| **Unit 1** | **Unit 2** | **Unit 3** | **Unit 4** | **Unit 5** | | **Unit 6** | **Unit 7** | **Unit 8** | **Unit 9** | | **Unit 10** | **Unit 11** | **Unit 12** | **Unit 13** |
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| **Properties of Functions through Quadratics** | **Complex Number System** | **Polynomial Expressions and Equations** | **Rational Expressions and Equations** | **Radical Expressions and Equations** | | **Polynomial Functions** | **Systems of Equations** | **Rational Exponents** | **Geometric Series/Sequence to Functions** | | **Exponential and Logarithmic Equations and Functions** | **Trigonometric Functions** | **Univariate Statistics** | **Probability** |
| **14 days** | **9 days** | **11 days** | **9 days** | **13 days** | | **14 days** | **11 days** | **11 days** | **12 days** | | **19 days** | **19 days** | **11 days** | **14 days** |
| F-IF.B.6 | A-REI.B.4b | A-SSE.A.2 | A-REI.A.1 | A-REI.A.1 | | A-APR.B.2 | A-REI.D.11 | N-RN.A.1 | A-SSE.B.4 | | N-Q.A.2 | F-IF.B.4 | S-IC.B.3 | S-CP.A.1 |
| F-BF.B.3 | N-CN.A.1 | A-APR.C.4 | A-REI.A.2 | A-REI.A.2 | | A-APR.B.3 | A-REI.C.6 | N-RN.A.2 | F.BF.A.1a | | F-IF.C.7e | F-IF.C.7e | S-IC.B.4 | S-CP.A.2 |
| F-BF.B.4a | N-CN.A.2 |  | A-APR.D.6 |  | | F-IF.B.4 | A-REI.C.7 | A-SSE.B.3c | F.BF.A.2 | | F-IF.C.8b | F-TF.A.1 | S-IC.B.5 | S-CP.A.3 |
| G-GPE.A.2 | N-CN.C.7 |  | A-CED.A.1 |  | | F-IF.C.7c |  |  | F.IF.A.3 | | F-LE.A.4 | F-TF.A.2 | S-IC.B.6 | S-CP.A.4 |
|  |  |  |  |  | | F-IF.C.9 |  |  | F-LE.A.2 | | S-ID.B.6a | F-TF.B.5 | S-IC.A.1 | S-CP.A.5 |
|  |  |  |  |  | |  |  |  |  | | F-LE.B.5 | F-TF.C.8 | S-IC.A.2 | S-CP.B.6 |
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| **Major Clusters** | | | | | **Supporting Clusters** | | | | | **Additional Clusters** | | | | |
| **N-RN** The Real Number System (1, 2)  **A-SSE** Seeing Structure in Expressions (2, 3, 4)  **A-APR** Arithmetic with Polynomials and Rational Expressions (2, 3)  **A-REI** Reasoning with Equations and Inequalities (1, 2, 11)  **F-IF** Interpreting Functions (4, 6)  **F-BF** Building Functions (1, 2)  **S-IC** Making Inferences and Justifying Conclusions (3, 4, 5, 6) | | | | | **N-Q** Quantities (2)  **A-APR** Arithmetic with Polynomials and Rational Expressions (6)  **A-CED** Creating Equations (1)  **A-REI** Reasoning with Equations and Inequalities (4)  **F-IF** Interpreting Functions (3, 7, 8, 9)  **F-LE** Linear, Quadratic, and Exponential Models (2, 4)  **S-ID** Interpreting Categorical and Quantitative Data (6)  **S-IC** Making Inferences and Justifying Conclusions (1, 2) | | | | | **N-CN** The Complex Number System (1, 2, 7)  **A-APR** Arithmetic with Polynomials and Rational Expressions (4)  **A-REI** Reasoning with Equations and Inequalities (6, 7)  **F-BF** Building Functions (3, 4a)  **F-LE** Linear, Quadratic, and Exponential Models (5)  **F-TF** Trigonometric Functions (1, 2, 5, 8)  **G-GPE** Expressing Geometric Properties with Equations (2)  **S-ID** Interpreting Categorical & Quantitative Data (4)  **S-CP** Conditional Probability and the Rules of Probability (1, 2, 3, 4, 5, 6, 7) | | | | |

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| **Summary of Year for Algebra II** |
| This course is designed to extend and apply students’ understanding of functions and the connection between algebraic skills and functions. Students come into this course with a solid foundation of linear and quadratic functions and having been exposed to exponential, absolute value, piecewise, and simple radical functions. This course emphasizes polynomial, exponential, logarithmic, and trigonometric functions. Students will also have the opportunity to master more sophisticated algebraic skills and apply many of them to their work with functions. In addition to the algebraic skills the students will apply to functions, the students will work with rational and radical expressions and equations. Furthermore, students will reengage with univariate statistics and apply their work of inferences, conclusions, and possible solutions to real-world problems. Bivariate statistics will provide students a meaningful context in which to apply their understanding of functions to real-world problems and data sets and use those functions to interpolate and extrapolate data points. Finally, students will extend their understanding of probability from the 7th grade to work with conditional probabilities and probability rules. |
| **Standards Clarification for Algebra II** |
| Some standards may be revisited several times during the course; others may by only partially addressed in different units, depending on the focus of the unit. Comments are included throughout the document in the **Standards Clarification** column to clarify and provide additional background for each unit.  Please refer to the [PARCC Model Content Frameworks](http://www.parcconline.org/parcc-model-content-frameworks) for more information concerning the Assessment Limits for Standards Assessed on More Than One End-of-Course Test. This information can be found on pages 55-59 of the frameworks. |
| **Mathematical Practices Recommendations for Algebra II** |
| * + While all of the mathematical practice standards are important in all three high school courses, the four below are especially important in Algebra II:   + MP.3: Construct viable arguments and critique the reasoning of others.   + MP.6: Attend to precision.   + MP.7: Look for and make use of structure   + MP.8: Look for and express regularity in repeated reasoning. * PARCC also makes the following recommendations for additional mathematical practices relative to Algebra II:   + MP.1: Make sense of problems and persevere in solving them. Algebra II is a course in which students can learn some technical methods for performing algebraic calculations and transformations, but sense-making is still paramount   + MP.4: Model with mathematics. Specific modeling standards appear throughout the high school standards indicated by a star symbol (★).   + MP.5: Use appropriate tools strategically. Computer algebra systems provide students with a tool for modeling all kinds of phenomena, experimenting with algebraic objects (e.g., sequences of polynomials), and reducing the computational overhead needed to investigate many classical and useful areas of algebra. |
| **Fluency Requirements for Algebra II** |
| * HSA-APR.D.6 This standard sets an expectation that students will divide polynomials with remainder by inspection in simple cases. For example, one can view the rational expression as . * HSA-SSE.A.2 The ability to see structure in expressions and to use this structure to rewrite expressions is a key skill in everything from advanced factoring (e.g., grouping) to summing series to the rewriting of rational expressions to examine the end behavior of the corresponding rational function. * HSF-IF.A.3 Fluency in translating between recursive definitions and closed forms is helpful when dealing with many problems involving sequences and series, with applications ranging from fitting functions to tables to problems in finance. |

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| **Unit 1: Properties of Functions Through Quadratics** | | | **Possible time frame**:  14 days |
| There are several concepts that students will apply to all of the major function types discussed in this course. Because students will come into Algebra II with a good understanding of quadratic functions and their graphs, quadratics will be the starting point to introduce these function concepts. This unit will be a place to reengage students with the Algebra I concepts of factoring, completing the square, and solving quadratic equations (to find x-intercepts). Beyond graphing, students will derive the equation of quadratic functions given a focus and directrix. While working with quadratics the students will use function transformations, estimate rates of change, calculate average rates of change on an interval, and begin to explore the idea of inverse functions. | | | |
| **Major Cluster Standards** | | **Standards Clarification** | |
| **Interpret functions that arise in applications in terms of the context**  **HSF-IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ | | ★Modeling standard  **HSF-IF.B.6** This skill (finding average rate of change on an interval) should appear again in the units on polynomial, exponential, logarithmic, and trig functions (Units 6, 10, and 11). However, this standard is only listed in Unit 1 because the skill needed will be fully taught in this unit and will then be applied to the other function types throughout the year. | |
| **Additional Cluster Standards** | | **Standards Clarification** | |
| **Build new functions from existing functions**  **HSF-BF.B.3** Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*  **HSF-BF.B.4** Find inverse functions.   1. Solve an equation of the form f(x) = c for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x–1) for x* ≠ *1.*   **Translate between the geometric description and the equation for a conic section**  **HSG-GPE.A.2** Derive the equation of a parabola given a focus and directrix. | | **HSF-BF.B.3**The idea of function transformations is first introduced in Algebra I and will continue to be developed through quadratics. An understanding of function transformations will aid students in graphing all function types throughout the course.  **HSF-BF.B.4a** This standard is only a very basic introduction to inverse functions. | |
| **Applying Mathematical Practices to CCSS** | | | |
| **MP.4** Model with mathematics. | Work with modeling and fitting quadratic data in contextual situations will help students demonstrate **MP.4**. As students derive the equation of a parabola using the focus and directrix, they will need to make use of geometric structures such as the distance formula (**MP.7**). | | |
| **MP.7** Look for and make use of structure. |

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| **Unit 2: Complex Number System** | | **Possible time frame**:  9 days |
| To connect with Unit 1, students will begin by exploring quadratics with no real zeros to highlight the need for numbers beyond real numbers. The set of imaginary numbers will be introduced and combined with real numbers to form the set of complex numbers. Students will understand the concept of the imaginary unit *i* such that *i*2 = -1 and solve quadratic equations with complex solutions. Additionally, students will use these concepts to perform operations with complex numbers (excluding division). | | |
| **Supporting Cluster Standards** | | **Standards Clarification** |
| **Solve equations and inequalities in one variable**  **HSA-REI.B.4** Solve quadratic equations in one variable.   1. Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. | |  |
| **Additional Cluster Standards** | | **Standards Clarification** |
| **Perform arithmetic operations with complex numbers.**  **HSN-CN.A.1** Know there is a complex number *i* such that *i*2 = –1, and every complex number has the form *a* + *bi* with *a* and *b* real.  **HSN-CN.A.2** Use the relation *i*2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.  **Use complex numbers in polynomial identities and equations.**  **HSN-CN.C.7** Solve quadratic equations with real coefficients that have complex solutions. | |  |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.2** Reason abstractly and quantitatively**.** | As students are introduced to the complex numbers they will need to decontextualize in order to represent and operate on complex numbers (**MP.2**). As students solve quadratic equations, they should learn to recognize the structures that would allow them to determine the type of solutions prior to obtaining the final solution (**MP.7**). | |
| **MP.7** Look for and make use of structure. |

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| **Unit 3: Polynomial Expressions and Equations** | | **Possible time frame**:  11 days |
| In this unit working with polynomials will move beyond quadratics. Students will begin to work with polynomial expressions and equations and will prove polynomial identities. A common idea throughout the middle/high school math standards is using the structure of expressions to gain insight into equivalent expressions and insight into the meaning of the expressions. This will be the first application of this idea in the Algebra II course. Additionally, students will continue to develop and extend their understanding of factoring and will use the zero product property to solve polynomial equations. | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Interpret the structure of expressions**  **HSA-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. *For example, see x*4 – *y*4 *as* (*x*2)2 – (*y*2)2, *thus recognizing it as a difference of squares that can be factored as* (*x*2 – *y*2)(*x*2 + *y*2). | | **HSA-SSE.A.2** Students will continue to use structure to provide insight into expressions when working with rational and exponential expressions in Units 4 and 10. |
| **Additional Cluster Standards** | | **Standards Clarification** |
| **Use polynomial identities to solve problems**  **HSA-APR.C.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* (*x*2 + *y*2)2 = (*x*2 – *y*2)2 + (2*xy*)2 *can be used to generate Pythagorean triples.* | |  |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.3** Construct viable arguments and critique the reasoning of others. | Students will use the structure of expressions (**MP.7**) to assist them in proving the polynomial identities, and opportunity to provide convincing arguments and critique others proofs (**MP.3**). | |
| **MP.7** Look for and make use of structure. |

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| **Unit 4: Rational Expressions and Equations** | | **Possible time frame**:  9 days |
| In this unit students will begin to work with simple rational expressions and equations. Long division and inspection will be used to rewrite rational expressions. Rational equations will be created and used to solve problems. Additionally, students will explain the process of and justify the steps in solving rational equations. They will explain and provide examples of situations in which extraneous solutions may arise. (Students will not be expected to add, subtract, multiply, or divide rational expressions.) | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Understand solving equations as a process of reasoning and explain the reasoning**  **HSA-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  **HSA-REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | | Students will continue to use structure to provide insight into expressions in the context of rational expressions. (HSA-SSE.A.2 from Unit 3)  **HSA-REI.A.2** Work in this unit is limited to rational expressions and equations. |
| **Supporting Cluster Standards** | | **Standards Clarification** |
| **Rewrite rational expressions**  **HSA-APR.C.6** Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.  **Create equations that describe numbers or relationships**  **HSA-CED.A.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* ★ | | ★Modeling standard  **HSA-CED.A.1** Students will solve problems with exponential equations in Unit 10. |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.1** Make sense of problems and persevere in solving them. | Students are making sense of problems and persevering in their solutions (**MP.1**) when creating equations to solve problems (**HAS-CED.A.1**). Students may use technology to verify extraneous solutions (**MP.5**) as they describe how extraneous solutions may arise. Students will also practice looking for and making use of structure (**MP.7**) as they rewrite rational functions. | |
| **MP.5** Use appropriate tools strategically. |
| **MP.7** Look for and make use of structure. |

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| **Unit 5: Radical Expressions and Equations** | | **Possible time frame**:  13 days |
| In this unit students will continue to master more sophisticated algebra skills. While the students will be familiar with simple radical functions from Algebra I, now they will work with more complicated radical equations. The expectations in this unit are analogous to Unit 4. Students will explain the process of and justify the steps in solving radical equations. They will explain and provide examples of situations in which extraneous solutions may arise. | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Understand solving equations as a process of reasoning and explain the reasoning**  **HSA-REI.A.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  **HSA-REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | | **HAS-REI.A.2** The focus in this unit is on radical equations. |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.2** Reason abstractly and quantitatively. | Students build proficiency with **MP.2** and **MP.5** as they operate, solve, and evaluate the reasonableness of solutions. Students may critique the work of others or other students’ reasoning as they describe their solution methods (**MP.3**). | |
| **MP.3** Construct viable arguments and critique the reasoning of others. |
| **MP.5** Use appropriate tools strategically. |

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| **Unit 6: Polynomial Functions** | **Possible time frame**:  14 days |
| Now that students have more skill with algebraic manipulation, the focus will shift back to functions. In this unit students start to explore polynomial functions. The connection between zeros of a function and factors of that function will be explored. For quartic functions (and those with lesser degree), students will use intercepts (when suitable factorizations are available) and end behavior to create rough graphs of the functions. Additionally, for a function modeling real-life quantities, students will interpret key features of the function in terms of the provided context. Key features could include intercepts, intervals where the function is increasing/decreasing/positive/negative, relative extreme, symmetries, end behavior, etc. | |
| **Major Cluster Standards** | **Standards Clarification** |
| **Understand the relationship between zeros and factors of polynomials**  **HSA-APR.B.2** Know and apply the Remainder Theorem: For a polynomial *p*(*x*) and a number *a*, the remainder on division by *x* – *a* is *p*(*a*), so *p*(*a*) = 0 if and only if (*x* – *a*) is a factor of *p*(*x*).  **HSA-APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.  **Interpret functions that arise in applications in terms of the context**  **HSF-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ | ★Modeling standard  In this unit students should continue to find average rate of change on a given interval. (HSF-IF.B.6 from Unit 1)  **HSF-IF.B.4** Students will continue to interpret key features of functions which relate quantities in terms of the provided context for exponential, logarithmic, and trigonometric functions in Units 10 and 11. |
| **Supporting Cluster Standards** | **Standards Clarification** |
| **Analyze functions using different representations**  **HSF-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★   1. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.   **HSF-IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* | ★Modeling standard  **HSF-IF.C.9** Students will continue to compare properties of functions represented in different ways in Units 10 and 11. |

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| **Applying Mathematical Practices to CCSS** | |
| **MP.5** Use appropriate tools strategically. | Students will investigate and build polynomial functions that precisely communicate different characteristics using technology (**MP.5**). With their work with factoring polynomials and proving polynomial identities, students will become proficient with making use of structure and repeated reasoning (**MP.7** and **MP.8**). |
| **MP.7** Look for and make use of structure. |
| **MP.8** Look for and express regularity in repeated reasoning. |

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| **Unit 7: Systems of Equations** | | **Possible time frame**:  11 days |
| Systems of equations are first introduced in 8th grade with 2x2 systems of linear equations. In Algebra I this concept is extended to include 2x2 systems of linear inequalities. In this unit the concept of systems will be continued and students will solve 3x3 systems of linear equations and systems consisting of one linear and one quadratic equation. Additionally, students will explain why the x-coordinates of the points of intersection of y=f(x) and y=g(x) are the solutions to the equation f(x)=g(x). Graphical and algebraic methods will be used for linear/quadratic systems. Approximation techniques will be used for other function types. | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Represent and solve equations and inequalities graphically**  **HSA-REI.D.11** Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ | | ★Modeling standard  **HSA-REI.D.11** Functions should be limited to only those studied thus far. |
| **Additional Cluster Standards** | | **Standards Clarification** |
| **HSA-REI.C.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.  **HSA-REI.C.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y* = –*3x* and the circle *x*2 + *y*2 = *3.* | |  |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.2** Reason abstractly and quantitatively. | Students will use technology as one way to find the solutions to systems of equations (**MP.5**). Students will also need to attend to precision and determine whether the solutions they find make sense as they approximate solutions to these systems (**MP.2** and **MP.6**). | |
| **MP.5** Use appropriate tools strategically. |
| **MP.6** Attend to precision. |

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| **Unit 8: Rational Exponents** | | **Possible time frame**:  11 days |
| In Algebra I students work with linear and exponential relationships; however, students do not graph exponential functions. Algebra I students can only work with exponential functions with integer domains because the concept of rational exponents has not yet been introduced. In this unit students will be introduced to the concept of rational exponents and their connection to radicals to allow for work with exponential and logarithmic functions in future units. The primary focus of this unit is to extend the properties of integer exponents to rational exponents, use those properties to rewrite expressions in equivalent forms, and use the structure of expressions and equations with radicals or rational exponents in a real-world context. | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Extend the properties of exponents to rational exponents.**  **HSN–RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5.*  **HSN–RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.  **Write expressions in equivalent forms to solve problems**  **HSA-SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★   1. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* (1.151/12)12*t* ≈ 1.01212t *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* | | **HSA-SSE.B.3c** Students will continue to rewrite expression in different forms to solve problems involving a geometric series in Unit 9 and involving exponential functions in Unit 10. |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.3** Construct viable arguments and critique the reasoning of others. | Students will use the structure of expressions to solve problems (**MP.7**) and explain the connection between radicals and rational exponents (**MP.3**). | |
| **MP.7** Look for and make use of structure. |

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| **Unit 9: Geometric Series/Sequence to Functions** | | **Possible time frame**:  12 days |
| In this unit students will work with arithmetic and geometric sequences. The work with geometric sequences will be a basic introduction to exponential functions with various whole number bases. Students will recognize that sequences are functions (sometimes defined recursively) whose domain is a subset of the integers. The focus will be on determining recursively and/or explicitly defined sequences from information about a real-world context or to solve real-world problems. Students will also convert between recursively and explicitly defined sequences. Additionally, students will derive the formula for the sum of a finite geometric series and use the formula to solve problems. | | |
| **Major Cluster Standards** | **Standards Clarification** | |
| **Write expressions in equivalent forms to solve problems**  **HSA-SSE.B.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★  **Build a function that models a relationship between two quantities**  **HSF-BF.A.1** Write a function that describes a relationship between two quantities.★   1. Determine an explicit expression, a recursive process, or steps for calculation from a context.   **HSF-BF.A.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ | ★Modeling standard  Students will continue to rewrite expressions involving exponents in different forms to solve problems. (A-SSE.B.3c from Unit 8)  **HSF.BF.A.1a** Students will continue to determine an explicit expression, a recursive process, or steps for calculation from a given context as they work with exponential functions in Unit 10. | |
| **Supporting Cluster Standards** | **Standards Clarification** | |
| **Understand the concept of a function and use function notation**  **HSF-IF.A.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n* ≥ *1.*  **Construct and compare linear, quadratic, and exponential models and solve problems**  **HSF-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★ | ★Modeling standard  **HSF.LE.A.2** Students will continue to construct exponential functions given a graph, description, or table of values in Unit 10. | |

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| **Applying Mathematical Practices to CCSS** | |
| **MP.6** Attend to precision. | Encourage students to use precise vocabulary with discussions of the formulas they create (**MP.6**). To write functions that generate sequences, students must look for common ratios and constant addendum (**MP.7**). In derivations for the explicit formulas, recursive formulas, and sums of finite series students will benefit from the practice of examining repeated calculations and look for shortcuts (**MP.8**). |
| **MP.7** Look for and make use of structure. |
| **MP.8** Look for and express regularity in repeated reasoning. |

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| **Unit 10: Exponential and Logarithmic Equations and Functions** | | | **Possible time frame**:  19 days |
| Information from previous units culminates in fully utilizing exponential and logarithmic functions. Students should be familiar with rational exponents and experimenting with geometric sequences that will give them insight into functions with a variable as the exponent. Many of the skills applied to functions throughout the year will be reinforced in this unit. These standards are not listed again as they are not the focus of this unit, but instead they will help to support creating, graphing, and using exponential and logarithmic functions to solve problems. Students will explore the connection between exponential and logarithmic functions and will use logarithms to express solutions to exponential equations with bases 2, 10, or *e*. Students will also graph exponential and logarithmic functions and will determine appropriate key features of the function and/or graph to solve problems. Additionally, students will begin to fit functions to data and will informally assess the function type and fit it to the data. | | | |
| **Supporting Cluster Standards** | | **Standards Clarification** | |
| **Reason quantitatively and use units to solve problems.**  **HSN-Q.A.2** Define appropriate quantities for the purpose of descriptive modeling. ★  **Analyze functions using different representations**  **HSF-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★   1. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.   **HSF-IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.   1. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.*   **Construct and compare linear, quadratic, and exponential models and solve problems**  **HSF-LE.A.4** For exponential models, express as a logarithm the solution to *ab*ct = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. ★  **Summarize, represent, and interpret data on two categorical and quantitative variables**  **HSS-ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.   1. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* | | ★Modeling standard  Students will continue to apply the concepts of the following standards from previous units:   * HSA-SSE.A.2 * HSA-SSE.B.3 * HSA-CED.A.1 * HSF-IF.B.4 * HSF-IF.B.6 * HSF-IF.C.9 * HSF-BF.A.1a * HSF-LE.A.2   **HSN-Q.A.2** Students will continue to define appropriate quantities for descriptive modeling (by autonomously selecting the characteristics of a graph most appropriate to solve specific problems) in Unit 11.  **HSS-ID.B.6a** Students will continue to fit function to data including Trigonometric functions in Unit 11. | |
| **Additional Cluster Standards** | | **Standards Clarification** | |
| **Interpret expressions for functions in terms of the situation they model**  **HSF-LE.B.5** Interpret the parameters in a linear or exponential function in terms of a context. ★ | | ★Modeling standard  **HSF-LE.B.5** Students continue to interpret parameters of functions in terms of the provided real-world context in Unit 11. | |
| **Applying Mathematical Practices to CCSS** | | | |
| **MP.2** Reason abstractly and quantitatively. | Students will investigate data that can be modeled with exponential and logarithmic functions (**MP.4**). When calculating logarithms and exponents, students must recognize the importance of maintaining the mantissa and significant digits (**MP.6**). Students will understand how to flexibly utilize properties of logarithmic operations while operating on exponential and logarithmic equations (**MP.2**). | | |
| **MP.4** Model with mathematics. |
| **MP.6** Attend to precision. |

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| **Unit 11: Trigonometric Functions** | | **Possible time frame**:  19 days |
| As in Unit 10 students will continue to use many of the skills developed in previous units as they explore trigonometric functions. In Geometry students will have used basic trigonometric ratios to solve problems involving right triangles. This unit will be the first introduction to the concept of a radian as an angle measure. Students will understand the radian measure of an angle as the length of the arc on the unit circle subtended by the angle. Students will understand the unit circle and its usefulness to extend trigonometric functions to all real numbers. Additionally, students will prove the Pythagorean identity sin2(θ) + cos2(θ) = 1. Finally, students will graph trigonometric functions, interpret key characteristics of the function and/or graph, use the functions and/or graphs to solve problems, and fit trigonometric functions to given data. | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Interpret functions that arise in applications in terms of the context**  **HSF-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ | | ★Modeling standard  Students will continue to apply the concepts of the following standards from previous units:   * HSN-Q.A.2 * HSF-IF.B.4 * HSF-IF.B.6 * HSF-IF.C.9 * HSF-LE.B.5 * HSS-ID.B.6a |
| **Supporting Cluster Standards** | | **Standards Clarification** |
| **Analyze functions using different representations**  **HSF-IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★   1. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | |  |
| **Additional Cluster Standards** | | **Standards Clarification** |
| **Extend the domain of trigonometric functions using the unit circle**  **HSF-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.  **HSF-TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.  **Model periodic phenomena with trigonometric functions**  **HSF-TF.B.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★  **Prove and apply trigonometric identities**  **HSF-TF.C.8** Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. | |  |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.4** Model with mathematics. | In the extension of the trigonometric functions to the unit circle, proficient students must use repeated reasoning (**MP.8**). Students will model real-world situations with trigonometric functions (**MP.4**). Use of trigonometric vocabulary, (such as amplitude, frequency, period, midline, degree, and radian) aid in communicating precisely (**MP.6**). | |
| **MP.6** Attend to precision. |
| **MP.8** Look for and express regularity in repeated reasoning. |

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| **Unit 12: Univariate Statistics** | | **Possible time frame**:  11 days |
| Students are first introduced to univariate statistics in 6th grade and work with univariate and/or bivariate statistics in 6th grade, 7th grade, 8th grade, and Algebra I. In Algebra II students will work with bivariate statistics as they fit data with different types of functions in Units 10 and 11. In Unit 12 students will focus on univariate statistics. Students will explain the basis of statistics as predicting population characteristics from a sample and will use sample data to make predictions about a population for normally distributed data. Students will understand the differences and similarities among sample surveys, experiments, and observational studies and will explain the role of randomization in each. Also, students will compare and evaluate given models with experimental data and use data and their knowledge of statistics to evaluate reports. Additionally, students will use data to determine if a treatment has resulted in a significant difference between parameters. | | |
| **Major Cluster Standards** | | **Standards Clarification** |
| **Make inferences and justify conclusions from sample surveys, experiments, and observational studies**  **HSS-IC.B.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.  **HSS-IC.B.4.** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.  **HSS-IC.B.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.  **HSS-IC.B**.**6** Evaluate reports based on data. | |  |
| **Supporting Cluster Standards** | | **Standards Clarification** |
| **Understand and evaluate random processes underlying statistical experiments**  **HSS-IC.A.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.  **HSS-IC.A.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* | |  |
| **Additional Cluster Standards** | | Standards Clarification |
| **HSS-ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.** | |  |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.1** Make sense of problems and persevere in solving them. | Students must follow a reasoning process in order to develop a sound statistical study and to plan and evaluate simulations (**MP.1**). Have students communicate, justify, and ask questions to improve the designs of statistical studies (**MP.3**). Students will also critique the reasoning of others as they work with data to make inferences and justify conclusions (**MP.3**). Students will continue to increase their technology proficiencies as they consider the role of technology in randomization and perform simulations. | |
| **MP.3** Construct viable arguments and critique the reasoning of others. |
| **MP.5** Use appropriate tools strategically. |

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| **Unit 13: Probability** | | **Possible time frame**:  14 days |
| Students will be introduced to simple and compound probability in 7th grade. Probability is not present in any other grade until Algebra II. Students will describe simple and compound events as subsets of the sample space; however, the focus of probability in this course is on independence and conditional probability. Students will explain the concept of independent events and conditional probability in everyday language and using real-world examples. Students will understand that events A and B are independent if P(A)·P(B) = P(A and B), and that events A and B are independent if P(A|B) = P(A). Students will explain the conditional probability of A given B as P(A and B) / P(B) and calculate conditional probabilities. Students should also use and interpret the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B). Additionally, students will construct and interpret two-way frequency table (based on a sample) and use then to determine independence of events and approximate conditional probabilities of the population from which the sample is drawn. (Students will *not* be expected use the general Multiplication Rule or combination/permutations to solve problems.) | | |
| **Additional Cluster Standards** | | **Standards Clarification** |
| **Understand independence and conditional probability and use them to interpret data**  **HSS-CP.A.1** Describe events as subsets of a sample space (the set of outcomes)using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).  **HSS-CP.A.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.  **HSS-CP.A.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.  **HSS-CP.A.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*  **HSS-CP.A.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*  **Use the rules of probability to compute probabilities of compound events in a uniform probability model**  **HSS-CP.B.6** Find the conditional probability of *A* given *B* as the fraction of *B*’s outcomes that also belong to *A,* and interpret the answer in terms of the model.  **HSS-CP.B.7** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. | |  |
| **Applying Mathematical Practices to CCSS** | | |
| **MP.1** Make sense of problems and persevere in solving them. | Students should be encouraged to persevere when problem solving in this unit. Multiple solutions are common and should be recognized. Students can often make sense of complex contextual probabilities by considering a simpler analogous probability situation (**MP.1**). As students work to identify events for which probabilities are to be determined and rules to apply, encourage students to verify and critique the thinking of their classmates (**MP.3**). Students have the opportunity to demonstrate proficiency with **MP.6** by paying close attention to precise use of new vocabulary and writing complete sentences describing probabilities. | |
| **MP.3** Construct viable arguments and critique the reasoning of others. |
| **MP.6** Attend to precision. |