CCGPS Frameworks Teacher Edition

Mathematics

Second Grade

Grade Level Overview

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

Second Grade
Grade Level Overview

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Common Core Georgia Performance Standards
Second Grade

Common Core Georgia Performance Standards: Curriculum Map						
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Extending Base Ten Understanding	**Becoming Fluent with Addition and Subtraction**	**Understanding Measurement, Length, and Time**	**Applying Base Ten Understanding**	**Understanding Plane and Solid Figures**	**Developing Multiplication**	**Show What We Know**
MCC2.NBT.1 **MCC2.NBT.2** **MCC2.NBT.3** **MCC2.NBT.4** **MCC2.MD.10**	**MCC2.OA.1** **MCC2.OA.2** **MCC2.NBT.5** **MCC2.MD.8** **MCC2.MD.10**	**MCC2.MD.1** **MCC2.MD.2** **MCC2.MD.3** **MCC2.MD.4** **MCC2.MD.5** **MCC2.MD.6** **MCC2.MD.7** **MCC2.MD.9** **MCC2.MD.10**	**MCC2.NBT.6** **MCC2.NBT.7** **MCC2.NBT.8** **MCC2.NBT.9** **MCC2.MD.8** **MCC2.MD.10**	**MCC2.G.1** **MCC2.G.2** **MCC2.G.3** **MCC2.MD.10**	**MCC2.OA.3** **MCC2.OA.4** **MCC2.MD.10**	**ALL**
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain.						

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades K-2 Key: CC = Counting and Cardinality, G= Geometry, MD=Measurement and Data, NBT= Number and Operations in Base Ten, OA = Operations and Algebraic Thinking.

STANDARDS OF MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.

> *The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections.*
>
> *The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).*

Students are expected to:

1. Make sense of problems and persevere in solving them.
In second grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. They may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They make conjectures about the solution and plan out a problem-solving approach.

2. Reason abstractly and quantitatively.
Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. Second graders begin to know and use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.
Second graders may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?", "Explain your thinking," and "Why is that true?" They not only explain their own thinking, but listen to others' explanations. They decide if the explanations make sense and ask appropriate questions.

4. Model with mathematics.
In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed.

5. Use appropriate tools strategically.
In second grade, students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be better suited. For instance, second graders may decide to solve a problem by drawing a picture rather than writing an equation.

6. Attend to precision.
As children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning.

7. Look for and make use of structure.
Second graders look for patterns. For instance, they adopt mental math strategies based on patterns (making ten, fact families, doubles).

8. Look for and express regularity in repeated reasoning.
Students notice repetitive actions in counting and computation, etc. When children have multiple opportunities to add and subtract, they look for shortcuts, such as rounding up and then adjusting the answer to compensate for the rounding. Students continually check their work by asking themselves, "Does this make sense?"

******Mathematical Practices 1 and 6 should be evident in EVERY lesson******

CONTENT STANDARDS

OPERATIONS AND ALGEBRAIC THINKING (OA)

CLUSTER #1: REPRESENT AND SOLVE PROBLEMS INVOLVING ADDITION AND SUBTRACTION.

MCC2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

This standard calls for students to add and subtract numbers within 100 in the context of one and two step word problems. Students should have ample experiences working on various types of problems that have unknowns in all positions, including:

Addition Examples:

Result Unknown	Change Unknown	Start Unknown
There are 29 students on the playground. Then 18 more students showed up. How many students are there now? *(29 + 18 = ___)*	There are 29 students on the playground. Some more students show up. There are now 47 students. How many students came? *(29 + ___ = 47)*	There are some students on the playground. Then 18 more students came. There are now 47 students. How many students were on the playground at the beginning? *(___ + 18 = 47)*

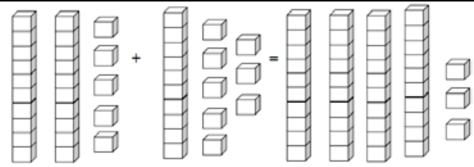
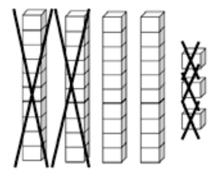
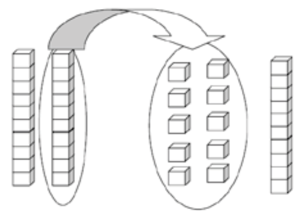
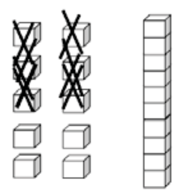
See Table 1 at the end of this document for more addition examples as well as subtraction examples.

This standard also calls for students to solve one- and two-step problems using drawings, objects and equations. Students can use place value blocks or hundreds charts, or create drawings of place value blocks or number lines to support their work. Examples of one-step problems with unknowns in different places are provided in Table 1. Two step-problems include situations where students have to add and subtract within the same problem.

Example:

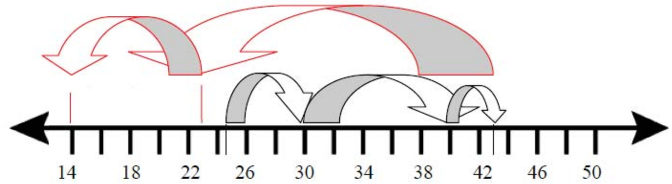
> In the morning there are 25 students in the cafeteria. 18 more students come in. After a few minutes, some students leave. If there are 14 students still in the cafeteria, how many students left the cafeteria? Write an equation for your problem.

Student 1

Step 1	I used place value blocks and made a group of 25 and a group of 18. When I counted them I had 3 tens and 13 ones which is 43.	
Step 2	I then wanted to remove blocks until there were only 14 left. I removed blocks until there were 20 left.	
Step 3	Since I have two tens I need to trade a ten for 10 ones.	
Step 4	After I traded it, I removed blocks until there were only 14 remaining.	
Step 5	My answer was the number of blocks that I removed. I removed 2 tens and 9 ones. That's 29. My equation is 25 + 18 – ___ = 14.	

Student 2

I used a number line. I started at 25 and needed to move up 18 spots so I started by moving up 5 spots to 30, and then 10 spots to 40, and then 3 more spots to 43. Then I had to move backwards until I got to 14 so I started by first moving back 20 spots until I got to 23. Then I moved to 14 which were an additional 9 places. I moved back a total of 29 spots. Therefore there were a total of 29 students left in the cafeteria. My equation is 25 + 18 – ___ = 14.



Student 3

<table>
<tr><td>Step 1</td><td>I used a hundreds board. I started at 25. I moved down one row which is 10 more, then moved to the right 8 spots and landed on 43. This represented the 18 more students coming into the cafeteria.</td><td>
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<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
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</td></tr>
<tr><td>Step 2</td><td>Now starting at 43, I know I have to get to the number 14 which represents the number of students left in the cafeteria so I moved up 2 rows to 23 which is 20 less. Then I moved to the left until I land on 14, which is 9 spaces. I moved back a total of 29 spots. That means 29 students left the cafeteria.</td><td>
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<tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
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</table>
</td></tr>
<tr><td>Step 3</td><td colspan="2">My equation to represent this situation is 25 + 18 – ___ = 14.</td></tr>
</table>

CLUSTER #2: ADD AND SUBTRACT WITHIN 20.

<u>MCC2.OA.2 Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.</u>

This standard mentions the word *fluently* when students are adding and subtracting numbers within 20. Fluency means accuracy (correct answer), efficiency (within 4-5 seconds), and flexibility (using strategies such as making 10 or breaking apart numbers). Research indicates that teachers' can best support students' memorization of sums and differences through varied experiences making 10, breaking numbers apart and working on mental strategies, rather than repetitive timed tests.

Example: 9 + 5 = ___

Student 1: *Counting On*	Student 2: *Decomposing a Number Leading to a Ten*
I started at 9 and then counted 5 more. I landed at 14.	I know that 9 and 1 is 10, so I broke 5 into 1 and 4. 9 plus 1 is 10. Then I have to add 4 more, which gets me to 14.

Example: 13 – 9 = ___

Student 1: *Using the Relationship between Addition and Subtraction*	Student 2: *Creating an Easier Problem*
I know that 9 plus 4 equals 13. So 13 minus 9 equals 4.	I added 1 to each of the numbers to make the problem 14 minus 10. I know the answer is 4. So 13 minus 9 is also 4.

CLUSTER #3: WORK WITH EQUAL GROUPS OF OBJECTS TO GAIN FOUNDATIONS FOR MULTIPLICATION.

MCC2.OA.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

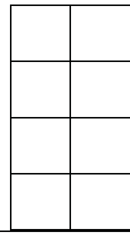
This standard calls for students to apply their work with doubles addition facts to the concept of odd or even numbers. Students should have ample experiences exploring the concept that if a number can be decomposed (broken apart) into two equal addends (e.g., 10 = 5 +5), then that number (10 in this case) is an even number. Students should explore this concept with concrete objects (e.g., counters, place value cubes, etc.) before moving towards pictorial representations such as circles or arrays.

Example: Is 8 an even number? Prove your answer.

Student 1	Student 2
I grabbed 8 counters. I paired counters up into groups of 2. Since I didn't have any counters left over, I know that 8 is an even number.	I grabbed 8 counters. I put them into 2 equal groups. There were 4 counters in each group, so 8 is an even number.

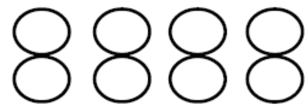
Student 3

I drew 8 boxes in a rectangle that had two columns. Since every box on the left matches a box on the right, I know 8 is even.



Student 4

I drew 8 circles. I matched one on the left with one on the right. Since they all match up, I know that 8 is an even number.

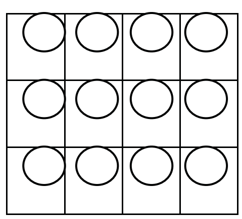


Student 5 I know that 4 plus 4 equals 8. So 8 is an even number.

Find the total number of objects below.

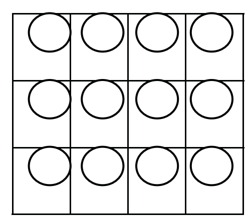
Student 1

I see 3 counters in each column and there are 4 columns. So I added: 3 + 3 + 3+ 3. That equals 12.



Student 2

I see 4 counters in each row and there are 3 rows. So I added 4 + 4 + 4. That equals 12.



NUMBER AND OPERATIONS IN BASE TEN (NBT)

CLUSTER #1: UNDERSTAND PLACE VALUE.

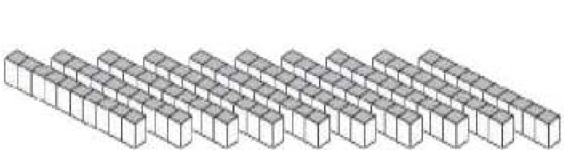
Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

MCC2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
This standard calls for students to work on decomposing numbers by place. Students should have ample experiences with concrete materials and pictorial representations examining that numbers all numbers between 100 and 999 can be decomposed into hundreds, tens, and ones.
Interpret the value of a digit (1-9 and 0) in a multi-digit numeral by its position within the number with models, words and numerals.

Use 10 as a benchmark number to compose and decompose when adding and subtracting whole numbers.

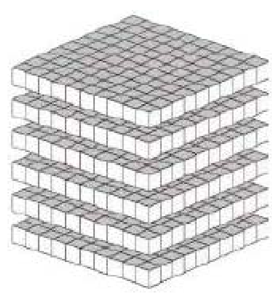
a. 100 can be thought of as a bundle of ten tens — called a "hundred."
MCC2.NBT.1a calls for students to extend their work from 1st Grade by exploring a hundred as a unit (or bundle) of ten tens.

b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
MCC2.NBT.1b builds on the work of MCC2.NBT.1a. Students should explore the idea that numbers such as 100, 200, 300, etc., are groups of hundreds that have no tens or ones. Students can represent this with place value (base 10) blocks.



is the same as

6 hundreds are the same as 600



MCC2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s.
This standard calls for students to count within 1,000. This means that students are expected to ―count on‖ from any number and say the next few numbers that come afterwards.

Understand that counting by 2s, 5s and 10s is counting groups of items by that amount.

Example:
What are the next 3 numbers after 498? 499, 500, 501.
When you count back from 201, what are the first 3 numbers that you say? 200, 199, 198.
This standard also introduces skip counting by 5s and 100s. Students are introduced to skip counting by 10s in First Grade. Students should explore the patterns of numbers when they skip count. When students skip count by 5s, the ones digit alternates between 5 and 0. When students skip count by 100s, the hundreds digit is the only digit that changes, and it increases by one number.

MCC2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
This standard calls for students to read, write and represent a number of objects with a written numeral (number form or standard form). These representations can include place value (base 10) blocks, pictorial representations or other concrete materials. Please be cognizant that when reading and writing whole numbers, the word "and" should not be used.

Example:
235 is written as two hundred thirty-five.

MCC2.NBT.4 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

This standard builds on the work of MCC2.NBT.1 and MCC2.NBT.3 by having students compare two numbers by examining the amount of hundreds, tens and ones in each number. Students are introduced to the symbols greater than (>), less than (<) and equal to (=) in First Grade, and use them in Second Grade with numbers within 1,000. Students should have ample experiences communicating their comparisons in words before using only symbols in this standard.
Example: 452 __ 455

Student 1	Student 2
452 has 4 hundreds, 5 tens, and 2 ones. 455 has 4 hundreds, 5 tens, and 5 ones. They have the same number of hundreds and the same number of tens, but 455 has 5 ones and 452 only has 2 ones. 452 is	452 is less than 455. I know this because when I count up I say 452 before I say 455. 452 < 455.

less than 455. 452 < 455.	

CLUSTER #2: USE PLACE VALUE UNDERSTANDING AND PROPERTIES OF OPERATIONS TO ADD AND SUBTRACT.

Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

MCC2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

This standard mentions the word fluently when students are adding and subtracting numbers within 100. Fluency means accuracy (correct answer), efficiency (basic facts computed within 4-5 seconds), and flexibility (using strategies such as making 10 or breaking numbers apart).

This standard calls for students to use pictorial representations or strategies to find the solution. Students who are struggling may benefit from further work with concrete objects (e.g., place value blocks).

Example: 67 + 25 = __

Place Value Strategy	Counting On and Decomposing a Number Leading to Ten	Commutative Property
I broke both 67 and 25 into tens and ones. 6 tens plus 2 tens equals 8 tens. Then I added the ones. 7 ones plus 5 ones equals 12 ones. I then combined my tens and ones. 8 tens plus 12 ones equals 92.	I wanted to start with 67 and then break 25 apart. I started with 67 and counted on to my next ten. 67 plus 3 gets me to 70. Then I added 2 more to get to 72. I then added my 20 and got to 92.	I broke 67 and 25 into tens and ones so I had to add 60 + 7 + 20 + 5. I added 60 and 20 first to get 80. Then I added 7 to get 87. Then I added 5 more. My answer is 92.

Example: 63 – 32 = __

Relationship between Addition and Subtraction
I broke apart both 63 and 32 into tens and ones. I know that 2 plus 1 equals 3, so I have 1 left in the ones place. I know that 3 plus 3 equals 6, so I have a 3 in my tens place. My answer has a 1 in the ones place and 3 in the tens place, so my answer is 31.

MCC2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.

This standard calls for students to add a string of two-digit numbers (up to four numbers) by applying place value strategies and properties of operations.

Example: 43 + 34 + 57 + 24 = __

Student 1: ***Associative Property***

I saw the 43 and 57 and added them first, since I know 3 plus 7 equals 10. When I added them 100 was my answer. Then I added 34 and had 134. Then I added 24 and had 158.

Student 2: ***Place Value Strategies***

I broke up all of the numbers into tens and ones. First I added the tens. 40 + 30 + 50 + 20 = 140. Then I added the ones. 3 + 4 + 7 + 4 = 18. Then I combined the tens and ones and had 158 as my answer.

Student 3: ***Place Value Strategies and Associative Property***

I broke up all the numbers into tens and ones. First I added up the tens: 40 + 30 + 50 + 20. I changed the order of the numbers to make adding them easier. I know that 30 plus 20 equals 50 and 50 more equals 100. Then I added the 40 and got 140. Then I added up the ones: 3 + 4 + 7 + 4. I changed the order of the numbers to make adding easier. I know that 3 plus 7 equals 10 and 4 plus 4 equals 8. 10 plus 8 equals 18. I then combined my tens and ones. 140 plus 18 equals 158.

MCC2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

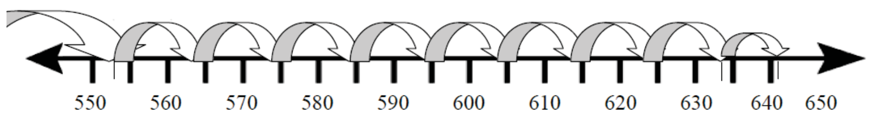
This standard builds on the work from 2.NBT.5 by increasing the size of numbers (two 3-digit numbers). Students should have ample experiences to use concrete materials (place value blocks) and pictorial representations to support their work.

This standard also references composing and decomposing a ten. This work should include strategies such as making a 10, making a 100, breaking apart a 10, or creating an easier problem. While the standard algorithm could be used here, students' experiences should extend beyond only working with the algorithm.

Example: 354 + 287 = ___

Student 1

I started at 354 and jumped 200. I landed on 554. Then I made 8 jumps of 10 and landed on 634. I then jumped 7 and landed on 641



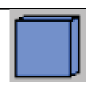
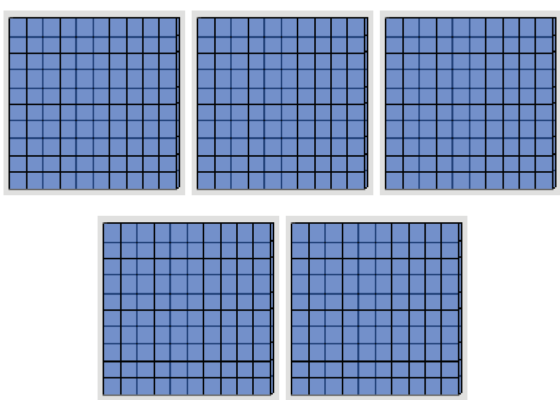
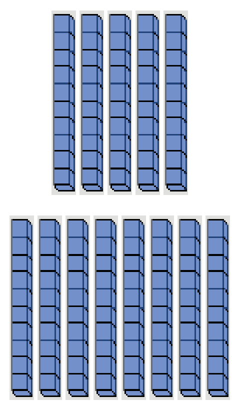
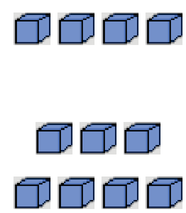
Student 2

I broke all of the numbers up by place using a place value chart.
I first added the ones.4 + 7 = 11.
I then added the tens. 50 + 80 = 130.
I then added the hundreds. 300 + 200 = 500.
I then combined my answers. 500 + 130 = 630. 630 + 11= 641

Student 2

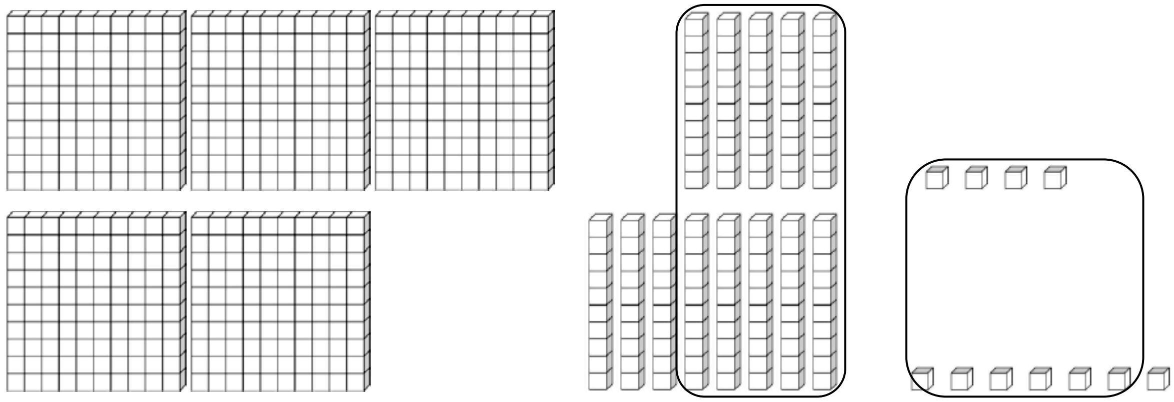
I broke all of the numbers up by place using a place value chart.

- I first added the ones: 4 + 7 = 11.
- Then I added the tens: 50 + 80 = 130.
- Then I added the hundreds: 300 + 200 = 500.
- Then I combined my answers: 500 + 130 = 630; 630 + 11 = 641.

 **Hundreds**	 **Tens**	 **Ones**
		

Student 3

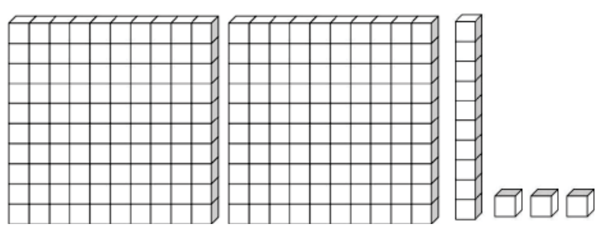
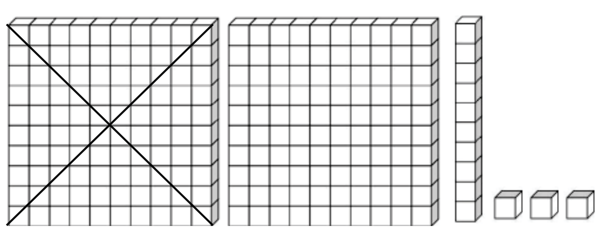
I used place value blocks. I made a pile of 354. I then added 287. That gave me 5 hundreds, 13 tens and 11 ones. I noticed that I could trade some pieces. I had 11 ones, and I traded 10 ones for a ten. I then had 14 tens, so I traded 10 tens for a hundred. I ended up with 6 hundreds, 4 tens, and 1 ones.

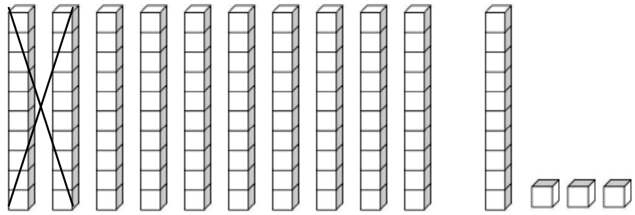
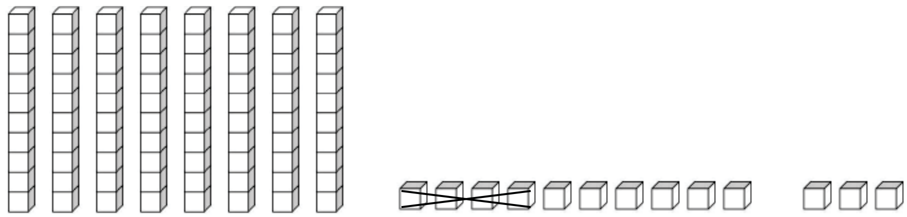
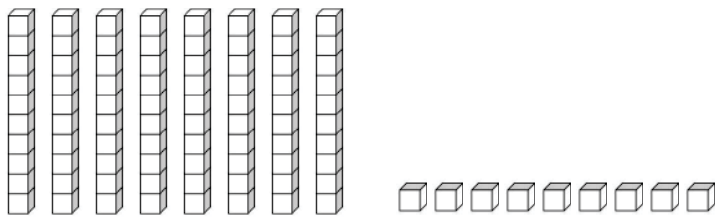


Example: 213 – 124 = ___

Student 1

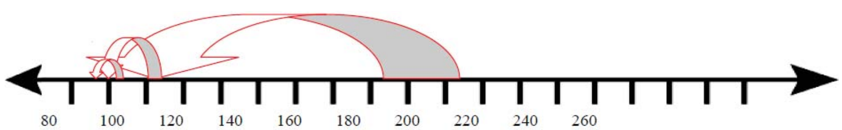
I used place value blocks. I made a pile of 213. Then I started taking away blocks. First I took away a hundred, which left me with 1 hundred and thirteen. I need to take away 2 tens but I only had 1 ten so I traded in my last hundred for 10 tens. Then I took 2 tens away, leaving me with no hundreds, 9 tens, and 3 ones. Then I had to take 4 ones away but I only have 3 ones. I traded in a ten for 10 ones. Then I took away 4 ones. This left me with no hundreds, 8 tens, and 9 ones. My answer is 89.

Step 1 **213**	
Step 2 **113**	

Step 3 **93**	
Step 4 **89**	
Final Answer **89**	

Student 2

I started at 213 and moved backwards 100 and landed on 113. Then I moved back 2 jumps of ten and landed on 93. Then I moved back 4 and landed on 89.



Student 3

I noticed that I was taking 124 away from 213. I changed 213 into 224 by adding 11. That made my problem 224 – 124. I know the answer to that problem is 100. Then I had to take away the 11 that I added. 100 – 11 = 89. My answer is 89.

<u>MCC2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.</u>

This standard calls for students to mentally add or subtract multiples of 10 or 100 to any number between 100 and 900. Students should have ample experiences working with the concept that when you add or subtract multiples of 10 or 100 that you are only changing the tens place (multiples of ten) or the digit in the hundreds place (multiples of 100).
In this standard, problems in which students cross centuries should also be considered.
Example: 273 + 60 = 333.

MCC2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations.

This standard calls for students to explain using concrete objects, pictures and words (oral or written) to explain why addition or subtraction strategies work. The expectation is that students apply their knowledge of place value and the properties of operations in their explanation. Students should have the opportunity to solve problems and then explain why their strategies work.

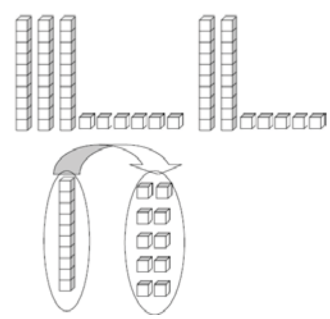
Example: There are 36 birds in the park. 25 more birds arrive. How many birds are there? Solve the problem and show your work.

Student 1

I broke 36 and 25 into tens and ones and then added them. 30 + 6 + 20 + 5. I can change the order of my numbers, so I added 30 + 20 and got 50. Then I added on 6 to get 56. Then I added 5 to get 61. This strategy works because I broke all the numbers up by their place value.

Student 2

I used place value blocks and made a pile of 36. Then I added 25. I had 5 tens and 11 ones. I had to trade 10 ones for 1 10. Then I had 6 tens and 1 one. That makes 61. This strategy works because I added up the tens and then added up the ones and traded if I had more than 10 ones.



Students could also have experiences examining strategies and explaining why they work. Also include incorrect examples for students to examine.

Example: One of your classmates solved the problem 56 - 34 = __ by writing ―I know that I need to add 2 to the number 4 to get 6. I also know that I need to add 20 to 30 to get 20 to get to 50. So, the answer is 22. Is their strategy correct? Explain why or why not?

Example: One of your classmates solved the problem 25 + 35 by adding 20 + 30 + 5 + 5. Is their strategy correct? Explain why or why not?

MEASUREMENT AND DATA (MD)

CLUSTER #1: MEASURE AND LENGTHS IN STANDARD UNITS.

Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

MCC2.MD.1 Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
This standard calls for students to measure the length of objects in both customary (inches and feet) and metric (centimeters and meters). Students should have ample experiences choosing objects, identifying the appropriate tool and unit, and then measuring the object. The teacher should allow students to determine which tools and units to use.

Foundational understandings to help with measure concepts:

- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principle).
- Understand the measuring of two-dimensional space (area) using non-standard units.

MCC2.MD.2 Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
This standard calls for students to measure an object using two units of different lengths.

Example: A student measures the length of their desk and finds that it is 3 feet and 36 inches. Students should explore the idea that the length of the desk is larger in inches than in feet, since inches are smaller units than feet. This concept is referred to as the compensatory principle. Note: this standard does not specify whether the units have to be within the same system.

MCC2.MD.3 Estimate lengths using units of inches, feet, centimeters, and meters.
This standard calls for students to estimate the lengths of objects using inches, feet, centimeters, and meters. Students should make estimates after seeing a benchmark unit, such as the length of one inch, before making their estimate.

Example: Look at your ruler to see how long one inch is. Now, estimate the length of this paper in inches.

MCC2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

This standard calls for students to determine the difference in length between two objects. Students should choose objects, identify appropriate tools and units, measure both objects, and then determine the differences in lengths.

CLUSTER #2: RELATE ADDITION AND SUBTRACTION TO LENGTH.

MCC2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
This standard applies the concept of length to solve addition and subtraction word problems with numbers within 100. Students should use the same unit in these problems.

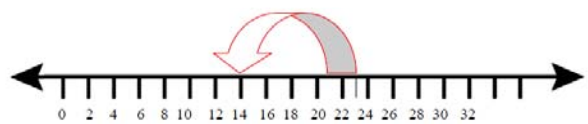
Example: In P.E. class Kate jumped 14 inches. Mary jumped 23 inches. How much farther did Mary jump than Kate? Write an equation and then solve the problem.

Student 1

My equation is 14 + ___ = 23 since I am trying to find out the difference between Kate and Mary's jumps. I used place value blocks and counted out 14. Then I added blocks until I got to 23. I needed to add 9 blocks. Mary jumped 9 more inches than Kate.

Student 2

My equation is 23 – 14 = ___. I drew a number line. I started at 23. I moved back to 14 and counted how far I moved. I moved back 9 spots. Mary jumped 9 more inches than Kate.



MCC2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
This standard calls for students to create number lines using numbers within 100 to solve addition and subtraction problems. Students should create the number line with evenly spaced points corresponding to the numbers.

CLUSTER #3: WORK WITH TIME AND MONEY.

MCC2.MD.7 Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
This standard calls for students to tell (orally and in writing) and write time after reading analog and digital clocks. Time should be to 5 minute intervals, and students should also use the terms a.m. and p.m. Teachers should help students make the connection between skip counting by 5s (MCC2.NBT.2) and telling time on an analog clock.

<u>MCC2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.</u> *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*
This standard calls for students to solve word problems involving either dollars or cents. Since students have not been introduced to decimals, problems should either have only dollars or only cents.

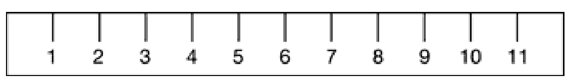
Example: What are some possible combinations of coins (pennies, nickels, dimes, and quarters) that equal 37 cents?

Example: What are some possible combinations of dollar bills ($1, $5 and $10) that equal 12 dollars?

CLUSTER #4: REPRESENT AND INTERPRET DATA.

<u>MCC2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</u>
This standard calls for students to represent the length of several objects by making a line plot. Students should round their lengths to the nearest whole unit.

Example: Measure objects in your desk to the nearest inch, display data collected on a line plot. How many objects measured 2 inches? 3 inches? Which length had the most number of objects? How do you know?



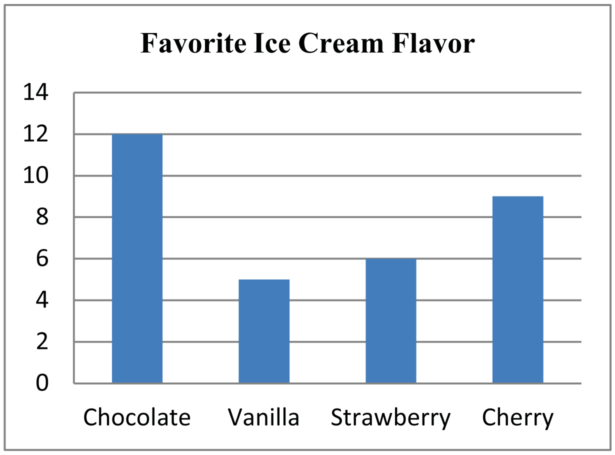
<u>MCC2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.</u>
This standard calls for students to work with categorical data by organizing, representing and interpreting data. Students should have experiences posing a question with 4 possible responses and then work with the data that they collect.

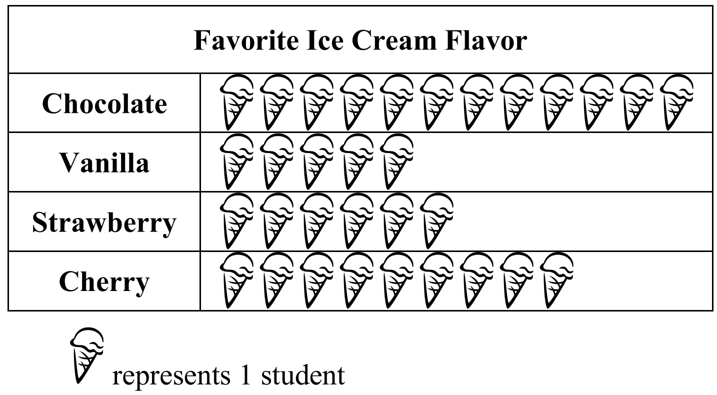
Example: Students pose a question and the 4 possible responses. Which is your favorite flavor of ice cream: Chocolate, vanilla, strawberry, or cherry?

Students collect their data by using tallies or another way of keeping track. Students organize their data by totaling each category in a chart or table. Picture and bar graphs are introduced in 2nd Grade.

Flavor	Number of People
Chocolate	12
Vanilla	5
Strawberry	6
Cherry	9

Students display their data using a picture graph or bar graph using a single unit scale.





Students answer simple problems related to addition and subtraction that ask them to put together, take apart, and compare numbers. See Table 1 at the end of this document for examples of these.

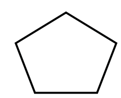
GEOMETRY (G)

CLUSTER #1: REASON WITH SHAPES AND THEIR ATTRIBUTES.
Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

MCC2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
This standard calls for students to identify (recognize) and draw shapes based on a given set of attributes. These include triangles, quadrilaterals (squares, rectangles, and trapezoids), pentagons, hexagons and cubes.

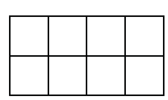
Example: Draw a closed shape that has five sides. What is the name of the shape?

Student 1
I drew a shape with 5 sides. It is called a pentagon.



MCC2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
This standard calls for students to partition a rectangle into squares (or square-like regions) and then determine the total number of squares. This relates to the standard 2.OA.4 where students are arranging objects in an array of rows and columns.

Example: Split the rectangle into 2 rows and 4 columns. How many small squares did you make?



MCC2.G.3 Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.
This standard calls for students to partition (split) circles and rectangles into 2, 3 or 4 equal shares (regions). Students should be given ample experiences to explore this concept with paper strips and pictorial representations. Students should also work with the vocabulary terms halves, thirds, half of, third of, and fourth (or quarter) of. While students are working on this standard, teachers should help them to make the connection that a ―whole‖ is composed of two halves, three thirds, or four fourths.

This standard also addresses the idea that equal shares of identical wholes may not have the same shape.

Example: Divide each rectangle into fourths a different way.

ARC OF LESSON (OPENING, WORK SESSION, CLOSING)

"When classrooms are workshops-when learners are inquiring, investigating, and constructing- there is already a feeling of community. In workshops learners talk to one another, ask one another questions, collaborate, prove, and communicate their thinking to one another. The heart of math workshop is this: investigations and inquiries are ongoing, and teachers try to find situations and structure contexts that will enable children to mathematize their lives- that will move the community toward the horizon. Children have the opportunity to explore, to pursue inquiries, and to model and solve problems on their own creative ways. Searching for patterns, raising questions, and constructing one's own models, ideas, and strategies are the primary activities of math workshop. The classroom becomes a community of learners engaged in activity, discourse, and reflection." *Young Mathematicians at Work- Constructing Addition and Subtraction* by Catherine Twomey Fosnot and Maarten Dolk.

"Students must believe that the teacher does not have a predetermined method for solving the problem. If they suspect otherwise, there is no reason for them to take risks with their own ideas and methods." *Teaching Student-Centered Mathematics, K-3* by John Van de Walle and Lou Ann Lovin.

Opening: Set the stage
Get students mentally ready to work on the task
Clarify expectations for products/behavior
How?

- Begin with a simpler version of the task to be presented
- Solve problem strings related to the mathematical idea/s being investigated
- Leap headlong into the task and begin by brainstorming strategies for approaching the task
- Estimate the size of the solution and reason about the estimate

Make sure everyone understands the task before beginning. Have students restate the task in their own words. Every task should require more of the students than just the answer.

Work session: Give 'em a chance
Students- grapple with the mathematics through sense-making, discussion, concretizing their mathematical ideas and the situation, record thinking in journals
Teacher- Let go. Listen. Respect student thinking. Encourage testing of ideas. Ask questions to clarify or provoke thinking. Provide gentle hints. Observe and assess.

Closing: Best Learning Happens Here
Students- share answers, justify thinking, clarify understanding, explain thinking, question each other
Teacher- Listen attentively to all ideas, ask for explanations, offer comments such as, "Please tell me how you figured that out." "I wonder what would happen if you tried…"
Anchor charts
Read Van de Walle K-3, Chapter 1

BREAKDOWN OF A TASK (UNPACKING TASKS)

How do I go about tackling a task or a unit?

1. Read the unit in its entirety. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks. Collaboratively complete the culminating task with your grade level colleagues. As students work through the tasks, you will be able to facilitate their learning with this end in mind. The structure of the units/tasks is similar task to task and grade to grade. This structure allows you to converse in a vertical manner with your colleagues, school-wide. The structure of the units/tasks is similar task to task and grade to grade. There is a great deal of mathematical knowledge and teaching support within each grade level guide, unit, and task.

2. Read the first task your students will be engaged in. Discuss it with your grade level colleagues. Which parts do you feel comfortable with? Which make you wonder? Brainstorm ways to implement the tasks.

3. If not already established, use the first few weeks of school to establish routines and rituals, and to assess student mathematical understanding. You might use some of the tasks found in the unit, or in some of the following resources as beginning tasks/centers/math tubs which serve the dual purpose of allowing you to observe and assess.

   Additional Resources:
   Math Their Way: http://www.center.edu/MathTheirWay.shtml
   NZMaths- http://www.nzmaths.co.nz/numeracy-development-projects-books?parent_node=
   K-5 Math Teaching Resources- http://www.k-5mathteachingresources.com/index.html (this is a for-profit site with several free resources)
   Winnepeg resources- http://www.wsd1.org/iwb/math.htm
   Math Solutions- http://www.mathsolutions.com/index.cfm?page=wp9&crid=56

4. Points to remember:
   - Each task begins with a list of the standards specifically addressed in that task, however, that does not mean that these are the only standards addressed in the task. Remember, standards build on one another, and mathematical ideas are connected.
   - Tasks are made to be modified to match your learner's needs. If the names need changing, change them. If the materials are not available, use what is available. If a task doesn't go where the students need to go, modify the task or use a different resource.
   - The units are not intended to be all encompassing. Each teacher and team will make the units their own, and add to them to meet the needs of the learners.

ROUTINES AND RITUALS

Teaching Math in Context and Through Problems

"By the time they begin school, most children have already developed a sophisticated, informal understanding of basic mathematical concepts and problem solving strategies. Too often, however, the mathematics instruction we impose upon them in the classroom fails to connect with this informal knowledge" (Carpenter et al., 1999). The 8 Standards of Mathematical Practices (SMP) should be at the forefront of every mathematics lessons and be the driving factor of HOW students learn.

One way to help ensure that students are engaged in the 8 SMPs is to construct lessons built on context or through story problems. "Fosnot and Dolk (2001) point out that in story problems children tend to focus on getting the answer, probably in a way that the teacher wants. "Context problems, on the other hand, are connected as closely as possible to children's lives, rather than to 'school mathematics'. They are designed to anticipate and to develop children's mathematical modeling of the real world."

Traditionally, mathematics instruction has been centered around a lot of problems in a single math lesson, focusing on rote procedures and algorithms which do not promote conceptual understanding. Teaching through word problems and in context is difficult however, "kindergarten students should be expected to solve word problems" (Van de Walle, K-3).

A *problem* is defined as any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific correct solution method. A problem for learning mathematics also has these features:

- *The problem must begin where the students are which makes it accessible to all learners.*
- *The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.*
- *The problem must require justifications and explanations for answers and methods.*

It is important to understand that mathematics is to be taught *through* problem solving. That is, problem-based tasks or activities are the vehicle through which the standards are taught. Student learning is an outcome of the problem-solving process and the result of teaching within context and through the Standards for Mathematical Practice. (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 11).

Use of manipulatives

"It would be difficult for you to have become a teacher and not at least heard that the use of manipulatives, or a "hands-on approach," is the recommended way to teach mathematics. There is no doubt that these materials can and should play a significant role in your classroom. Used correctly they can be a positive factor in children's learning. But they are not a cure-all that some educators seem to believe them to be. It is important that you have a good perspective on how manipulatives can help or fail to help children construct ideas. We can't just give students a ten-

frame or bars of Unifix cubes and expect them to develop the mathematical ideas that these manipulatives can potentially represent. When a new model of new use of a familiar model is introduced into the classroom, it is generally a good idea to explain how the model is used and perhaps conduct a simple activity that illustrates this use."
(Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 6).

Once you are comfortable that the models have been explained, you should not force their use on students. Rather, students should feel free to select and use models that make sense to them. In most instances, not using a model at all should also be an option. The choice a student makes can provide you with valuable information about the level of sophistication of the student's reasoning.

Whereas the free choice of models should generally be the norm in the classroom, you can often ask students to model to show their thinking. This will help you find out about a child's understanding of the idea and also his or her understanding of the models that have been used in the classroom.

The following are simple rules of thumb for using models:

- Introduce new models by showing how they can represent the ideas for which they are intended.
- Allow students (in most instances) to select freely from available models to use in solving problems.
- Encourage the use of a model when you believe it would be helpful to a student having difficulty." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 8-9)

Use of strategies and effective questioning

Teachers ask questions all the time. They serve a wide variety of purposes: to keep learners engaged during an explanation; to assess their understanding; to deepen their thinking or focus their attention on something. This process is often semi-automatic. Unfortunately, there are many common pitfalls. These include:

- asking questions with no apparent purpose;
- asking too many closed questions;
- asking several questions all at once;
- poor sequencing of questions;
- asking rhetorical questions;
- asking 'Guess what is in my head' questions;
- focusing on just a small number of learners;
- ignoring incorrect answers;
- not taking answers seriously.

In contrast, the research shows that effective questioning has the following characteristics:

- Questions are planned, well ramped in difficulty.
- Open questions predominate.
- A climate is created where learners feel safe.
- A ‘no hands’ approach is used, for example when all learners answer at once using mini-whiteboards, or when the teacher chooses who answers.
- Probing follow-up questions are prepared.
- There is a sufficient ‘wait time’ between asking and answering a question.
- Learners are encouraged to collaborate before answering.
- Learners are encouraged to ask their own questions.

There are many types of questioning that promote mathematical thinking. On these pages we can offer a sample of what can be done.

0-99 Chart or 1-100 Chart
(Adapted information from About Teaching Mathematics A K–8 RESOURCE MARILYN BURNS 3rd edition and Van de Walle)

Both the 0-99 Chart and the 1-100 Chart are valuable tools in the understanding of mathematics. Most often these charts are used to reinforce counting skills. Counting involves two separate skills: (1) ability to produce the standard list of counting words (i.e. one, two, three) and (2) the ability to connect the number sequence in a one-to-one manner with objects (Van de Walle, 2007). The counting sequence is a rote procedure. The ability to attach meaning to counting is “the key conceptual idea on which all other number concepts are developed” (Van de Walle, p. 122). Children have greater difficulty attaching meaning to counting than rote memorization of the number sequence. Although both charts can be useful, the focus of the 0-99 chart should be at the forefront of number sense development in early elementary.

A 0-99 Chart should be used in place of a 1-100 Chart when possible in early elementary mathematics for many reasons, but the overarching argument for the 0-99 is that it helps to develop a deeper understanding of place value. Listed below are some of the benefits of using the 0-99 Chart in your classroom:

- A 0-99 Chart begins with zero where as a hundred’s chart begins with 1. It is important to include zero because it is a digit and just as important as 1-9.
- A 1-100 chart puts the decade numerals (10, 20, 30, etc.) on rows without the remaining members of the same decade. For instance, on a hundred’s chart 20 appears at the end of the teens’ row. This causes a separation between the number 20 and the numbers 21-29. The number 20 is the beginning of the 20’s family; therefore it should be in the beginning of the 20’s row like in a 99’s chart to encourage students to associate the quantities together.
- A 0-99 chart ends with the last two digit number, 99, this allows the students to concentrate their understanding using numbers only within the ones’ and tens’ place values. A hundred’s chart ends in 100, introducing a new place value which may change the focus of the places.

- The understanding that 9 units fit in each place value position is crucial to the development of good number sense. It is also very important that students recognize that zero is a number, not merely a placeholder. This concept is poorly modeled by a typical 1-100 chart, base ten manipulatives, and even finger counting. We have no "zero" finger, "zero" block, or "zero" space on typical 1-100 number charts. Whereas having a zero on the chart helps to give it status and reinforces that zero holds a quantity, a quantity of none. Zero is the answer to a question such as, "How many elephants are in the room?".
- Including zero presents the opportunity to establish zero correctly as an even number, when discussing even and odd. Children see that it fits the same pattern as all of the other even numbers on the chart.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

While there are differences between the 0-99 Chart and the 1-100 Chart, both number charts are valuable resources for your students and should be readily available in several places around the classroom. Both charts can be used to recognize number patterns, such as the increase or decrease by multiples of ten. Provide students the opportunity to explore the charts and communicate the patterns they discover.

The number charts should be placed in locations that are easily accessible to students and promote conversation. Having one back at your math calendar/bulletin board area provides you the opportunity to use the chart to engage students in the following kinds of discussions. Ask students to find the numeral that represents:

- the day of the month
- the month of the year
- the number of students in the class
- the number of students absent or any other amount relevant to the moment.

Using the number is 21, give directions and/or ask questions similar to those below.

- Name a number greater than 21.
- Name a number less than 21.
- What number is 3 more than/less than 21?
- What number is 5 more than/less than 21?
- What number is 10 more than/less than 21?
- Is 21 even or odd?

- What numbers live right next door to 21?

Ask students to pick an even number and explain how they know the number is even. Ask students to pick an odd number and explain how they know it is odd. Ask students to count by 2's, 5's or 10's. Tell them to describe any patterns that they see. *(Accept any patterns that students are able to justify. There are many right answers!)*

Games

"A game or other repeatable activity may not look like a problem, but it can nonetheless be problem based. The determining factor is this: Does the activity cause students to be reflective about new or developing relationships? If the activity merely has students repeating procedure without wrestling with an emerging idea, then it is not a problem-based experience. However the few examples just mentioned and many others do have children thinking through ideas that are not easily developed in one or two lessons. In this sense, they fit the definition of a problem-based task.

Just as with any task, some form of recording or writing should be included with stations whenever possible. Students solving a problem on a computer can write up what they did and explain what they learned. Students playing a game can keep records and then tell about how they played the game- what thinking or strategies they used." (Van de Walle and Lovin, Teaching Student-Centered Mathematics: K-3, page 26)

Journaling

"Students should be writing and talking about math topics every day. Putting thoughts into words helps to clarify and solidify thinking. By sharing their mathematical understandings in written and oral form with their classmates, teachers, and parents, students develop confidence in themselves as mathematical learners; this practice also enables teachers to better monitor student progress." NJ DOE

"Language, whether used to express ideas or to receive them, is a very powerful tool and should be used to foster the learning of mathematics. Communicating about mathematical ideas is a way for students to articulate, clarify, organize, and consolidate their thinking. Students, like adults, exchange thoughts and ideas in many ways—orally; with gestures; and with pictures, objects, and symbols. By listening carefully to others, students can become aware of alternative perspectives and strategies. By writing and talking with others, they learn to use more-precise mathematical language and, gradually, conventional symbols to express their mathematical ideas. Communication makes mathematical thinking observable and therefore facilitates further development of that thought. It encourages students to reflect on their own knowledge and their own ways of solving problems. Throughout the early years, students should have daily opportunities to talk and write about mathematics." NCTM

When beginning math journals, the teacher should model the process initially, showing students how to find the front of the journal, the top and bottom of the composition book, how to open to the next page in sequence (special bookmarks or ribbons), and how to date the page. Discuss the usefulness of the book, and the way in which it will help students retrieve their math thinking whenever they need it.
When beginning a task, you can ask, "What do we need to find out?" and then, "How do we figure it out?" Then figure it out, usually by drawing representations, and eventually adding words, numbers, and symbols. During the closing of a task, have students show their journals with a document camera or overhead when they share their thinking. This is an excellent opportunity to discuss different ways to organize thinking and clarity of explanations.

Use a composition notebook (the ones with graph paper are terrific for math) for recording or drawing answers to problems. The journal entries can be from Frameworks tasks, but should also include all mathematical thinking. Journal entries should be simple to begin with and become more detailed as the children's problem-solving skills improve. Children should always be allowed to discuss their representations with classmates if they desire feedback. The children's journal entries demonstrate their thinking processes. Each entry could first be shared with a "buddy" to encourage discussion and explanation; then one or two children could share their entries with the entire class. Don't forget to praise children for their thinking skills and their journal entries! These journals are perfect for assessment and for parent conferencing. The student’s thinking is made visible!

GENERAL QUESTIONS FOR TEACHER USE

Adapted from *Growing Success* and materials from Math GAINS and *TIPS4RM*

Reasoning and Proving

- How can we show that this is true for all cases?
- In what cases might our conclusion not hold true?
- How can we verify this answer?
- Explain the reasoning behind your prediction.
- Why does this work?
- What do you think will happen if this pattern continues?
- Show how you know that this statement is true.
- Give an example of when this statement is false.
- Explain why you do not accept the argument as proof.
- How could we check that solution?
- What other situations need to be considered?

Reflecting

- Have you thought about…?
- What do you notice about…?
- What patterns do you see?
- Does this problem/answer make sense to you?
- How does this compare to…?

- What could you start with to help you explore the possibilities?
- How can you verify this answer?
- What evidence of your thinking can you share?
- Is this a reasonable answer, given that…?
- How can you change . . .

Selecting Tools and Computational Strategies

- How did the learning tool you chose contribute to your understanding/solving of the problem? assist in your communication?
- In what ways would [name a tool] assist in your investigation/solving of this problem?
- What other tools did you consider using? Explain why you chose not to use them.
- Think of a different way to do the calculation that may be more efficient.
- What estimation strategy did you use?

Connections

- What other math have you studied that has some of the same principles, properties, or procedures as this?
- How do these different representations connect to one another?
- When could this mathematical concept or procedure be used in daily life?
- What connection do you see between a problem you did previously and today's problem?

Representing

- What would other representations of this problem demonstrate?
- Explain why you chose this representation.
- How could you represent this idea algebraically? graphically?
- Does this graphical representation of the data bias the viewer? Explain.
- What properties would you have to use to construct a dynamic representation of this situation?
- In what way would a scale model help you solve this problem?
- Show me an example of . . .

QUESTIONS FOR TEACHER REFLECTION

- How did I assess for student understanding?
- How did my students engage in the 8 mathematical practices today?
- How effective was I in creating an environment where meaningful learning could take place?
- How effective was my questioning today? Did I question too little or say too much?
- Were manipulatives made accessible for students to work through the task?
- Name at least one positive thing about today's lesson and one thing you will change.
- How will today's learning impact tomorrow's instruction?

MATHEMATICS DEPTH-OF-KNOWLEDGE LEVELS

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a Level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret skills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

DOK cont'd…

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas *within* the content area or *among* content areas—and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

DEPTH AND RIGOR STATEMENT

By changing the way we teach, we are not asking children to learn less, we are asking them to learn more. We are asking them to mathematize, to think like mathematicians, to look at numbers before they calculate, to think rather than to perform rote procedures. Children can and do construct their own strategies, and when they are allowed to make sense of calculations in their own ways, they understand better. In the words of Blaise Pascal, "We are usually convinced more easily by reasons we have found ourselves than by those which have occurred to others."

By changing the way we teach, we are asking teachers to think mathematically, too. We are asking them to develop their own mental math strategies in order to develop them in their students.

Catherine Twomey Fosnot and Maarten Dolk, *Young Mathematicians at Work.*

While you may be tempted to explain and show students how to do a task, much of the learning comes as a result of making sense of the task at hand. Allow for the productive struggle, the grappling with the unfamiliar, the contentious discourse, for on the other side of frustration lies understanding and the confidence that comes from "doing it myself!"

Problem Solving Rubric (K-2)

SMP	1-Emergent	2-Progressing	3- Meets/Proficient	4-Exceeds
Make sense of problems and persevere in solving them.	The student was unable to explain the problem and showed minimal perseverance when identifying the purpose of the problem.	The student explained the problem and showed some perseverance in identifying the purpose of the problem, and selected and applied an appropriate problem solving strategy that lead to a partially complete and/or partially accurate solution.	The student explained the problem and showed perseverance when identifying the purpose of the problem, and selected an applied and appropriate problem solving strategy that lead to a generally complete and accurate solution.	The student explained the problem and showed perseverance by identifying the purpose of the problem and selected and applied an appropriate problem solving strategy that lead to a thorough and accurate solution.
Attends to precision	The student was unclear in their thinking and was unable to communicate mathematically.	The student was precise by clearly describing their actions and strategies, while showing understanding and using appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using grade-level appropriate vocabulary in their process of finding solutions.	The student was precise by clearly describing their actions and strategies, while showing understanding and using above-grade-level appropriate vocabulary in their process of finding solutions.
Reasoning and explaining	The student was unable to express or justify their opinion quantitatively or abstractly using numbers, pictures, charts or words.	The student expressed or justified their opinion either quantitatively OR abstractly using numbers, pictures, charts OR words.	The student expressed and justified their opinion both quantitatively and abstractly using numbers, pictures, charts and/or words.	The student expressed and justified their opinion both quantitatively and abstractly using a variety of numbers, pictures, charts and words.
Models and use of tools	The student was unable to select an appropriate tool, draw a representation to reason or justify their thinking.	The student selected an appropriate tools or drew a correct representation of the tools used to reason and justify their response.	The student selected an efficient tool and/or drew a correct representation of the efficient tool used to reason and justify their response.	The student selected multiple efficient tools and correctly represented the tools to reason and justify their response. In addition this students was able to explain why their tool/ model was efficient
Seeing structure and generalizing	The student was unable to identify patterns, structures or connect to other areas of mathematics and/or real-life.	The student identified a pattern or structure in the number system and noticed connections to other areas of mathematics or real-life.	The student identified patterns or structures in the number system and noticed connections to other areas of mathematics and real-life.	The student identified various patterns and structures in the number system and noticed connections to multiple areas of mathematics and real-life.

SUGGESTED LITERATURE

- *A Place for Zero* by Angeline LoPresti
- *A Remainder of One* by Elinor J. Pinczes
- *Bunny Money* by Rosemary Wells
- *Captain Invincible and the Space Shapes* by Stuart J. Murphy
- *Each Orange has 8 slices: A Counting Book* by P. Giganti
- *Gator Pie* by Louise Mathews
- *How Big Is a Foot?* by Rolf Myller
- *Inch by Inch* by Leo Lionni
- *It's About Time!* By Stuart J. Murphy
- *Jim and the Beanstalk* by Raymond Briggs
- *Lemonade for Sale* by Stuart Murphy
- *Math Appeal* by Greg Tang
- *Measuring Penny* by Loreen Leedy
- *Mouse Count* by Ellen Stoll Walsh
- *Ready, Set, Hop* by Stuart Murphy
- *The Greedy Triangle* by Marilyn Burns
- *The Grouchy Lady Bug* by Eric Carle
- *The Shape of Things* by Dayle Ann Dodds
- *Twelve Snails to One Lizard* by Susan Highower
- *What Comes in 2's, 3's, and 4's?* by Suzanne Aker
- *When a Line Bends, a Shape Begins* by Rhonda G. Greene

TECHNOLOGY LINKS

- http://www.gpb.org/education/common-core
- http://catalog.mathlearningcenter.org/apps
- http://nzmaths.co.nz/digital-learning-objects
- http://www.fi.uu.nl/toepassingen/00203/toepassing_rekenweb.xml?style=rekenweb&language=en&use=game
- https://www.georgiastandards.org/resources/Pages/Tools/LearningVillage.aspx
- https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
- https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

RESOURCES CONSULTED

Content:
Ohio DOE
http://www.ode.state.oh.us/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704
Arizona DOE
http://www.azed.gov/standards-practices/mathematics-standards/
Nzmaths
http://nzmaths.co.nz/

Teacher/Student Sense-making:
http://www.youtube.com/user/mitcccnyorg?feature=watch
http://www.insidemathematics.org/index.php/video-tours-of-inside-mathematics/classroom-teachers/157-teachers-reflect-mathematics-teaching-practices
https://www.georgiastandards.org/Common-Core/Pages/Math.aspx
or http://secc.sedl.org/common_core_videos/

Journaling:
http://www.kindergartenkindergarten.com/2010/07/setting-up-math-problemsolving-notebooks.html

Community of Learners:
http://www.edutopia.org/math-social-activity-cooperative-learning-video
http://www.edutopia.org/math-social-activity-sel
http://www.youtube.com/user/responsiveclassroom/videos
http://www.responsiveclassroom.org/category/category/first-weeks-school