Kansas Guide to Learning:
Reasoning and Quantitative Literacy

July 2012 – July 2013

The Kansas State Department of Education in conjunction with many math leaders across the state recently completed work on the Kansas Guide to Learning: Reasoning and Quantitative Literacy project. The primary focus of this project was creating rubrics aimed at evaluating whether instructional and professional development resources are aligned with the Kansas College and Career Ready Standards for Mathematics as well as instructional shifts necessary in their implementation and training. The final project contains two rubrics with both an extended and quick version of each. The Kansas Mathematics Professional Development Review Tool provides a lens for educators to use when both creating and evaluating the many professional development resources available. Likewise, the Kansas Mathematics Instructional Review Tool is a rubric for educators to utilize when both creating and evaluating the many instructional resources available.

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Kansas Mathematics Professional Development Quick Review Tool

Professional Development Title:_____ Copyright Date:_____

Author:_____ Publisher:_____

Components
☐ Includes Facilitator Guide for professional learning experience which is organized and follows a logical progression. ☐ Includes all materials and instructions for presenter, such as: o Technology needs o Materials to be printed o Instructions for activities o Resources that participants may need during training o Suggested time allotment for professional learning and activities o Participant feedback form
Characteristics of High Quality Mathematics Professional Development
☐ Is grounded in the KCCRS and mathematically correct. ☐ Embeds key instructional shifts that are found in the KCCRS: o **Focus:** Centers on the concepts, foundational knowledge, and level of rigor that are prioritized in the standards at the targeted grade level(s). o **Coherence:** Connections and transfer of knowledge and skills within and across domains and learning progressions. o **Rigor:** Challenging mathematics including *conceptual understanding, fluency, and application*. ☐ Utilizes the Math Practice Standards within the content of the professional development. ☐ Aims to deepen teacher content knowledge and/or pedagogical knowledge. ☐ Uses either the Learning Forward Professional Development Standards and/or the NCTM's Professional Standards for Teaching Mathematics and where applicable the recommendations from Mathematical Education for Teachers (MET II).
Effective Practices Based on Scientific Research
☐ Customizes to match the participants' needs (i.e., teacher's needs in relationship to their students' needs) ☐ Advances teacher use of effective mathematical instructional strategies. ☐ Involves active learning by participants (e.g., hands-on learning, inquiry-based learning). ☐ Involves participants working in collaborative groups appropriate for the goals of the learning. ☐ Guides present and future practice through reflective strategies. ☐ Provides extended follow-up opportunities. ☐ Provides archives of resources for teachers to use.
Additional District/School Considerations
☐ There are no copyright issues apparent and resource is available as an OER. ☐ Professional learning opportunity should be applicable across states. ☐ Aligns with broader goals (e.g., school or district improvement goals, Professional Development Plan goals). ☐ Uses data to inform future professional development. ☐ Supported by active involvement of school and/or district leadership. ☐ Embedded within the school day and school year.

Item Descriptors and Bibliography

Characteristics of High Quality Mathematics Professional Development

Focus

"Teaching less, learning more" means a greater depth of understanding of mathematics, and a rich classroom environment in which reasoning, sense making, applications, and a range of mathematical practices all thrive. Focus requires that we significantly narrow the scope of content in each grade or course so students more deeply experience the content that remains. Faithfully implementing the Standards requires moving some topics traditionally taught in earlier grades up to higher grades entirely, sometimes to much higher grades. Furthermore, some topics, such as fractions, shifted to earlier grades.

The overwhelming focus of the Standards in early grades is the conceptual development of number sense and arithmetic along with the components of measurement that support it. Arithmetic in the K–5 standards includes the concepts underlying arithmetic; the number sense needed to develop arithmetic understanding, the skills of arithmetic computation, and the ability to apply arithmetic to solve problems and put arithmetic to engaging uses as a rehearsal for algebra in the middle grades and high school.

Focus on conceptual development continues in the middle and high school grades in order to prepare students for college and careers; including additional development of algebra, functions, geometry, statistics, probability, and trigonometry. Surveys suggest that postsecondary instructors value greater mastery of prerequisites over shallow exposure to a wide array of topics, and the National Mathematics Advisory Panel report (2008) supports this approach as well.

Coherence

High quality professional development should recognize that coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics, but an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations. The standards define progressions of learning that leverage these principles as they build knowledge over grades and courses.

High quality professional development should help teachers make connections between topics. The most important connections are vertical: the links from one grade (or course) to the next that allow students to progress in their mathematical education. That is why it is critical to think across grades and courses, and examine the progressions in the standards to see how major content develops over time.

High quality professional development recognizes that connections at a single grade level (or course) can be used to improve focus, by tightly linking secondary topics to the major work of the grade/course. For example, in grade 3, bar graphs are not "just another topic to cover." Rather, the standard about bar graphs asks students to use information presented in bar graphs to solve word problems using the four operations of arithmetic. Instead of allowing bar graphs to detract from the focus on arithmetic, the standards are showing how bar graphs can be positioned in support of the major work of the grade. In this way coherence can support focus.

High quality professional development resources cannot match the contours of the Standards by approaching each individual content standard as a separate event. Nor can resources align to the Standards by approaching each individual grade/course as a separate event. From the K-8 Publisher's Criteria Appendix (2012): "The standards were not so much assembled out of topics as woven out of progressions. Maintaining these

progressions in the implementation of the standards will be important for helping all students learn mathematics at a higher level. … For example, the properties of operations, learned first for simple whole numbers, then in later grades extended to fractions, play a central role in understanding operations with negative numbers, expressions with letters and later still the study of polynomials. As the application of the properties is extended over the grades, an understanding of how the properties of operations work together should deepen and develop into one of the most fundamental insights into algebra. The natural distribution of prior knowledge in classrooms should not prompt abandoning instruction in grade level content, but should prompt explicit attention to connecting grade level content to content from prior learning. To do this, instruction should reflect the progressions on which the CCSSM are built."

Rigor

To help students meet the expectations of the Standards, educators will need to pursue high quality professional development, with equal intensity, in three aspects of rigor in the major work of each grade/course: **conceptual understanding, procedural skill** and **fluency**, and **applications**. The Standards set high expectations for all three components of rigor in the major work of each grade/course, and professional development should do the same.

The word "understand" is used in the Standards to set explicit expectations for conceptual understanding, the word "fluently" is used to set explicit expectations for being reasonably fast and accurate, and the phrase "real-world problems" and the star symbol (*) is used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a content category in High School).

High quality professional development is balanced in its approach to these three aspects of rigor. Some professional development materials stress fluency in computation (or worse yet, memorization alone), without acknowledging the role of developing conceptual understanding prior to attaining fluency. Some stress conceptual understanding, without acknowledging that fluency requires separate classroom work of a different nature. Some stress pure mathematics, without acknowledging first of all that applications can be highly motivating for students, and moreover, that a mathematical education should make students fit for more than just their next mathematics course. At another extreme, some focus on applications, without acknowledging that math doesn't teach itself.

Math Practice Standards

High quality professional development provides experiences and strategies that relate to the Standards for Mathematical Practice (KCCRS), demonstrating the behaviors educators seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

High quality professional development should attend to the need to connect the mathematical practices to mathematical content in mathematics instruction. The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the

subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.
The Standards for Mathematical Practice from the Kansas College and Career Ready Standards for Mathematics are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Application

High quality professional development provides teachers with strategies for helping their students apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

High quality professional development engages teachers in opportunities to apply the mathematics they know to solve problems in the same way that their students are expected. Application of mathematics is interpreted not as a collection of isolated topics but rather in relation to other standards.

Deep Understanding

Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? High quality professional development helps teachers address this question.

High quality professional development has a hallmark of mathematical understanding as the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a+b)(x+y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a+b+c)(x+y)$. High quality professional development helps teachers develop strategies to facilitate this level of understanding in student learning.

Uses Learning Forward Standards or NCTM Standards

Learning Forward Standards	NCTM Standards
Learning Communities **Resources** **Learning Designs** **Outcomes** **Leadership** **Data** **Implementation** (Learning Forward, 2011)	Experiencing Good Mathematics Teaching Knowing Mathematics and School Mathematics Knowing Students as Learners Knowing Mathematical Pedagogy Developing As a Teacher of Mathematics The Teachers Role in Professional Development (NCTM, 1991 & 2000)
See pages _____ of the _____for further information.	**See pages _____ of the _____for further information.**

Elements of Effective Practice(s)

In the study titled *Professional Learning in the Learning Profession: A Status Report on Teacher Development in the U.S. and Abroad, A Technical Report* (Wei, Darling-Hammond, Andree, Richardson & Orphanos, 2009) provides a review of what research says regarding the relationship between teacher professional development and student learning, the availability of the kinds of professional learning opportunities that research finds most effective in the United States and in high-achieving nations around the world, and illustrates with examples how key features of effective professional development contexts and strategies operate in these systems.

Some key features of effective professional development provide opportunities and contexts for active learning to engage teachers in hands-on-work which enhances their knowledge of the content to be taught to students and how to teach it. These activities develop understanding of complex concepts by asking teachers to participate in sense making activities where they experience mathematics at a deeper level, teach in new ways, and report back to a collaborative group on the effectiveness of lessons. These activities involve modeling practices, and constructing opportunities for teachers to practice and reflect on new strategies. In addition teachers are engaged in analyzing and evaluating student work to determine the effectiveness of instruction.

"Student achievement improves most when teachers are engaged in sustained collaborative professional development that specifically focuses on deepening teachers' content knowledge and instructional practices p. 5." Additionally, curriculum, assessment, standards, and professional learning should be seamlessly linked between what teachers learn and what they are able to implement for substantial change in teaching practices to occur.

Considerations for District/Schools

Wei et. al, (2009) identified the common features characterizing professional development practices in high achieving countries as:

- Extensive opportunities for both formal and informal in-service development;
- Time for professional learning and collaboration built into teachers' work hours;
- Professional development activities that are embedded in teachers' contexts and that are ongoing over a period of time;
- School governance structures that support the involvement of teachers in decisions regarding curriculum and instructional practice;
- Teacher induction programs for new teachers with release time for new teachers and mentor teachers, and formal training for mentors.

A second resource *titled Job-Embedded Professional Development: What It Is, Who Is Responsible, and How to Get It Done Well* (Croft, Coggshall, Dolan, Powers, with Killion, J., 2010) establishes that effective professional development should begin with an analysis of school needs in terms of both student and teacher learning based on formative evidence of their performance. Through an analysis of these data, learning goals can be developed and aligned with professional development methods.

Powerful and practical connections also can be made between district and school improvement plans, resulting in greater coherence across the system. These locally based plans are highly conducive to adult learning through its focus on concrete acts of teaching that are highly relevant to teachers while requiring their active participation and construction of professional knowledge. Teachers as learners benefit from multiple opportunities to learn. Those opportunities are created when teachers are afforded the time, space, structures, and support to engage in professional development. Moreover, the school's professional culture significantly affects teachers' opportunity to learn. Adults learn best when they are self-directed, building new knowledge upon preexisting knowledge, and aware of the relevance and personal significance of what they are learning.

School leaders are instrumental in fostering an organizational culture of continuous learning and teamwork through venues such as professional learning communities and professional norms, including, for example, open-door policies for observing each other's classrooms.

Learning Forward Standards

Learning Communities
Professional learning that increases educator effectiveness and results for all students occurs within learning communities committed to continuous improvement, collective responsibility, and goal alignment.

Resources
Professional learning that increases educator effectiveness and results for all students requires prioritizing, monitoring, and coordinating resources for educator learning.

Learning Designs
Professional learning that increases educator effectiveness and results for all students integrates theories, research, and models of human learning to achieve its intended outcomes.

Outcomes
Professional learning that increases educator effectiveness and results for all students aligns its outcomes with educator performance and student curriculum standards.

Leadership
Professional learning that increases educator effectiveness and results for all students requires skillful leaders who develop capacity, advocate, and create support systems for professional learning.

Data
Professional learning that increases educator effectiveness and results for all students uses a variety of sources and types of student, educator, and system data to plan, assess, and evaluate professional learning.

Implementation
Professional learning that increases educator effectiveness and results for all students applies research on change and sustains support for implementation of professional learning for long term change.

NCTM Standards for the Professional Development for Teachers of Mathematics

Standard 1 - Experiencing Good Mathematics Teaching
Mathematics and mathematics education instructors in preservice and continuing education programs should model good mathematics teaching by-

- posing worthwhile mathematical tasks;
- engaging teachers in mathematical discourse;
- enhancing mathematical discourse through the use of a variety of tools, including calculators, computers, and physical and pictorial models;
- creating learning environments that support and encourage mathematical reasoning and teachers' dispositions and abilities to do mathematics;
- expecting and encouraging teachers to take intellectual risks in doing mathematics and to work independently and collaboratively;
- representing mathematics as an ongoing human activity;
- affirming and supporting full participation and continued study of mathematics by all students.

Standard 2 - Knowing Mathematics and School Mathematics
The education of teachers of mathematics should develop their knowledge of the content and discourse of mathematics; including-

- mathematical concepts and procedures and the connections among them;
- multiple representations of mathematical concepts and procedures;
- ways to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality;

and, in addition, develop their perspectives on-

- the nature of mathematics, the contributions of different cultures toward the development of mathematics, and the role of mathematics in culture and society;
- the changes in the nature of mathematics and the way we teach, learn, and do mathematics resulting from the availability of technology,
- school mathematics within the discipline of mathematics;
- the changing nature of school mathematics, its relationships to other school subjects, and its applications in society.

Standard 3 - Knowing Students as Learners of Mathematics
The preservice and continuing education of teachers of mathematics should provide multiple perspectives on students as learners of mathematics by developing teachers' knowledge of-

- research on how students learn mathematics;
- the effects of students' age, abilities, interests, and experience on learning mathematics;
- the influences of students' linguistic, ethnic, racial, and socioeconomic backgrounds and gender on learning mathematics;
- ways to affirm and support full participation and continued study of mathematics by all students.

Standard 4 - Knowing Mathematical Pedagogy
The preservice and continuing education of teachers of mathematics should develop teachers' knowledge of and ability to use and evaluate-

- instructional materials and resources, including technology;
- ways to represent mathematics concepts and procedures;
- instructional strategies and classroom organizational models;
- ways to promote discourse and foster a sense of mathematical community;
- means for assessing student understanding of mathematics.

Standard 5 - Developing as a Teacher of Mathematics
The preservice and continuing education of teachers of mathematics should provide them with opportunities to-

- examine and revise their assumptions about the nature of mathematics, how it should be taught, and how students learn mathematics;
- observe and analyze a range of approaches to mathematics teaching and learning, focusing on the tasks, discourse, environment, and assessment;
- work with a diverse range of students individually, in small groups, and in large class settings with guidance from and in collaboration with mathematics education professionals;
- analyze and evaluate the appropriateness and effectiveness of their teaching;
- develop dispositions toward teaching mathematics.

Standard 6 - The Teacher's Role in Professional Development
Teachers of mathematics should take an active role in their own professional development by accepting responsibility for-

- experimenting thoughtfully with alternative approaches and strategies in the classroom;
- reflecting on learning and teaching individually and with colleagues;
- participating in workshops, courses, and other educational opportunities specific to mathematics;
- participating actively in the professional community of mathematics educators;
- reading and discussing ideas presented in professional publications;
- discussing with colleagues issues in mathematics and mathematics teaching and learning;
- participating in proposing, designing, and evaluating programs for professional development specific to mathematics;
- participating in school, community, and political efforts to effect positive change in mathematics education.
- Schools and school districts must support and encourage teachers in accepting these responsibilities.

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23. Add the K-8 Publisher's Criteria document
24. Add the Appendix to the K-8 Publisher's Criteria document by Daro, McCallum, and Zimba.