*Literacy Design Collaborative*

<http://www.literacydesigncollaborative.org/>

with the

Iowa Core Mathematics

<http://www.educateiowa.gov>.

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Overview

Iowa Core Mathematics defines what students should understand and be able to do in their study of mathematics. Iowa Core Mathematics involves Content Standards and Practice Standards. The Standards for Mathematical Content are a balanced combination of procedure and understanding. “Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut.” (pg. 10, Iowa Core)

“The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education.” (pg. 8, Iowa Core; these practices are listed at the end of this document). The Practice Standards are important in math education. It is the Standards for Mathematical Practices that capture the habits of mind, the thinking skills.

The procedures and understanding needed for mathematics, those found in the blending of the Content Standards and the Practice Standards, are skills similar to reading and writing skills and, when taught together, reinforce each other. The Literacy Design Collaborative [LDC] offers an approach to incorporating literacy into middle and high school mathematics that focus on procedures and understanding. “Each LDC task is a reading and writing prompt, asking middle or high school students to take on an important issue in science, history, ELA or another subject.” (pg. 8, ***1.0 Guidebook to LDC***). The following resource is an attempt to integrate literacy into the area of mathematics. Several of the LDC template tasks were utilized using information found in the Iowa Core. Each LDC template task is accompanied with supporting documentation. It will be imperative that when teachers focus on these tasks that they focus on two things: how can the Mathematical Practices help students understand and manipulate the content found in the template tasks and how can the understanding and manipulation of the content through the template tasks help students develop the expertise found in the Mathematical Practices.

**Samples of Math in LDC Task Templates**

**Template Tasks - Argumentation**

**Task 3 (Argumentation/Comparison):**

After researching \_\_\_\_\_\_\_\_ (informational texts) on \_\_\_\_\_\_\_\_ (content), write a/an \_\_\_\_\_\_\_\_ (essay or substitute) that compares \_\_\_\_\_\_\_\_ (content) and argues \_\_\_\_\_\_\_\_ (content). Be sure to support your position with evidence from the texts.

After researching mathematical sources on making inferences and justifying conclusions, write a report that compares the purposes of and differences among sample surveys, experiments, and observational studies and argues how randomization relates to each.

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| Supporting documentation for this task can be found on page 7 | Mathematical Practices…page 17  |

**Task 4 Template (Argumentation/Comparison):**

[Insert question] After reading \_\_\_\_\_\_\_\_ (literature or informational texts), write a/an \_\_\_\_\_\_\_\_ (essay or substitute) that compares \_\_\_\_\_\_\_\_ (content) and argues \_\_\_\_\_\_\_\_ (content). Be sure to support your position with evidence from the texts.

What is the fairest voting method? After reading articles on a variety of voting methods*,* write a proposal that compares different voting methods and how different voting methods can give very different results and argues which voting method would be best in conducting an election at your school for Homecoming King and Queen. Be sure to support your position with evidence from the texts.

Alternative: What is the fairest voting method? After reading articles on a variety of voting methods and completing the *Mathematics of Preferential Voting* task*,* write a proposal that compares different voting methods and how different voting methods can give very different results and argues which voting method would be best in conducting an election at your school for Homecoming King and Queen. Be sure to support your position with evidence from the texts and the task.

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| Supporting documentation for this task can be found on page 8 | Mathematical Practices…page 17 |

**LDC Template Tasks - Informational or Explanatory**

**Task 12 Template (Informational or Explanatory/Definition L1):**

[Insert question] After reading \_\_\_\_\_\_\_\_ (literature or informational texts), write \_\_\_\_\_\_\_\_ (essay, report, or substitute) that defines \_\_\_\_\_\_\_\_ (term or concept) and explains \_\_\_\_\_\_\_\_ (content). Support your discussion with evidence from the text(s). L2 What \_\_\_\_\_\_\_\_ (conclusions or implications) can you draw?

What happens to medication in the body? After reading articles on antibiotics, prescribing medicine, and preventing errors write a summary report that defines recursion and explains how mathematical models can be used to describe the relationship between two quantities as it relates to the long-term effects of medication in the body. Support your discussions with evidence from the texts.

Alternative: How can we use recursion and iteration to predict events in the physical world and in our society? After reading articles on antibiotics, prescribing medicine, and preventing errors and after completing the ‘Medical Dosage’ task, write a summary report that defines recursion and explains the connections among the numeric, graphic, and symbolic models in analyzing the long-term effects of medication in the body. Support your discussions with evidence from the task and from the texts.

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| Supporting documentation for this task can be found on page 10 | Mathematical Practices…page 17 |

**Task 13 (Informational or** **Explanatory/Description):**

After researching \_\_\_\_\_\_\_\_ (informational texts) on \_\_\_\_\_\_\_\_ (content), write a \_\_\_\_\_\_\_\_ (report or substitute) that describes \_\_\_\_\_\_\_\_ (content). Support your discussion with evidence from your research.

After researching articles on information processing and the internet, write a paper that describes in general how a public-key cryptosystem works discussing basic number theory and modular arithmetic. Support your discussion with evidence from your research.

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| Supporting documentation for this task can be found on page 12 | Mathematical Practices…page 17 |

**Task 17 Template Math (Informational or Explanatory/Procedural-Sequential):**

After researching \_\_\_\_\_\_\_\_ (informational texts) on \_\_\_\_\_\_\_\_ (content), complete the task, examining \_\_\_\_\_\_\_\_ (content), write a report that explains your procedures and results and discusses connections to the real world.

After researching articles on graph theory - using diagrams consisting of vertices and edges to model and solve problems related to networks, and completing a mathematical task on paths and circuits with vertex edge graphs, write a how-to-article that explains your procedures and results for an efficient solution and discusses connections to the real world.

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| Supporting documentation for this task can be found on page 13 | Mathematical Practices…page 17 |

**Task 20 Template (Informational or Explanatory/Analysis):**

After researching \_\_\_\_\_\_\_\_ (informational texts) on \_\_\_\_\_\_\_\_ (content), write a \_\_\_\_\_\_\_\_ (report or substitute) that analyzes \_\_\_\_\_\_\_\_ (content), providing evidence to clarify your analysis. What \_\_\_\_\_\_\_\_ (conclusions or implications) can you draw? L2 In your discussion, address the credibility and origin of sources in view of your research topic. L3 Identify any gaps or unanswered questions. Optional: Include \_\_\_\_\_\_\_\_ (e.g., bibliography)

After researching mathematical documents and textbooks on patterns of association seen in bivariate categorical data, write a letter to your classmates, in which you analyze graphs, frequencies and relative frequencies in a two-way table to describe possible association between the two variables, providing evidence to clarify your analysis. What conclusions can you draw from the documents, textbook, and data? Include a bibliography.

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| Supporting documentation for this task can be found on page 15 | Mathematical Practices…page 17 |

**Task 22 (Informational or Explanatory/Comparison):**

After researching \_\_\_\_\_\_\_\_ (informational texts) on \_\_\_\_\_\_\_\_ (content), write a \_\_\_\_\_\_\_\_ (report or substitute) that compares \_\_\_\_\_\_\_\_ (content). L2 In your discussion, address the credibility and origin of sources in view of your research topic. L3 Identify any gaps or unanswered questions.

After researching mathematical sources on interpreting categorical and quantitative data, write a report that compares different data distributions or treatments. In your discussion, address the correlation and causation in the data and/or mathematical linear models.

Alternative: After researching mathematical sources on interpreting linear models, write a report that compares different data distributions or treatments. In your discussion, address the correlation and causation in the data and/or mathematical linear models.

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| Supporting documentation for this task can be found on page 16 | Mathematical Practices…page 17 |

**Supporting Materials**

**Task 3 – Math: Iowa Core Connections**

**Statistics and Probability**

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

**Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **(S-IC.3.) (DOK 1,2)Task 4 – Math: Iowa Core Connections**

**High School Number and Quantity**

Quantities- In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

**(IA) Understand and apply the mathematics of voting.**

IA.3.Understand, analyze, apply, and evaluate some common voting and analysis methods in addition to majority and plurality, such as runoff, approval, the so-called instant-runoff voting (IRV) method, the Borda method and the Condorcet method.

**Potential Resources**

Articles

* Bowles, Scott, *Oscars' new voting system is a real puzzler*, By Scott Bowles, USA TODAY 3/5/2010 <http://usatoday30.usatoday.com/life/movies/movieawards/oscars/2010-03-05-1Aoscar05_VA_N.htm>
* Saari, Donald G. *Chaotic Elections! A Mathematician Looks at Voting,* American Mathematical Society, Providence, R.I., 2001. This expository book begins with the 2000 presidential election and discusses a number of paradoxical results in voting.

Web Sites

* *Comparing Voting Systems*: <http://www.equalvote.org/Instant_Runoff_Comparisons.html>
* *Controversial Elections, Fair Vote*, <http://archive.fairvote.org/e_college/controversial.htm#2000>
* *Voting and Elections*, American Mathematical Society featured column, [*http://www.ams.org/samplings/feature-column/fcarc-voting-introduction*](http://www.ams.org/samplings/feature-column/fcarc-voting-introduction)
* *Voting Methods*: <http://www.ctl.ua.edu/math103/voting/voting.htm#Voting%20Methods>
* *Perplexing Mathematics of Presidential Elections* <http://www.maa.org/devlin/devlin_11_00.html>

Tasks

* *Mathematics of Preferential Voting (Adapted from Core-Plus Mathematics, Course 4, Glencoe, 2009.) -* <http://educateiowa.gov/esc/secure/links/hs/h-y4-d1_voting.pdf>

**Task 12 – Math: Iowa Core connections**

**Eighth Grade - Functions**

Critical Area 2: Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

 Cluster Heading: Define, evaluate, and compare functions

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[[1]](#footnote-1) **(8.F.1.) (DOK 1,2)**

Cluster Heading: Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **(8.F.4.) (DOK 1,2,3)**

**High School Functions**

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like *f*(*x*) = *a* + *bx*; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

**Understand the concept of a function and use function notation**

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.* **(F-IF.3.)**

**Build a function that models a relationship between two quantities**

1. Write a function that describes a relationship between two quantities.★

1. Determine an explicit expression, a recursive process, or steps for calculation from a context.
2. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★ **(F-BF.2.)**

**Potential Resources**

Articles

* *Prescribing Medicine & Preventing Errors*, NCTM’s Mathematics Teaching in the Middle School journal, Vol. 15, No. 1, August 2009

Websites

* Antibiotics Channel: <http://antibiotics.emedtv.com/>
* *Pandemics How are viruses spread*, NCTM Lessons and Resources, <http://www.nctm.org/resources/content.aspx?id=8496>

Tasks

* Achieve - <http://www.achieve.org/ccss-cte-classroom-tasks>, Under the Health Sciences heading select *Medication Dosage task*
* Iowa Department of Education: <http://www.educateiowa.gov/> (follow the links to the task), Select Priority Link: Iowa Core, Mathematics, Iowa Core Mathematics Support, Classroom Resources, Additional Sense Making Activities, High School, [**“Medicine Dosage”**](http://www.educateiowa.gov/index.php?option=com_docman&task=doc_download&gid=11820&Itemid=5111)**.**

**Task 13 – Math: Iowa Core Connections**

**Number and Quantity**

In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

**(IA) Understand and apply some basic mathematics of information processing and the Internet.**

IA. 6.(+) Understand and apply basic number theory, including modular arithmetic, for example, as used in keeping information secure through public-key cryptography. **(DOK 1,2)**

**Why is this mathematics important, and where did these particular standards come from?**

This is important mathematics because the Internet is an essential and ubiquitous part of contemporary life. Students living in the modern information age need some mathematical literacy related to the Internet and information processing. This begs the question, what mathematics related to the Internet is accessible and valuable for high school students to learn? This question does not yet have a standardized answer. The Iowa Additions provide a modest initial answer, by including a small amount of fundamental mathematics in set theory, number theory, and logic related to the Internet and information processing. Moreover, this mathematics is important in its own right, with applications in many other areas.

These particular standards (IA4 – IA 6) a rose from initial recommendations made in 2005 by the Iowa Core Project Lead Team, a group of Iowa leaders representing diverse stakeholders in Iowa education. The standards first appeared in the early Iowa Core at that time. In 2010, an alignment was made between the Iowa Core and the new Common Core State Standards for School Mathematics (CCSSM ) that Iowa adopted. There was strong alignment, except that an Iowa Core committee recommended that certain Additions be added, including the standards for the mathematics of information processing and the Internet. The Iowa State Board of Education officially approved these particular standards at a Board meeting in 2010.

**Potential Resources**

Website

* Resources for Teaching the Iowa Core Additional Standards in High School Mathematics, Iowa Department of Education, <https://www.softchalkcloud.com/lesson/serve/ICVxmbioO0wR62/html>

**Task 17 – Math: Iowa Core Connections**

**High School Geometry**

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

**(IA) Use diagrams consisting of vertices and edges (vertex-edge graphs) to model and solve problems related to networks.**

IA.8.Understand, analyze, evaluate, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.★

IA.9.Model and solve problems using at least two of the following fundamental graph topics and models: Euler paths and circuits, Hamilton paths and circuits, the traveling salesman problem (TSP), minimum spanning trees, critical paths, vertex coloring.★

IA.10.Compare and contrast vertex-edge graph topics and models in terms of:★

* properties
* algorithms
* optimization
* types of problems that can be solved

**Why is this mathematics important, and where did these particular standards come from?**

Vertex-edge graphs are broadly useful mathematical models. They help us understand and solve problems related to networks, paths, and relationships. And certainly there are many instances of networks and relationships in business, industry, government, and everyday life, particularly in our modern interconnected digital world.

The theory of vertex-edge graphs is called graph theory. Graph theory has been an active and important branch of mathematics since the eighteenth century, at which time the Swiss mathematician Leonhard Euler is credited with starting the field.

Out of all the many topics of graph theory, the six topics addressed in Iowa Core Mathematics

have been identified by experts in the field as the most important and appropriate for high school students to learn. For example, these topics are the same topics recommended in a recent book from the National Council of Teachers of Mathematics, Navigating through Discrete Mathematics in Grades 6– 12 (NCTM, 2008).

These standards (IA8–IA10) were first proposed in 2005 to the Iowa Core Project Lead Team, a group of Iowa leaders representing diverse stakeholders in Iowa education. This group supported the standards, and so they first appeared in the early Iowa Core at that time. In 2010, an alignment was done between the Iowa Core and the new Common Core State Standards for School Mathematics (CCSSM) that Iowa adopted. There was strong alignment, except that an Iowa Core committee recommended that a few standards be added, including the standards for vertex-edge graphs. The Iowa State Board of Education officially approved these particular standards at a Board meeting in 2010.

**Potential Resources**

Articles

* Biehl, L. Charles. “Massive Graphs, Power Laws, and the World Wide Web.” *Mathematics Teacher* 96 (September 2003): 434–39.
* Hart,Eric W. “[Algorithmic Problem Solving in Discrete Mathematics](file:///D%3A%5Cdocuments%5Creadings%5Cdm2arh.pdf)”, The Teaching and Learning of Algorithms in School Mathematics, 1998 NCTM Yearbook
* “Vertex-Edge Graphs: An Essential Topic in High School Geometry.” *Mathematics Teacher* (October, 2008)
* A Few Excerpts related to VERTEX-EDGE GRAPHS NCTM’s Principles and Standards for School Mathematics
* http://infinitemath.com/mathforallresources/Discrete%20Math/Vertex-Edge%20Graphs/VEGnctmPSSM.doc

Websites

* Resources for Teaching the Iowa Core Additional Standards in High School Mathematics: <https://www.softchalkcloud.com/lesson/serve/ICVxmbioO0wR62/html>
* www.infinitemath.com/mathforall/Home.html – See vertex-edge graph resources in the Geometry section

Tasks

* *Vertex-Edge Graphs in Grades 9-12*, Hart, Eric W., Margaret J. Kenney, Valerie A. DeBellis, Joseph G. Rosenstein, Navigating through Discrete Mathematics in Grades 6-12, pgs. 63 – 80.

**Task 20 – Math: Iowa Core Connections**

**Eighth Grade – Statistics and Probability**

Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m·A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

**Investigate patterns of association in bivariate data**

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* **(8.SP.4.) (DOK 1,2,3)**

**Task 22 – Math: Iowa Core Connections**

**Statistics and Probability**

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

**Interpret linear models**

Distinguish between correlation and causation. **(S-ID.9.) (DOK 1,2)**

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy). <http://www.educateiowa.org>

1. **Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

1. **Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

1. **Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

1. **Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

1. **Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

1. **Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

1. **Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression *x2* + 9x + 14, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see 5 – 3(*x – y*)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

Students who look for patterns in their environment expect things to make sense and develop a habit of finding relationships and making predictions. Students should investigate patterns in number, shape, data, change, and chance. They should be given opportunities to learn how to represent those patterns numerically, geometrically and/or algebraically.

1. **Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation (*y* – 2)/(*x* – 1) = 3. Noticing the regularity in the way terms cancel when expanding (*x* – 1)(*x* + 1), (*x* – 1)(*x*2 + *x* + 1), and (*x* – 1)(*x*3 + *x*2 + *x* + 1) might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

1. Function notation is not required in Grade 8. [↑](#footnote-ref-1)