other than circles, squares or rectangles to prevent students from overgeneralizing that all shapes can be divided the same way. For example, have students fold a triangle into eighths. Provide oral directions for folding the triangle:
1. Fold the triangle into half by folding the left vertex (at the base of the triangle) over to meet the right vertex.
2. Fold in this manner two more times.
3. Have students label each eighth using fractional notation. Then, have students count the fractional parts in the triangle (one-eighth, two-eighths, three-eighths, and so on).

MEASUREMENT AND DATA (MD)

MCC CLUSTER #1: SOLVE PROBLEMS INVOLVING MEASUREMENT AND ESTIMATION OF INTERVALS OF TIME, LIQUID VOLUMES, AND MASSES OF OBJECTS.

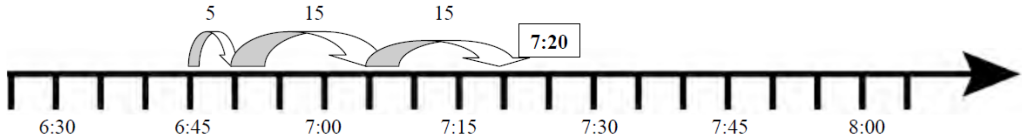
Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: estimate, time, time intervals, minute, hour, elapsed time, measure, liquid volume, mass, standard units, metric, gram (g), kilogram (kg), liter (L).

MCC. 3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

This standard calls for students to solve elapsed time, including word problems. Students could use clock models or number lines to solve. On the number line, students should be given the opportunities to determine the intervals and size of jumps on their number line. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).

Example:

Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school?



MCC.3.MD.2 Measure and estimate liquid volumes and masses of objects using standardunits of grams (g), kilograms (kg), and liters (l).[1] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[2]

This standard asks for students to reason about the units of mass and volume. Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter. Word problems should only be one-step and include the same units.

Example:

Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams so 10 boxes would weigh one kilogram.

Example:

A paper clip weighs about a) a gram, b) 10 grams, c) 100 grams?

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration). Understand the relationship between the size of a unit and the number of units needed (compensatory principle).

Common Misconceptions

Students may read the mark on a scale that is below a designated number on the scale as if it was the next number. For example, a mark that is one mark below 80 grams may be read as 81 grams. Students realize it is one away from 80, but do not think of it as 79 grams.

MCC CLUSTER #2: REPRESENT AND INTERPRET DATA.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with

[1] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[2] Excludes multiplicative comparison problems (problems involving notions of "times as much"). See Table 2 at the end of this document.

increasing precision with this cluster are: scale, scaled picture graph, scaled bar graph, line plot, data.

MCC.3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. ***For example, draw a bar graph in which each square in the bar graph might represent 5 pets.***

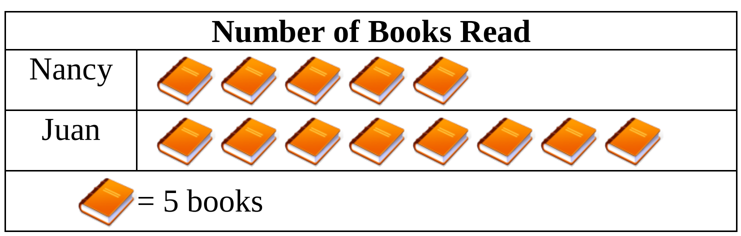
Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts. While exploring data concepts, students should **P**ose a question, **C**ollect data, **A**nalyze data, and **I**nterpret data (PCAI). Students should be graphing data that is relevant to their lives

Example:

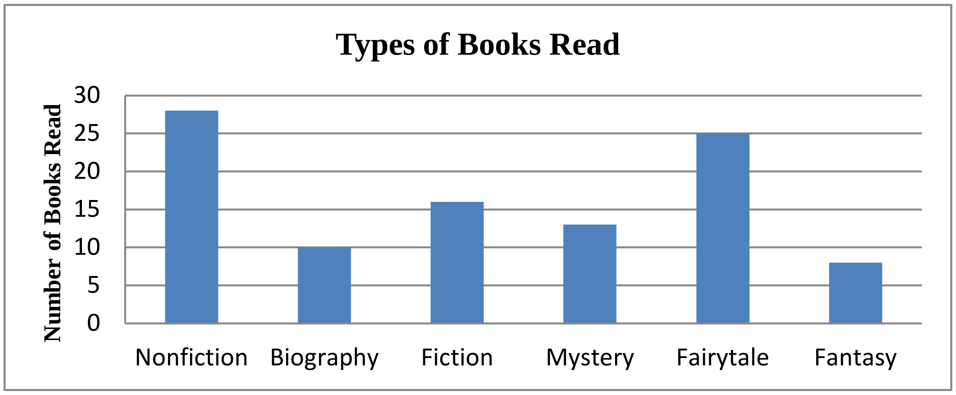
Pose a question: Student should come up with a question. What is the typical genre read in our class?

Collect and organize data: student survey

Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data. How many more books did Juan read than Nancy?



Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.

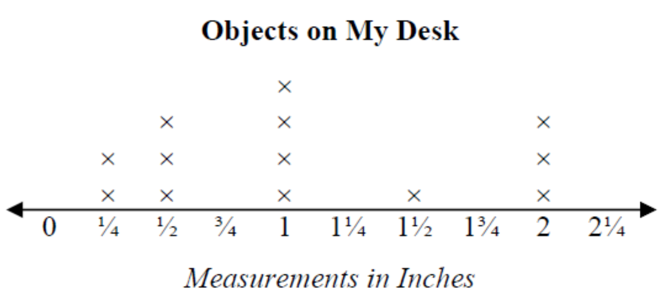


Analyze and Interpret data:

- How many more nonfiction books where read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library which would be the best genre? Why?

MCC.3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Students in second grade measured length in whole units using both metric and U.S. customary systems. It is important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch. Third graders need many opportunities measuring the length of various objects in their environment.This standard provides a context for students to work with fractions by measuring objects to a quarter of an inch.Example: Measure objects in your desk to the nearest ½ or ¼ of an inch, display data collected on a line plot. How many objects measured ¼? ½? etc. …



Common Misconceptions

Although intervals on a bar graph are not in single units, students count each square as one. To avoid this error, have students include tick marks between each interval. Students should begin each scale with 0. They should think of skip-counting when determining the value of a bar since the scale is not in single units.

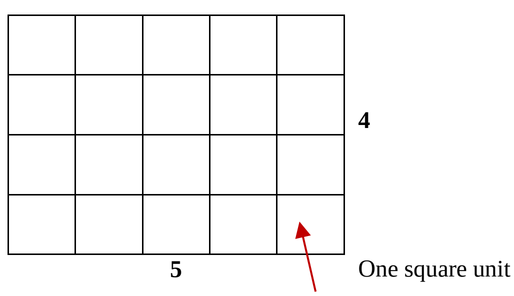
MCC CLUSTER #3: GEOMETRIC MEASUREMENT – UNDERSTAND CONCEPTS OF AREA AND RELATE AREA TO MULTIPLICATION AND TO ADDITION.

Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: attribute, area, square unit, plane figure, gap, overlap, square cm, square m , square in., square ft, nonstandard units, tiling, side length, decomposing.

MCC.3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. **A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.**
b. **A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.**

These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper.



MCC.3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches.

MCC.3.MD.7 Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Students should tile rectangles then multiply their side lengths to show it is the same.

To find the area, one could count the squares or multiply $3 \times 4 = 12$.

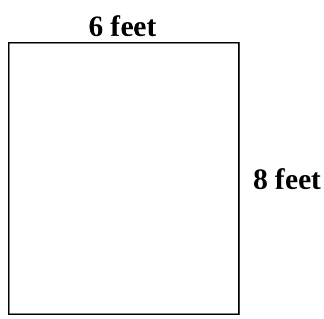
1	2	3	4
5	6	6	8
9	10	11	12

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Students should solve real world and mathematical problems

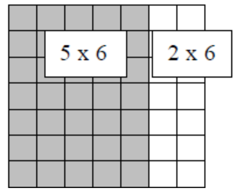
Example:

Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?

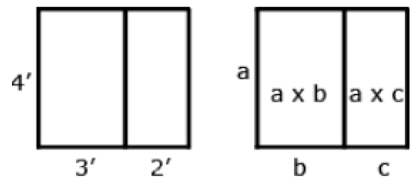


c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

This standard extends students' work with the distributive property. For example, in the picture below the area of a 7×6 figure can be determined by finding the area of a 5×6 and 2×6 and adding the two sums.



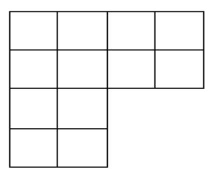
Example:



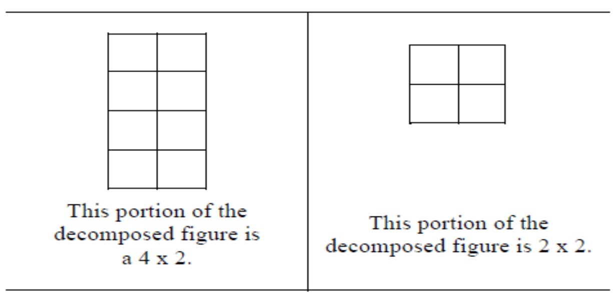
4 x 3 + 4 x 2 = 20
4 (3 + 2) = 20
4 x 5 = 20

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.



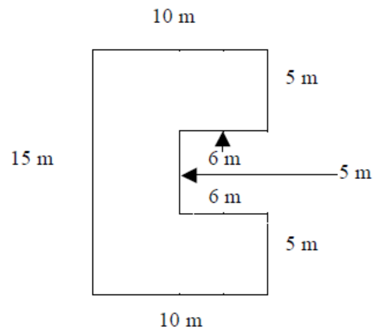
How could this figure be decomposed to help find the area?



4 x 2 = 8 and 2 x 2 = 4
So 8 + 4 = 12
Therefore the total area of this figure is 12 square units

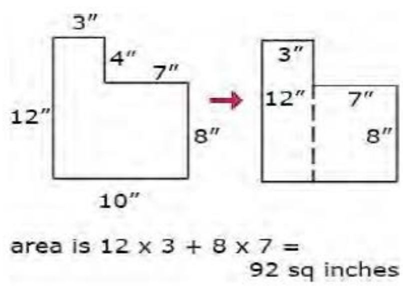
Example:

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



Example:

Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



<u>Common Misconceptions</u>

Students may confuse perimeter and area when they measure the sides of a rectangle and then multiply. They think the attribute they find is length, which is perimeter. Pose problems situations that require students to explain whether they are to find the perimeter or area.

<u>MCC CLUSTER #4</u>: GEOMETRIC MEASUREMENT – RECOGNIZE PERIMETER AS AN ATTRIBUTE OF PLANE FIGURES AND DISTINGUISH BETWEEN LINEAR AND AREA MEASURES.

Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: ***attribute, perimeter, plane figure, linear, area, polygon, side length.***

MCC.3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects; use addition to find perimeters; and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.

Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in	3 in.	4 in.	14 in.
12 sq. in	4 in.	3 in.	14 in.
12 sq. in	6 in.	2 in.	16 in.
12 sq. in	12 in.	1 in.	26 in.

Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g. find the rectangles that have an area of 12 square units.) They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

Common Misconceptions

Students think that when they are presented with a drawing of a rectangle with only two of the side lengths shown or a problem situation with only two of the side lengths provided, these are the only dimensions they should add to find the perimeter. Encourage students to include the appropriate dimensions on the other sides of the rectangle. With problem situations, encourage students to make a drawing to represent the situation in order to find the perimeter.

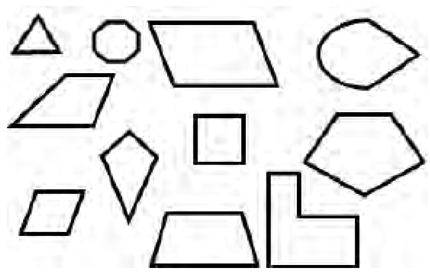
GEOMETRY (G)

MCC CLUSTER: REASON WITH SHAPES AND THEIR ATTRIBUTES.

Students describe, analyze, and compare properties of two dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole. Mathematically proficient students communicate precisely by engaging in discussion about their reasoning using appropriate mathematical language. The terms students should learn to use with increasing precision with this cluster are: attributes, *properties, quadrilateral, open figure, closed figure , three-sided, 2-dimensional, 3-dimensional, rhombi, rectangles, and squares are subcategories of quadrilaterals, cubes, cones, cylinders, and rectangular prisms are subcategories of 3-dimensional figures shapes: polygon, rhombus/rhombi, rectangle, square, partition, unit fraction. From previous grades: triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle, cone, cylinder, sphere.*

MCC.3.G.1 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

In second grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. Third graders build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombuses as quadrilaterals.



Students should classify shapes by attributes and drawing shapes that fit specific categories. For example, parallelograms include: squares, rectangles, rhombi, or other shapes that have two pairs of parallel sides. Also, the broad category quadrilaterals include all types of parallelograms, trapezoids and other four-sided figures.

Example:

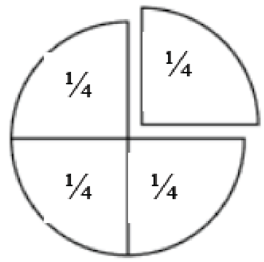
Draw a picture of a quadrilateral. Draw a picture of a rhombus. How are they alike? How are they different? Is a quadrilateral a rhombus? Is a rhombus a quadrilateral? Justify your thinking.

MCC.3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. ***For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.***

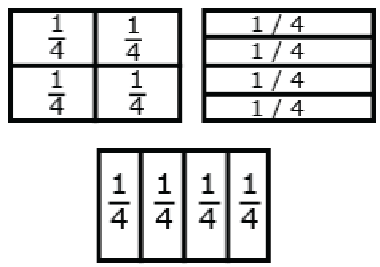
This standard builds on students' work with fractions and area. Students are responsible for partitioning shapes into halves, thirds, fourths, sixths and eighths.

Example:

This figure was partitioned/divided into four equal parts. Each part is ¼ of the total area of the figure.



Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.



Common Misconceptions

Students may identify a square as a "nonrectangle" or a "nonrhombus" based on limited images they see. They do not recognize that a square is a rectangle because it has all of the properties of a rectangle. They may list properties of each shape separately, but not see the interrelationships between the shapes. For example, students do not look at the properties of a square that are characteristic of other figures as well. Using straws to make four congruent figures have students change the angles to see the relationships between a rhombus and a square. As students develop definitions for these shapes, relationships between the properties will be understood.

Table 1

Common Addition and Subtraction Situations

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**[3]
Put together/ Take apart[4]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[5]	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[3] Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[4] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

[5] For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Table 2

Common Multiplication and Division Situations

The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

	Unknown Product	Group Size Unknown ("How many in each group? Division)	Number of Groups Unknown ("How many groups?" Division)
	3 × 6 = ?	**3 × ? – 18, and 18 ÷3 = ?**	**? × 6 = 18, and 18 ÷ 6 = ?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[6], Area[7]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

[6] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[7] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

"When classrooms are workshops-when learners are inquiring, investigating, and constructing- there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

Opening: Set the stage
Get students mentally ready to work on the task
Clarify expectations for products/behavior
How?

- Begin with a simpler version of the task to be presented
- Solve problem strings related to the mathematical idea/s being investigated
- Leap headlong into the task and begin by brainstorming strategies for approaching the task
- Estimate the size of the solution and reason about the estimate

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

Work session: Give 'em a chance
Students- grapple with the mathematics through sense-making, discussion, concretizing their mathematical ideas and the situation, record thinking in journals
Teacher- Let go. Listen. Respect student thinking. Encourage testing of ideas. Ask questions to clarify or provoke thinking. Provide gentle hints. Observe and assess.

Closing: Best Learning Happens Here
Students- share answers, justify thinking, clarify understanding, explain thinking, question each other
Teacher- Listen attentively to all ideas, ask for explanations, offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"
Anchor charts
Read Van de Walle K-3, Chapter 1

BREAKDOWN OF A TASK (UNPACKING TASKS)

How do I go about tackling a task or a unit?

1. Read the unit in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units/tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school-wide. The structure of the units/tasks is similar task to task and grade to grade. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

2. Read the first task your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

3. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks/centers/math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=
   K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html (this is a for-profit site with several free resources)
   Winnepeg resources- http://www.wsd1.org/iwb/math.htm
   Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

4. Points to remember:
   - Each task begins with a list of the standards specifically addressed in that task, however, that does not mean that these are the only standards addressed in the task. Remember, standards build on one another, and mathematical ideas are connected.
   - Tasks are made to be modified to match your learner's needs. If the names need changing, change them. If the materials are not available, use what is available. If a task doesn't go where the students need to go, modify the task or use a different resource.
   - The units are not intended to be all encompassing. Each teacher and team will make the units their own, and add to them to meet the needs of the learners.

ROUTINES AND RITUALS

Teaching Math in Context and Through Problems

"By the time they begin school; most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practices (SMP) should be at the forefront of every mathematics lessons and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMPs is to construct lessons built on context or through story problems. It is important for you to understand the difference between story problems and context problems. "Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around many problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult however; there are excellent reasons for making the effort.

- Problem solving focuses students' attention on ideas and sense making
- Problem solving develops the belief in students that they are capable of doing the mathematics and that mathematics makes sense
- Problem solving provides on going assessment data
- Problem solving is an excellent method for attending to a breadth of abilities
- Problem solving engages students so that there are few discipline problems
- Problem solving develops "mathematical power"
  (Van de Walle 3-5 pg. 15 and 16)

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are, which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 11 and 12

Use of Manipulatives

Used correctly manipulatives can be a positive factor in children's learning. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 6

When a new model of new use of a familiar model is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use.

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty. (Van de Walle and Lovin, Teaching Student-Centered Mathematics3-5 pg. 9

Use of Strategies and Effective Questioning

Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose;
- asking too many closed questions;
- asking several questions all at once;
- poor sequencing of questions;
- asking rhetorical questions;
- asking 'Guess what is in my head' questions;
- focusing on just a small number of learners;

- ignoring incorrect answers;
- not taking answers seriously.

In contrast, the research shows that effective questioning has the following characteristics:

- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A 'no hands' approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient 'wait time' between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

Mathematize the World through Daily Routines

The importance of continuing the established classroom routines cannot be overstated. Daily routines must include such obvious activities such as taking attendance, doing a lunch count, determining how many items are needed for snack, lining up in a variety of ways (by height, age, type of shoe, hair color, eye color, etc.), and daily questions. They should also include less obvious routines, such as how to select materials, how to use materials in a productive manner, how to put materials away, and have productive discourse about the mathematics in which students are engaged. An additional routine is to allow plenty of time for children to explore new materials before attempting any directed activity with these new materials. The regular use of the routines are important to the development of students' number sense, flexibility, and fluency, which will support students' performances on the tasks in this unit.

Number Talks

Though the current understanding of mathematics may have been appropriate years ago, it is no longer sufficient to succeed in today's society. "Our students must have the ability to reason about quantitative information, possess number sense, and check for the reasonableness of solutions and answers (Parrish, 2010 – Number Talks*: Helping Children Build Mental Math and Computation Strategies K-5,* p. 4-5)." Students need to be encouraged and given plenty of opportunities to mentally compute and explain their strategy.

For example, if you are focusing on friendly numbers, you may include a computation problem such as 50-28. Students may look in a number of ways and given the opportunity to share.

Student 1 Strategy:	Student 2 Strategy:	Student 3 Strategy:	Student 4 Strategy:
I see that 28 is two away from 30. Then, I can just add 20 more to get to 50, so my answer is 22.	I pretended that 28 was 25 and I know that 25 + 25 = 50. But if I added 28 + 25 that would be 53 so took 3 away from 25 and that equals 22.	I jumped back 2 from 50 to 48 and jumped back another 20 to 28 to find the difference. I know that 2 and 20 more is 22.	I know that 28 + 30 is 58 and that is too much so I know I need to remove 8 from 30 and that is 22.

When providing a solution, students should always be required to justify, even if it is not correct. Designating as little as 10-15 minutes a day for mental computation and talking about numbers will help students look and think about numbers flexibly.

In a classroom number talk, students begin to share the authority of determining whether answers are accurate, and are expected to think through all solutions and strategies carefully (Parrish, 2010). During the number talk, the teacher is not the definitive authority. The teacher maintains the role of the facilitator, and is listening and learning for and from the students' natural mathematical thinking. The discussions should maintain a focus, assist students in learning appropriate ways to structure comments and misunderstandings, and the conversation should flow in a meaningful and natural way (Parrish, 2010).

Workstations and Learning Centers

When thinking about developing work stations and learning centers you want to base them on student readiness, interest, or learning profile such as learning style or multiple intelligence. This will allow different students to work on different tasks. Students should be able to complete the tasks within the stations or centers independently, with a partner or in a group.

It is important for students to be engaged in purposeful activities within the stations and centers. Therefore, you must careful consider the activities selected to be a part of the stations and centers. When selecting an activity, you may want to consider the following questions:

- Will the activity reinforce or extend a concept that's already been introduced?
- Are the directions clear and easy to follow?
- Are materials easy to locate and accessible?
- Can students complete this activity independently or with minimal help from the teacher?
- How will students keep a record of what they've completed?
- How will students be held accountable for their work?

(Laura Candler, *Teaching Resources*)

When implementing work stations and learning centers within your classroom, it is important to consider when the stations and centers will be used. Will you assign students to specific stations or centers to complete each week or will they be able to select a station or center of their choice? Will this opportunity be presented to all students during particular times of your math block or to students who finish their work early?

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned.

Games
"A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience.

Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: 3-5 pg. 28

Journaling

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.

When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student’s thinking is made visible!

GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?
- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?

Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem? assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use?

Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

Representing

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?

QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK cont'd…

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others."

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students. Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from "doing it myself!"

Problem Solving Rubric (3-5)

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, and selected and applied an appropriate problem solving strategy that lead to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, and selected an applied and appropriate problem solving strategy that lead to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem solving strategy that lead to a thorough and accurate solution. In addition, student will check answer using another method.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words. Student is able to make connections between models and equations.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words. The student connects quantities to written symbols and create a logical representation with precision.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected an appropriate tools or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition this students was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

SUGGESTED LITERATURE

-
- *Amanda Bean's Amazing Dream (A mathematical story)* by Cindy Neuschwander
- *Bigger, Better, Best!* by Stuart J. Murphy
- *Divide and Ride* by Stuart J. Murphy
- *Give Me Half!* By Stuart J. Murphy
- *Grandfather Tang's Story* by Ann Tompert
- *Grapes of Math* by Greg Tang
- *How Many Seeds in a Pumpkin?* by Margaret McNamara
- *Lemonade for Sale* by Stuart J. Murphy
- *One Hundred Hungry Ants*, by Elinor J. Pinczes
- *Picture Pie 2: A Drawing Book and Stencil* by Ed Emberely
- *Picture Pie: A Circle Drawing Book* by Ed Emberely
- *Racing Around* by Stuart J. Murphy
- *Spaghetti and Meatballs For All* by Marilyn Burns
- *The Best of Times* by Greg Tang
- *The Great Graph Contest* by Loreen Leedy
- *The Greedy Triangle* by Marilyn Burns
- *The Long Wait* by Annie Cobb
- *The Mathemagician's Apprentice* by Brian Boyd

TECHNOLOGY LINKS

Unit 1

- http://www.shodor.org/interactivate/activities/EstimatorFour/?version=1.6.0_02&browser=MSIE&vendor=Sun_Microsystems_Inc. A "Four in a Row" game where players get checkers when they quickly and efficiently estimate a sum to two numbers.
- http://www.oswego.org/ocsd-web/games/Estimate/estimate.html Students estimate the number indicated on a number line.
- http://olc.spsd.sk.ca/de/math1-3/p-mentalmath.html Teacher background information as well as student practice materials on the topic of elementary mental math strategies
- http://www.curriculumsupport.education.nsw.gov.au/primary/mathematics/numeracy/mental/index.htm Mental computation strategies with some fun graphics to demonstrate the strategies.
- http://www.oswego.org/ocsd-web/games/SpeedGrid/Addition/urikaadd2res.html A fun race against time using mental addition skills.
- http://letsplaymath.wordpress.com/tag/mental-math/ Offers ideas for other games and links to additional math sites.

- http://www.aaamath.com/pro.html Students practice identification and application of arithmetic properties
- http://www.cdli.ca/CITE/math_problems.htm A source for teachers to get ideas for additional word problems for students
- Graphs can be created using templates such as the pictograph template below: http://www.beaconlearningcenter.com/documents/2351_5255.pdf
- Pictographs can be created using excel following the directions below: http://faculty.kutztown.edu/schaeffe/Excel/Vallone/Vallone_Excel.pdf
- Pictographs can be created using a website such as: http://gwydir.demon.co.uk/jo/numbers/pictogram/pictogram.htm
- Bar graphs can be created using a website such as:
  - http://nces.ed.gov/nceskids/createagraph/ or
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=63
- If students are having difficulty thinking of a question, these websites have many ideas:
  - http://www.canteach.ca/elementary/numbers13.html
  - http://www.uen.org/lessonplan/upload/10867-2-14587-graphing_ideas.pdf
- http://www.shodor.org/interactivate/activities/BarGraphSorter/ Create bar graphs based on random sets of shapes.

Unit 2

- http://mason.gmu.edu/~mmankus/whole/base10/asmdb10.htm#div A site for teachers and parents provides information on using base 10 blocks to solve division problems with an area model.

- http://mcq.wrdsb.on.ca/Admin/Documents/WORC/PDFs/LESSON%20PrimaryMath.pdf
- http://www.lessonplanspage.com/MathLAMultiplicationDivisionUsingTheDoorbellRang23.htm provide teacher resources for the book *The Doorbell Rang* by Pat Hutchins.
- http://www.stuartjmurphy.com/activities/activity_ideas.php Stuart Murphy website with activity suggestions for *Divide and Ride.* (Click on level 3 and then click on the title of the book.)
- http://www.softschools.com/math/games/division_practice.jsp Division practice; the student or teacher can determine the parameters for the divisor, dividend, and number of problems
- Directions for origami frog http://www.ljhs.sandi.net/faculty/mteachworth/avid-information/origami-frog-lab-avid.pdf
- http://www.frogtown.org/ This is a YouTube video of a frog jumping contest
- http://www.youtube.com/watch?v=luG7_nzBHjI&feature=fvo&ad=21937675894 This is a YouTube video that provides instructions for making an origami frog.

Unit 3

- http://www.multiplication.com/ Practice games for multiplication facts as well as teacher resource pages with instructional ideas on how to introduce multiplication. NOTE: this site contains advertising

- http://illuminations.nctm.org/LessonDetail.aspx?id=U109 Numerous ideas for introducing multiplication, including the array model.
- http://www.eduplace.com/math/mw/background/3/08/te_3_08_overview.html Provides background information on the relationship between multiplication and division.
- http://www.arcytech.org/java/patterns/patterns_d.shtml Allows students to work with pattern blocks in an interactive applet and easily print their work.

Unit 4

- http://www.brainpopjr.com/math/measurement/area/grownups.weml
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=204
- http://illuminations.nctm.org/ActivityDetail.aspx?ID=204
- http://www.youtube.com/watch?v=G6eTMRXHhmE
- http://www.youtube.com/watch?v=G6eTMRXHhmE
- http://mrsrenz.net/mathsites.htm#area-perim

Unit 5

- http://www.harcourtschool.com/activity/buzz/buzz.html
- http://www.ablongman.com/vandewalleseries/Vol_2_BLM_PDFs/BLM20-26.pdf
- http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities&from=category_g_2_t_3.html Interactive geoboard from the National Library of Virtual Manipulatives
- http://nlvm.usu.edu/
- http://www.mathcats.com/explore/polygons.html - Explore Polygons
- http://www.math-play.com/Polygon-Game.html - Name the Shape
- http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html?open=activities&from=category_g_2_t_3.html – Compose Shapes
- http://www.learner.org/courses/learningmath/video/geometry/wmp/geo_10_k5_ch1.html - Video of teacher using Venn Diagram to sort polygons with whole class
- http://www.regentsprep.org/regents/math/geometry/GP9/LQuad.htm
- http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html - allows students to move corners to make sizes.
- http://www.interactivestuff.org/match/maker.phtml?featured=1&id=24 – matching game
- http://www.mathplayground.com/matching_shapes.html - matching games - has kite

- http://wps.ablongman.com/wps/media/objects/3464/3547873/blackline_masters/BLM_40.pdf

- http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities This site offers an easy to use virtual geoboard.
- http://nlvm.usu.edu/en/nav/frames_asid_271_g_2_t_3.html?open=instructions&from=category_g_2_t_3.html Students follow a pattern to create attribute trains based on color,

shape, or the number on the shape (e.g. triangle, square pattern; red, red, blue pattern; triangle, square, square pattern; 2,3,1 pattern).

- http://www.regentsprep.org/regents/math/geometry/GP9/LQuad.htm A summary of the properties of several quadrilaterals.
- http://teams.lacoe.edu/documentation/classrooms/amy/geometry/6-8/activities/new_quads/quads.html A virtual "Quest" where students match properties with quadrilaterals.
- http://www.shodor.org/interactivate/lessons/IntroQuadrilaterals/ An introduction to Quadrilaterals
- http://www.mathsisfun.com/quadrilaterals.html - Use after completing the Quadrilateral Challenge; does allow for students to move the corners of the quadrilaterals
- http://illuminations.nctm.org/LessonDetail.aspx?id=L350 – Complete lesson on rectangles and parallelograms
- http://illuminations.nctm.org/LessonDetail.aspx?id=L813 – Shape Up Lesson from Illumination
- http://illuminations.nctm.org/LessonDetail.aspx?ID=L346
- http://ejad.best.vwh.net/java/patterns/patterns_j.shtml - Pattern Blocks
- http://nlvm.usu.edu/en/nav/frames_asid_170_g_2_t_3.html?open=activities&from=category_g_2_t_3.html – National Library of Virtual Manipulatives
- http://wps.ablongman.com/wps/media/objects/3464/3547873/blackline_masters/BLM_38.pdf
- http://etc.usf.edu/clipart/sitemap/fractions.php
- http://wps.ablongman.com/ab_vandewalle_math_6/0,12312,3547876-,00.html
- http://www.edemberley.com/pages/main.aspx?section=db&subSection=capPages, http://www.edemberley.com/pages/main.aspx?section=db
- http://www.edemberley.com/pages/main.aspx?section=legal
- http://www.kellyskindergarten.com/math/math_activities.htm
- http://nces.ed.gov/nceskids/createagraph/ - Create a bar graph online.
- http://www.softschools.com/math/data_analysis/pictograph/make_your_own_pictograph/ Create a Picture Graph
- http://www.eduplace.com/math/mthexp/g3/visual/pdf/vs_g3_144.pdf

Unit 6

- http://www.kidsnumbers.com/turkey-terminator-math-game.php

- http://www.visualfractions.com/
- http://nlvm.usu.edu/en/nav/frames_asid_103_g_1_t_1.html?from=topic_t_1.html
- http://nlvm.usu.edu/en/nav/frames_asid_274_g_2_t_1.html?open=activities&from=topic_t_1.html
- http://nlvm.usu.edu/en/nav/frames_asid_104_g_2_t_1.html?from=topic_t_1.html
- http://www.mathplayground.com/Scale_Fractions.html
- http://nlvm.usu.edu/en/nav/frames_asid_105_g_2_t_1.html?from=topic_t_1.html

- http://illuminations.nctm.org/ActivityDetail.aspx?ID=80
- http://www.gamequarium.com/fractions.html
- http://www.learningplanet.com/sam/ff/index.asp
- http://www.mathsisfun.com/numbers/fraction-number-line.html
- http://mrnussbaum.com/pizza_game/index.html
- http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/fractions/index.htm
- http://www.primarygames.com/fractions/2a.htm
- http://www.mathsisfun.com/numbers/fractions-match-words-pizza.html

Unit 7

- http://nlvm.usu.edu/en/nav/frames_asid_318_g_2_t_4.html Elapsed time problems using two clocks, both analog and digital
- http://www.shodor.org/interactivate/activities/ElapsedTime/?version=disabled&browser=MSIE&vendor=na&flash=10.0.32 Several activities surrounding elapsed time using analog or digital clocks.
- http://donnayoung.org/math/clock.htm Printable blank clock faces.

- *http://illuminations.nctm.org/ActivityDetail.aspx?ID=63* – Students can use this website to enter their data and create a bar graph.
- http://www.shodor.org/interactivate/activities/BarGraph/ – different link to the same program as above.
- http://nlvm.usu.edu/en/nav/frames_asid_114_g_2_t_3.html?open=activities
- http://people.rit.edu/mecsma/Professional/Puzzles/Pentominoes/P-A.html

- http://nlvm.usu.edu/en/nav/frames_asid_114_g_2_t_3.html?open=activities Interactive pentomino tasks
- http://people.rit.edu/mecsma/Professional/Puzzles/Pentominoes/P-A.html Provides several beginner problems with solutions for pentominos.
- http://puzzler.sourceforge.net/docs/pentominoes.html Solutions to several pentominos puzzles such as the one below.
- http://nlvm.usu.edu/en/nav/frames_asid_172_g_2_t_3.html?open=activities Geoboard with area/perimeter activity (Look for the activity titled, “Shapes with Perimeter 16.”)
- http://highered.mcgraw-hill.com/sites/0072532947/student_view0/grid_and_dot_paper.html Printable dot and graph paper
- http://www.shodor.org/interactivate/activities/ShapeExplorer/?version=1.6.0_07&browser=MSIE&vendor=Sun_Microsystems_Inc.&flash=10.0.32 Randomly generated rectangles for which the perimeter and the area can be found

- http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks2/maths/perimeter_and_area/index.html Instruction on finding area and perimeter of shapes with practice. Levels 2 and 3 for area require finding the area of several rectangles and adding the areas together.

- http://nlvm.usu.edu/en/nav/frames_asid_169_g_1_t_3.html?open=activities&from=category_g_1_t_3.html The squares from this virtual pattern blocks web site can be used. Students can use six squares from the pattern blocks and find different arrangements of the squares that meet the required conditions.
- http://www.learner.org/courses/learningmath/measurement/session9/part_a/index.html An interactive task for ***teachers*** to explore area and perimeter.
-
- http://public.doe.k12.ga.us/DMGetDocument.aspx/Grade%204%20Unit%203%20Creating%20Scales.pdf?p=6CC6799F8C1371F6CA1D4816B288AA4D386F6BD621BEA7BF4B4B9C6CFBBB4292&Type=D Link to directions on how to make a balance scale and a spring scale using common materials.
- http://www.mathsisfun.com/measure/metric-mass.html Provides some background on metric measures and lists items that weigh about one kilogram.
- http://gadoe.georgiastandards.org/mathframework.aspx?PageReq=MathHunt Is a link to a classroom video of this task. Teachers may want to view this video to see how one teacher implemented this task in his classroom.
- http://nces.ed.gov/nceskids/createagraph/default.aspx?ID=3525a186ca2b4d3e8589a61cbefa47ac.
- http://nces.ed.gov/nceskids/createagraph/default.aspx "Create A Graph" by the National Center for Education Statistics is a web-based program which allows students to create a bar graph.
- http://www.amblesideprimary.com/ambleweb/mentalmaths/grapher.html Limited in its use but very simple to use – most labels can be renamed.
- http://ellerbruch.nmu.edu/classes/cs255w03/cs255students/nsovey/P5/P5.html Provides instructions to create a line plot graph as well as features to look for in a line plot graph.
- http://www.thefutureschannel.com/dockets/realworld/teaching_zoo/ This FUTURES video is a great introduction to this task. Students see how important accurate measurements using both metric and standard weights are important in a zoo.

RESOURCES CONSULTED

Content:
Ohio DOE
http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704
Arizona DOE
http://www.azed.gov/standards-practices/mathematics-standards/
Nzmaths
http://nzmaths.co.nz/

Teacher/Student Sense-making:
http://www.youtube.com/user/mitcccnyorg?feature=watch
http://www.insidemathematics.org/index.php/video-tours-of-inside-mathematics/classroom-teachers/157-teachers-reflect-mathematics-teaching-practices
https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
or http://secc.sedl.org/common_core_videos/

Journaling:
http://www.mathsolutions.com/index.cfm?page=wp10&crid=3

Community of Learners:
http://www.edutopia.org/math-social-activity-cooperative-learning-video
http://www.edutopia.org/math-social-activity-sel
http://www.youtube.com/user/responsiveclassroom/videos
http://www.responsiveclassroom.org/category/category/first-weeks-school