Iowa Core Mathematics Content and Practice Shifts
Grades K-5

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The purpose of this document is to highlight **content** and **practice shifts** in the K-5 school curriculum as presented in Iowa Core Mathematics. **The magnitude of the identified shifts may vary depending on a district's past enacted curriculum and classroom practice.**

Iowa Core Mathematics emphasizes balance among conceptual understanding, fluency, and application. Instruction focuses on developing a deep conceptual understanding of mathematical content prior to expecting fluency. Students engage with meaningful real-world and mathematical problems while working to develop their conceptual understanding. With a growing conceptual understanding, students develop increasingly efficient and fluent methods to solve a greater variety of problems. This sequence and relationship among conceptual understanding, fluency, and application is evident throughout every domain of the K-5 standards. As you read and interpret this document it is important to understand the balance among these three components.

The following table presents highlighted content and practice shifts. The table consists of three columns: (1) Shifts in Content and Practice, (2) Contrasts to the Shifts, and (3) Domains and Standards. The Shifts in Content and Practice column includes descriptions of major shifts across the K-5 grade levels organized by mathematical topics. This column also includes links to related Learning Progressions documents. The Learning Progressions are a series of documents written by the Common Core authors intended to further describe the standards and how they progress across grade levels. The Contrasts to the Shifts column describes typical practice often observed in elementary mathematics classrooms in the United States. The Domains and Standards column gives reference to the specific Iowa Core Mathematics standards related to the addressed mathematical topic. Many of the content and instructional shifts are closely related and overlap; therefore, the same standard may be listed for more than one shift.

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Counting and Cardinality		
Shift to recognizing a robust understanding of counting and cardinality involves multiple interconnected components. Students need ample opportunities over time to experience the many facets of counting.	Students have lots of opportunities to count orally in large groups. Teachers assume students have a	Counting & Cardinality K.CC.1-7

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Four examples include: • Students understand each successive number refers to a quantity that is one more than the preceding number. • Students are able to count beginning with a number other than 1. • Students count collections in different arrangements, such as a straight line, circle, scattered, etc. • Students count sets of objects as well as counting a specific quantity from a larger set.	robust understanding of counting and number relationships when they know the counting sequence, and they are able to count a group of objects using one-to-one correspondence.	Number & Operations in Base Ten 1.NBT.1 2.NBT.2 Operations & Algebraic Thinking 1.OA.5
Shift to recognizing the importance of counting and cardinality in developing understanding of numbers and operations. Two examples include: • A natural extension of cardinality is subitizing, which is recognizing how many objects are in a small set without counting (perceptual subitizing). Once students can subitize they can began to see a larger collection as two or more subsets (conceptual subitizing). This understanding supports the development of basic fact strategies. • Other extensions of cardinality are the ability to count forward from any number and the understanding that each successive number refers to a quantity that is one more than the preceding number. This combined understanding relates counting to addition and subtraction. A student who solves 8 + 3 by counting on 9, 10, 11 understands three forward counts is equivalent to adding three. Similarly counting backwards relates to subtraction.	Teachers may not recognize the depth of connections between counting and future work with number and operations.	

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
For more information see the following Learning Progressions documents: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking[1] (also titled K, Counting and Cardinality; K–5, Operations and Algebraic Thinking)		
Properties of Operations		
Shift to using properties of operations as a tool to solve computational problems, both basic facts and multi-digit. Students develop an understanding of the properties of operations through problem solving. Teachers use questioning and discussions to help students connect strategies, and eventually algorithms, to the properties of operations, leading to generalizations of the properties of operations. Then students apply and extend their understanding of the properties of operations to fractions and decimals. The properties of operations are listed in Table 3 on page 94 of Iowa Core Mathematics. Three examples include: • A student may determine the sum of 29 + 18 by thinking "29 + 1 is 30; now add 17 more to get 47." This student is using the associative property of addition: 29 + (1+ 17) = (29 + 1) + 17. • A student might determine the product of 8 x 6 by thinking "8 x 5 = 40, plus 8 more is 48." This student is using the distributive property of multiplication over addition: 8(5 + 1) = 8(5) + 8 (1). • A student might determine the product of 8 x 6 by thinking 4 x 6 is 24, double that to get 48. This student is using the associative property of multiplication: (2 x 4) x 6 = 2 x (4 x 6).	Teachers expect students to identify properties of operations. For example, a teacher shows 4 + 5 = 5 + 4 and asks, "What property is this?" In many cases properties of operations have not been related to the rational number system which includes fractions and decimals.	Operations & Algebraic Thinking 1.OA.3, 6 2.OA.2 3.OA.5, 7, 9 Number & Operations in Base Ten 1.NBT.4, 6 2.NBT.5-7,9 3.NBT.2-3 4.NBT.5-6 5.NBT.6-7 Number & Operations - Fractions

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
For more information see the following Learning Progressions documents: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking[1] (also titled K, Counting and Cardinality; K–5, Operations and Algebraic Thinking) Draft K–5 Progression on Number and Operations in Base Ten[2] (also titled K–5, Number and Operations in Base Ten)		4.NF.3 5.NF.3-7
Addition, Subtraction, Multiplication, and Division Word Problems		
Shift to developing an understanding of addition, subtraction, multiplication, and division by solving word problems.	Students learn how to add, subtract, multiply and divide numbers stripped from context. After memorizing facts and procedures, students apply the skills to word problems.	Operations & Algebraic Thinking K.OA.1-2 1.OA.1-2 2.OA.1 3.OA.1-4, 8 4.OA.1-3
Shift to solving all problem situations. For students to gain a robust understanding of addition and subtraction and the relationship between the two, they need to experience all the different addition and subtraction problem situations described in Table 1 on page 92 of Iowa Core Mathematics. To develop a robust understanding of multiplication and division, students need to have experience with all the different problem situations described in Table 2 on p. 93 of Iowa Core Mathematics.	Overemphasis has been on "result unknown" problems for addition and subtraction. In multiplication an overemphasis has been on "equal groups" problems and in division the overemphasis has been on "how many in each group" problems.	
Shift to problem difficulty being determined by both the problem situation and the numbers involved.	Problem difficulty is determined by the size of the numbers.	

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Shift to analyzing the action and/or the relationship between the given quantities to make sense of word problems. Problems with the unknown in various positions require a deep understanding of the operations that goes beyond reliance on key words. Relying on key words does not develop mathematical reasoning. In addition, many problems do not have key words or key words may have different meanings in different contexts. **Shift to** developing understanding of multiplication as multiplicative along with understanding multiplication as repeated addition. In third grade multiplication starts with students interpreting multiplication as repeated addition through equal groups and array problems. In fourth grade word problems become more complex and students need to use multiplicative reasoning to solve some problems. Comparison and scaling problems reinforce this concept. Multiplicative reasoning is key to understanding proportional relationships in later grades. These two ways to interpret multiplication (repeated addition and multiplicative reasoning) apply to both word problems and numeric problems. • **Example of repeated addition:** 3 x 5 = 15 means there are 3 groups of 5 or 5 + 5 + 5. A word problem to match this situation is *There are three bags with five plums in each bag. How many plums are there in all?* • **Example of multiplicative thinking**: 3x5=15 means 15 is 3 times larger than 5. A word problem to match this situation is *A blue hat costs $5. A red hat costs three times as much as the blue hat. How much does the red hat cost?* For more information see the following Learning Progressions document:	The use of keywords has commonly been used to help students identify operations and solve word problems. For example the phrase “in all” would tell students they should add. In multiplication an overemphasis has been on understanding multiplication as repeated addition through equal grouping problems.	

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking[1] (also titled K, Counting and Cardinality; K–5, Operations and Algebraic Thinking)		
Basic Facts		
Shift to developing strategies for basic facts by solving word problems. (See the Mathematical Topic *Addition, Subtraction, Multiplication, and Division Word Problems.*) **Shift to** students applying the properties of operations and the relationship between operations to develop flexible and efficient mental strategies for basic facts. When students determine answers to basic fact problems which they don't know from memory, they engage in using the properties of operations. Three examples of efficient mental strategies include: • A student solves 9 + 7 by thinking 7 is 6 + 1, then 9 + 1 is 10 and 6 more is 16. This student is using the associative property of addition: 9 + (1 + 6) = (9 + 1) + 6. • A student solves 6 x 7 by thinking 3 x 7 is 21 and double 21 to get 42. This student is using the associative property of multiplication: (2 x 3) x 7 = 2 x (3 x 7). • A student solves 6 x 7 by thinking 5 x 7 is 35 and one more group of 7 is 42. This student is using the distributive property of multiplication over addition: (5 + 1) x 7 = (5 x 7) + (1 x 7). Repeated use of efficient mental strategies is the key to achieving fluency with facts and reaching automaticity. Automaticity is knowing facts by memory as a result of	Students learn basic fact strategies stripped from context. Common teaching practice is for teachers to introduce basic fact strategies (doubles plus 1, make a ten) to students as procedures to determine answers. Students may practice the strategies and then memorize facts through drill. Limited connections are made between the basic fact strategies and the properties of operations. Limited connections are made between the basic fact strategies and developing automaticity.	Operations & Algebraic Thinking K.OA.1-5 1.OA.3-6 2.OA.2 3.OA.5-7

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
repeated opportunities to engage in learning the facts in a meaningful way. Students do not need to memorize the basic facts through drill to achieve fluency and know the facts by memory. Page 8 of Iowa Core Mathematics describes procedural fluency as *skill in carrying out procedures flexibly, accurately, efficiently and appropriately*. Furthermore, the Common Core writing team makes the following statement on page 18 of Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking[1] (also titled K, Counting and Cardinality; K–5, Operations and Algebraic Thinking). *Fluency in each grade involves a mixture of just knowing some answers, knowing some answers from patterns (e.g., "adding 0 yields the same number"), and knowing some answers from the use of strategies. It is important to push sensitively and encouragingly toward fluency of the designated numbers at each grade level, recognizing that fluency will be a mixture of these kinds of thinking which may differ across students.* For more information see the following Learning Progressions document: Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking[1] (also titled K, Counting and Cardinality; K–5, Operations and Algebraic Thinking)		
Multi-digit Computation		
Shift to developing an understanding of multi-digit addition, subtraction, multiplication, and division by solving word problems. (See the Mathematical Topic *Addition, Subtraction, Multiplication, and Division Word Problems.*)	Students learn how to add, subtract, multiply and divide multi-digit numbers stripped from context.	Number & Operations in Base Ten 1.NBT.4-6 2.NBT.5-9

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Shift to delaying the teaching of the standard algorithms until conceptual understanding is developed.	Students learn the traditional standard algorithm as the primary way to solve computational problems. Too often students learn this algorithm without meaning and lack understanding of place value and properties of operations.	3.NBT.2-3 4.NBT.4-6 5.NBT.5-7
Shift to using concrete models or drawings and strategies based on place value, properties of operations, and the relationship among operations to solve problems prior to addressing a standard algorithm. The foundation of a standard algorithm is place value and properties of operations. **The standard algorithm is therefore defined as any efficient and generalizable algorithm grounded in place value and properties of operations.** Three examples of strategies based on place value, properties of operations, and the relationship among operations include: • A student might solve 36 + 49 by first adding the tens (30 + 40 = 70), adding the ones (6 + 9 = 15), and adding the two results (70 + 15 = 85). This student's thinking shows an understanding of place value, the commutative property of addition, and the associative property of addition. • A student might solve 94 - 36 by thinking 96 - 36 = 60 and 60 - 2 = 58. This student seems to understand when you add 2 and then subtract 2; you do not change the value. This is the additive inverse property: 2 + (-2) = 0. • A student might solve 128 ÷ 8 by thinking 10 x 8 = 80 and 6 x 8 = 48, so 16 x 8 = 128. Therefore 128 divided by 8 is 16. This student uses his or her understanding	Students identify the place value of the digits in a number and learn how to write a number in expanded form rather than using place value as a strategy to solve computational problems.	

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of the relationship between multiplication and division to solve a division problem with multiplication. The student also shows an understanding of the distributive property of multiplication over addition. See *Summary of K-5 Iowa Core Standards: Number and Operations in Base Ten Domain* for a table summarizing the Number and Operations in Base 10 standards. **This table indicates the grade level expectations when fluency with a standard algorithm is expected.** For more information see the following Learning Progressions document: Draft K–5 Progression on Number and Operations in Base Ten[2] (also titled K–5, Number and Operations in Base Ten)		
The Meaning of Fractions		
Shift to developing conceptual understanding of fractions through a variety of visual fraction models (such as area model, number line diagram, and set model). In first and second grade students began to develop understanding of fractions by partitioning area models into equal-sized parts. These standards are in the Geometry Domain. Students work with halves, thirds, and fourths. In third grade and beyond students focus on additional models as tools to further their understanding of fractions. **Shift to** emphasizing the relationship between the unit fraction and the composite fraction. A composite fraction $\frac{a}{b}$ is a multiple of $\frac{1}{b}$. In other words, the fraction $\frac{a}{b}$ is *a* iterations of the unit fraction $\frac{1}{b}$. In students' early work with number they understand	Teachers have not widely used the number line diagram to develop fraction concepts. The emphasis has been on using area models with shaded parts as the primary tool for developing understanding of fractions. This has resulted in a limited understanding of fractions and is prone to misconceptions. Here are three common misconceptions: • *Fractions are a brand new idea not*	Geometry 1.G.3 2.G.2-3 3.G.2 Number & Operations – Fractions 3.NF.1-3 4.NF.1-3

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the concept of any number being made up of iterations of the unit 1. For example, 4 = 1 + 1 + 1 + 1 or 4 is made up of four iterations of the unit 1. As students begin their work with fractions, they learn the unit is changing from 1 to $\frac{1}{b}$. For example, when the unit is $\frac{1}{4}$, $\frac{3}{4}$ is represented as $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$. In both whole numbers and fractions, the quantities are represented as iterations of the unit. The relationship between the unit fraction and the composite fraction is **key** to understanding fraction operations. **Shift to** understanding fraction as a quantity (number on the number line). Beginning in third grade, the number line diagram is emphasized as a tool to achieve this goal. For example, define the interval from 0 to 1 and partition it into 4 equal parts resulting in each part being equivalent to the unit fraction $\frac{1}{4}$. By representing $\frac{3}{4}$ on the number line diagram as 3 iterations of $\frac{1}{4}$, students understand $\frac{3}{4}$ as a quantity. All proper and improper fractions can be represented as a quantity on a number line using this reasoning. Following are two examples of contextual problems to help students deepen their understanding of the unit fraction and fractions as quantities. • **Using a number line diagram:** I have one yard of ribbon and want to make 4 equal-sized bows. How much ribbon will I need for 1 bow? How much ribbon will I need for 3 bows? Imagine a number line from 0 to 1 representing one yard of ribbon. Partition the number line into 4 equal-sized intervals, with each interval representing $\frac{1}{4}$ yard of ribbon, enough for 1 bow. To find out how much ribbon we need for three bows, we take three iterations of $\frac{1}{4}$ yard equaling $\frac{3}{4}$ yard of ribbon.	*connected to whole numbers.* • *Fractions are always less than one.* Area models predominantly show fractions less than one. The language "three out of four" compounds this issue as it doesn't make sense to have "five out of four." • *A fraction $\frac{a}{b}$ is two whole numbers rather than a single quantity.* Consider the fraction $\frac{3}{4}$. When the task is to shade three out of four equal-sized parts, students often think of three and four as separate quantities, rather than recognizing $\frac{3}{4}$ is 3 one-fourths or three iterations of $\frac{1}{4}$. Using the language "three out of four" compounds this issue.	

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• **Using an area model:** Four children share 3 brownies equally. How much brownie does each child get? Imagine 3 rectangles representing 3 brownies, each divided into 4 equal parts. Each child receives $\frac{1}{4}$ of each of the 3 brownies. Three pieces, each equal to $\frac{1}{4}$ brownie, results in each child receiving 3 iterations of $\frac{1}{4}$ brownie or an amount equal to $\frac{3}{4}$ brownie.		
Shift to developing flexibility and fluency in composing and decomposing fractions, including improper fractions and mixed numbers. Three examples include: • A student might represent $\frac{8}{5}$ as: $\frac{5}{5}+\frac{3}{5}$; $\frac{4}{5}+\frac{4}{5}$; $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$; $1+\frac{3}{5}$; etc. • A student might represent $1\frac{3}{4}$ as: $\frac{4}{4}+\frac{3}{4}$; $\frac{4}{4}+\frac{2}{4}+\frac{1}{4}$; $\frac{7}{4}$; $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$; $1+\frac{3}{4}$; etc. • A student might represent $\frac{5}{6}$ as: $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$; $\frac{3}{6}+\frac{2}{6}$; $\frac{4}{6}+\frac{1}{6}$; etc.	Emphasis has been on converting improper fractions to mixed numbers by learning a procedure.	
Shift to comparing and ordering fractions by reasoning about their size. Use strategies of common numerators, common denominators, and benchmark fractions. Students develop efficient and flexible strategies depending on the task and fractions. Three examples include: • **Common numerators:** $\frac{3}{4}$ is larger than $\frac{3}{5}$ because $\frac{1}{4}$ is larger than $\frac{1}{5}$ and we have 3 iterations of $\frac{1}{4}$ compared to 3 iterations of $\frac{1}{5}$.	Finding common denominators to compare or order fractions is the only strategy.	

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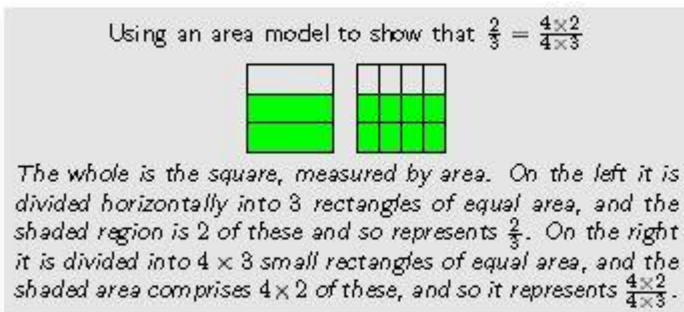
• **Common denominators:** $\frac{5}{7}$ is larger than $\frac{3}{7}$. • **Benchmark fractions:** $\frac{5}{8}$ is greater than $\frac{3}{7}$ because $\frac{5}{8}$ is greater than $\frac{1}{2}$ and $\frac{3}{7}$ is less than $\frac{1}{2}$. **Shift to** emphasizing each fraction has multiple equivalent fractions. Use visual fraction models to develop an understanding of equivalent fractions and why $\frac{a}{b} = \frac{an}{bn}$ (where n > 0). See the figure below for an example of a visual fraction model. The lowest term of any fraction is a **special case** of its family of equivalent fractions. The following figure is from page 5 of Draft 3–5 Progression on Number and Operations—Fractions[3] (also titled 3–5 Number and Operations—Fractions)  *The whole is the square, measured by area. On the left it is divided horizontally into 3 rectangles of equal area, and the shaded region is 2 of these and so represents $\frac{2}{3}$. On the right it is divided into 4×3 small rectangles of equal area, and the shaded area comprises 4×2 of these, and so it represents $\frac{4\times2}{4\times3}$.* For more information see the following Learning Progressions document: Draft 3–5 Progression on Number and Operations—Fractions[3] (also titled 3–5 Number and Operations—Fractions)	Emphasis has been on memorizing a procedure for determining equivalent fractions. Emphasis has also been on reducing fractions to lowest terms and requiring lowest terms regardless of the situation. As a result students often learn methods for reducing fractions without meaning and think answers not in lowest terms are incorrect.	
Fraction Computation		
Shift to the majority of fraction computation work being in 4th and 5th grade. Students in 6th, 7th, and 8th grade apply their knowledge of fraction computation as	Fraction computation is a major focus in 5^{th}, 6^{th} and 7^{th} grade.	Number & Operations –

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they solve algebraic equations. **Shift to** developing an understanding of fraction computation by solving word problems. (See the Mathematical Topic *Addition, Subtraction, Multiplication, and Division Word Problems.*)	 Students learn fraction computation stripped from context.	Fractions 4.NF.3-5 5.NF.1-7
Shift to developing conceptual understanding of fraction computation through visual models (number line diagrams, fraction strip models, and area models) and properties rather than a reliance on procedures. Developing an understanding of the meaning of fractions in 3rd and 4th grade with an emphasis on unit fractions and fraction equivalence provides the foundation for developing understanding of fraction computation. **Grade 4**: Addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers is grounded in the idea that all composite fractions are multiple iterations of the unit fraction. Students develop this understanding through the use of visual models (number line diagrams, area models, and fraction strip models). Three examples include: • $\frac{4}{3} + \frac{2}{3} = \frac{6}{3}$ because $(\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}) + (\frac{1}{3} + \frac{1}{3}) = \frac{6}{3}$. The unit is $\frac{1}{3}$. Four iterations of $\frac{1}{3}$ plus 2 iterations of $\frac{1}{3}$ equals 6 iterations of $\frac{1}{3}$ or $\frac{6}{3}$. This is the same as whole number addition of 4 and 2, but the unit is $\frac{1}{3}$ rather than 1. The same idea of multiple iterations of a unit fraction applies to subtraction. • $4 \times \frac{1}{3} = \frac{4}{3}$ because $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$.	The focus of teaching fraction computation is procedural. Students learn algorithms as the primary way to solve fraction computation problems. Too often students learn the algorithms without meaning and lack understanding of connections between composite fractions and unit fractions. The properties of operations are not connected to operations with fractions.	

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Four iterations of the unit fraction $\frac{1}{3}$ equals $\frac{4}{3}$. • $4 \times \frac{2}{3} = \frac{8}{3}$ because $(4 \times 2) \times \frac{1}{3} = 8 \times \frac{1}{3}$. Decompose $\frac{2}{3}$ into $2 \times \frac{1}{3}$. The associative property of multiplication shows $4 \times (2 \times \frac{1}{3}) = (4 \times 2) \times \frac{1}{3}$. This gives 8 iterations of $\frac{1}{3}$. **Grade 5:** Addition and subtraction of fractions with unlike denominators is grounded in a deep understanding of equivalent fractions and the need to add and subtract like units. Multiplication of a fraction multiplied by a fraction is developed through visual fraction models (fraction strip models, number line diagrams, and area models). Division of a whole number by a unit fraction and a unit fraction by a whole number is also developed through visual fraction models and the relationship between multiplication and division. Through extensive work with models students generalize methods to compute with fractions. Four examples include: • $\frac{3}{4} + \frac{3}{8} = 1\frac{1}{8}$ There are two common ways to reason through this problem. Method 1: Decompose $\frac{3}{8}$ into $\frac{1}{4}$ and $\frac{1}{8}$, and then add $\frac{3}{4} + \frac{1}{4} + \frac{1}{8}$ to get $1\frac{1}{8}$. Method 2: Rename $\frac{3}{4}$ to $\frac{6}{8}$ and add $\frac{3}{8} + \frac{6}{8}$ to get $\frac{9}{8}$, which is equivalent to $1\frac{1}{8}$. Understanding composite fractions as multiple iterations of unit fractions and understanding equivalent fractions is critical to both methods. • In 4th grade students reasoned that $4 \times \frac{2}{3} = 4(2 \times \frac{1}{3})$. The same reasoning is used to multiply two fractions in 5th grade. For example, $\frac{2}{3} \times \frac{3}{4} = (2 \times \frac{1}{3}) \times (3 \times \frac{1}{4})$. The associative property of multiplication shows that $(2 \times \frac{1}{3}) \times (3 \times \frac{1}{4}) = (2 \times 3) \times (\frac{1}{3} \times \frac{1}{4})$.		

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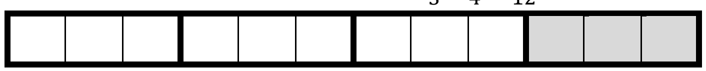
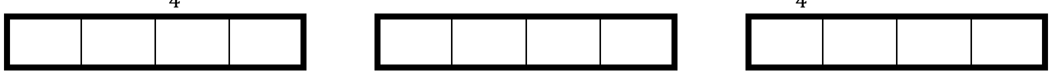
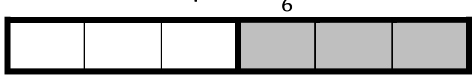
This shows that $\frac{2}{3}$ x $\frac{3}{4}$ = 6 x $\frac{1}{12}$ or $\frac{6}{12}$. In other words, 6 iterations of $\frac{1}{12}$. A visual model of $\frac{2}{3}$ x $\frac{3}{4}$ is used to help students make sense of multiplying fractions by fractions. The following models shows why $\frac{1}{3}$ x $\frac{1}{4}$ = $\frac{1}{12}$:  *The shaded region shows $\frac{1}{4}$ of the entire strip divided into 3 equal parts. One shaded part shows $\frac{1}{12}$ of the entire strip. One-third of $\frac{1}{4}$ is equal to $\frac{1}{12}$.* • 3 ÷ $\frac{1}{4}$ = 12 Students might use their understanding of unit fractions. If there are 4 iterations of $\frac{1}{4}$ in one, there are 3 times as many iterations of $\frac{1}{4}$ in three. 3 x 4 = 12.  *Each rectangular region represents 1 ÷ $\frac{1}{4}$ resulting in 4 iterations of $\frac{1}{4}$ for one unit. Therefore all three rectangular regions represent 3 ÷ $\frac{1}{4}$ resulting in 12 iterations of $\frac{1}{4}$ for 3 units.* • $\frac{1}{2}$ ÷ 3 = $\frac{1}{6}$ Students might reason if you divide $\frac{1}{2}$ into three equal parts, you create smaller units equal to $\frac{1}{6}$.  *The shaded region shows $\frac{1}{2}$ divided into 3 equal parts. Each shaded part equals $\frac{1}{6}$ of the entire strip.* For more information see the following Learning Progressions document: <u>Draft 3–5 Progression on Number and Operations—Fractions</u>[3] (also titled 3–5 Number and Operations—Fractions)		

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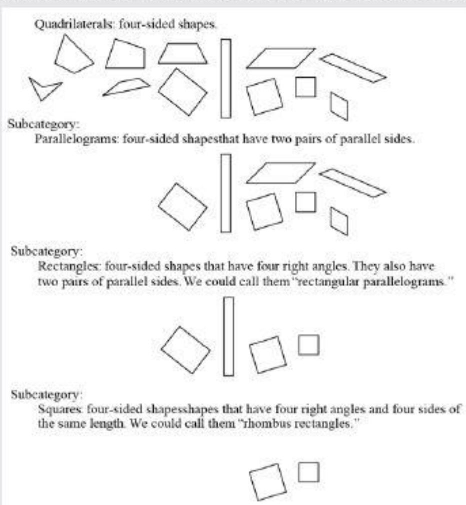
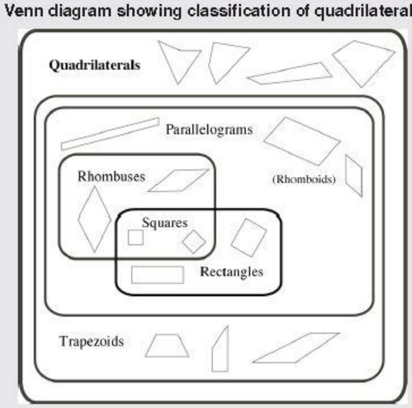
Geometry		
Shift to analyzing properties and understanding classes of shapes. Students characterize shapes by properties and determine which combinations of properties define a class of shapes. This leads to understanding relationships among classes of shapes and organizing two-dimensional shapes into hierarchies based on properties of shapes. For example, students reason all squares are parallelograms because they have all the properties of parallelograms. However, not all parallelograms are squares. The following figures are from pages 13 and 17 of Draft K–6 Progression on Geometry[4] (also titled K–6, Geometry).	Teachers focus instruction on recognizing and naming shapes. The emphasis is often on vocabulary rather than geometric concepts and properties. Students memorize definitions of shapes, but do not investigate the relationships among shapes in order to develop hierarchical classifications.	Geometry K.G.2-6 1.G.1-3 2.G.1-3 3.G.1-3 4.G.1-3 5.G.1-4

Quadrilaterals and some special kinds of quadrilaterals

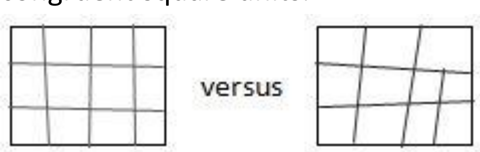


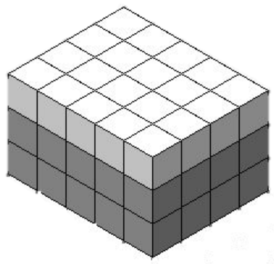
The representations above might be used by teachers in class. Note that the left-most four shapes in the first section at the top left have four sides but do not have properties that would place them in any of the other categories shown (parallelograms, rectangles, squares).

Venn diagram showing classification of quadrilaterals



Note that rhomboids are parallelograms that are not rhombuses or rectangles. This example uses the inclusive definition of trapezoid (see p. [pageref "T(E)"]).

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Shift to decomposing shapes into smaller shapes and composing larger shapes from smaller shapes. As teachers provide multiple experiences with composing and decomposing two- and three-dimensional shapes students began to focus on properties of shapes and *informally* investigate slides, flips, and turns. These experiences also help students better understand how to create a new shape by putting two or more shapes together and analyzing the new shape in relationship to its parts and the total. For example, students combine two right isosceles triangles in different ways in order to form a square, a parallelogram, and a larger right isosceles triangle. For a second example, students decompose a rectangle into fourths multiple ways, with and without congruent parts.	Work with manipulatives such as pattern blocks and tangrams may lack purpose. Students learn the vocabulary of transformations rather than use transformations to develop spatial reasoning.	
Shift to structuring two-dimensional shapes into arrays of unit squares and three-dimensional shapes into layers of arrays of unit cubes. This understanding develops spatial reasoning skills and builds understanding for measuring area and volume. It also connects to multiplication, fractions, and the coordinate plane. For example, students cover a rectangular region by drawing rows and columns of congruent square units.  For example, students visualize this rectangular prism with dimensions of 5 by 4 by 3 as 3 layers of 20 unit cubes. Each layer is a 4 by 5 array of unit cubes.	Teachers show shapes divided into equal parts and arrays rather than expecting students to partition shapes. For example, teachers often show a rectangular region divided into rows of unit squares and students count the unit squares to find the area.	

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
 For more information see the following Learning Progressions document: Draft K–6 Progression on Geometry[4] (also titled K–6, Geometry)		
Data		
Shift to data playing a supporting role to the other Iowa Core K-5 domains rather than being a separate topic of study. Specific direction is given to categorical and numerical data and the types of representations to be used for displaying and analyzing data. Data collected, displayed, analyzed, and used to solve problems appropriate for the grade levels. See Table 1 from Draft K-5 Progression on Measurement and Data (data part)[5] (also titled K–3, Categorical Data; Grades 2–5, Measurement Data) • The **Kindergarten** data standard supports the domain of Counting and Cardinality. The data standard involves collecting and classifying categorical data around two categories. This data standard supports the standards for counting as students sort and count objects. • The **First Grade** data standard supports the domain of Operations and Algebraic Thinking. The data standard involves collecting and classifying categorical data with up to three categories. This data standard supports the standards for addition and subtraction as students solve word problems about the data.	There has not been a consistent expectation for the study of data at the K-5 level. It has been wide and varied.	Measurement & Data K.MD.3 1.MD.4 2.MD.9-10 3.MD.3-4 4.MD.4 5.MD.2 Examples: K.CC.5-7 1.OA.1-2

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
• The **Second Grade** data standards support the domain of Operations and Algebraic Thinking. The data standards involve categorical data with up to four categories displayed with bar graphs and picture graphs. The standards also involve measurement data displayed with line plots. These data standards support standards for addition and subtraction where students solve word problems related to the data. In addition, these data standards support measurement standards where data is collected by measuring lengths. • The **Third - Fifth Grade** data standards support the domain of Number and Operations-Fractions. The data standards involve measurement data (linear, volume, mass, temperature, etc.) with an emphasis on fractional amounts displayed with line plots. These data standards support the standards for whole number operations, fraction concepts, and fraction operations. **Note:** When data standards connect to word problems, use the problem situations as identified in Tables 1 and 2 on pages 92 and 93 of Iowa Core Mathematics.		2.OA.1 2.MD.1-2, 6 3.NF.2 4.NF.3-4 5.NF.1-2, 4, 6-7
Shift to measures of center (mean and median) and variability (range) as an explicit focus in 6th grade. These concepts may come up informally in conversations about data in the elementary grades. In fifth grade students explore the concept of mean in the context of equal distribution.	Measures of center (mean, median, and mode) and variability (range) were typically taught around 4th and 5th grade.	
Shift to probability as an explicit focus in 7th grade. These concepts may come up informally in conversations about data in the elementary grades.	Aspects of probability were taught in upper elementary grades.	
For more information see the following Learning Progressions document: Draft K-5 Progression on Measurement and Data (data part)[5] (also titled K–3, Categorical Data; Grades 2–5, Measurement Data)		

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
Measurement		
Shift to indirect measurement as an additional way to compare lengths of objects. • **Examples of direct measurement:** Student A stands back to back with student B and finds student A is taller or students A, B, and C line up by height to determine who is tallest and shortest. • **Example of indirect measurement:** Using direct measurement student A stands back to back with student B and finds student A is taller. Then student A stands back to back with student C and finds student C is taller. Therefore, using indirect measurement, student C must be taller than student B since student A is taller than student B and student C is taller than student A. This is an example of the Transitivity Principle.	Comparisons of length have typically been with direct measurement only.	Measurement & Data K.MD.1-2 1.MD.1-2 2.MD.1-6 3.MD.5-8 4.MD.1-3 5.MD.1, 3-5
Shift to emphasizing standard units before nonstandard units of measurement in linear measurement instruction. Use standard units such as inch tiles or rulers from the beginning of instruction for measuring length in order to focus on what it means to measure. When measuring objects, students attend to the size of the unit being used to measure and realize iterations of the unit are being counted to determine the length of an object. See page 9-10 of Draft K-5 Progression on Measurement and Data (measurement part)[6] (also titled K–5, Geometric Measurement) for a full explanation.	Nonstandard units have typically been used to introduce measuring length. Instruction then moves to standard units.	
Shift to expectation of a robust understanding of area and perimeter of rectangles at fourth grade and volume of rectangular prisms at fifth grade. Area, perimeter, and volume of all other shapes are not part of the content in grades K-5.	Area, perimeter, and volume of rectangles/rectangular prisms are generally covered throughout several elementary grades and continue	

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
	beyond elementary. Areas of triangles and parallelograms are often included in elementary as well.	
Shift to connecting measurement to the other Iowa Core K-5 domains. For example teachers create word problems in the context of measurement to engage students in standards from the Measurement and Data domain, as well as the Operations and Algebraic Thinking, Number and Operations in Base Ten, or Number and Operations – Fractions domains. See specific examples for grades 2-5 throughout the Draft K-5 Progression on Measurement and Data (measurement part)[6] (also titled K–5, Geometric Measurement) **Note:** When measurement standards connect to word problems use the problem situations as identified in Tables 1 and 2 on pages 92 and 93 of Iowa Core Mathematics. For more information see the following Learning Progressions document: Draft K-5 Progression on Measurement and Data (measurement part)[6] (also titled K–5, Geometric Measurement)	Word problems in the context of measurement may not include all the problem situations. Word problems may not be strategically used to support the development measurement concepts.	
Patterns		
Shift to looking for mathematical structures and patterns. Patterns are not absent from Iowa Core Mathematics. Rather patterns are much more than "what comes next" activities and connect to deeper mathematical structures. This may be a broader view of patterns than many teachers have held in the past. Teachers embed	Students spend time in the lower elementary grades identifying ABAB or ABBABB patterns. Students at all grades spend time on "what comes	Multiple K-5 standards

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
work with patterns throughout the curriculum. Mathematical patterns and structures connect to counting, composing and decomposing numbers, place value, properties of operations, skip counting, addition and multiplication tables, multiplying powers of 10, area and perimeter formulas, etc. Explicit work on connecting a rule to a number or shape pattern begins in grade 4.	next" pattern activities. Students memorize rules rather than generalize patterns and explain why they occur.	
Examples from the domains: • **Operations and Algebraic Thinking:** Younger students find all the ways to decompose 10 or decompose two-digit numbers into tens and ones and describe the patterns involved. Older students study the pattern that occurs when multiplying by 0 or 1. • **Number and Operations in Base Ten:** Younger students explain the patterns of our written numerals. For example, any number from 11 to 19 is composed of one group of ten and some ones; any number from 21 to 29 is composed of two groups of ten and some ones, and so forth. Older students explain the pattern that occurs when you multiply by a multiple of a power of 10. For example, consider the relationship between the products of 6 x 7, 6 x 70, 6 x 700, and 6 x 7000. Relying on the associative property of multiplication, students can think of each as 42 times a power of ten. For example, 6 x 70 = 6 x (7 x 10) = (6 x 7) x 10 = 42 x 10 = 420. When you focus on "adding zeros" rather than understanding why this pattern of "adding zeros" occurs, students often get an incorrect answer with problems such as 5 x 400. This is because the product ends with three zeros rather than two. Understanding and applying the associative property of multiplication helps students see 5 x 400 = 5 x (4 x 100) = (5 x 4) x 100 = 20 x 100 = 2000. • **Number and Operations – Fractions:** Following are three examples. (1) Students recognize when comparing unit fractions, the one with the smaller denominator is the larger fraction. This is because the more pieces the whole is divided into the		Examples: K.OA.3-4 K.NBT.1 3.OA.5 K.NBT.1 3.NBT.3 3.NF.3 4.NF.1, 4

Shifts in Content and Practice	Contrasts to the Shifts	Domains and Standards
smaller the pieces. (2) Students are able to create equivalent fractions because they recognize $\frac{a}{b} = \frac{(a \, x \, n)}{(b \, x \, n)}$ where n > 0. This allows students to create an infinite number of fractions equivalent to $\frac{a}{b}$. (3) Students recognize and explain why multiplying a given number by a fraction less than one results in a product less than the given number.		5.NF.4-5
• **Geometry:** Students form patterns with shapes. The focus is using smaller shapes (the unit) to form larger shapes (unit of units). Students may make patterns with multiple iterations of a shape or combining different shapes. Students also decompose rectangular regions into rows and columns of smaller squares forming a pattern of repeated unit squares. This also occurs with three-dimensional shapes, but then students repeat unit cubes in arrays and repeat the arrays into layers.		K.G.6 1.G.2-3 2.G.2-3 3.G.2
• **Measurement:** Students develop understanding of the structure of systems of measurement. Students recognize the patterns of how larger units are made up of smaller units. For example, exploring the metric system helps students understand 1 m is equivalent to 100 cm; 2 m is equivalent to 200 cm; etc. The metric system also reinforces the structure of our base 10 system.		4.MD.1 5.MD.1
• **Data:** Students represent and interpret data with a focus on supporting standards from the other content domains. In the process, students may look for patterns in data to address questions such as, “What is the most common? What is the least common?” This experience in analyzing data prepares students to study patterns of association in middle school.		1.MD.4 5.MD.2

[1]Draft K-5 Progression on Counting and Cardinality and Operations and Algebraic Thinking (also titled K, Counting and Cardinality; K–5, Operations and Algebraic Thinking)
[2]Draft K–5 Progression on Number and Operations in Base Ten (also titled K–5, Number and Operations in Base Ten)
[3]Draft 3–5 Progression on Number and Operations—Fractions (also titled 3–5 Number and Operations—Fractions)
[4]Draft K–6 Progression on Geometry (also titled K–6, Geometry)
[5]Draft K–5 Progression on Measurement and Data (data part) (also titled K–3, Categorical Data; Grades 2–5, Measurement Data)
[6]Draft K–5 Progression on Measurement and Data (measurement part) (also titled K–5, Geometric Measurement)