Grade Four

In the years prior to grade four, students developed place value understandings, generalized written methods for addition and subtraction, and added and subtracted fluently within 1,000. They gained an understanding of single-digit multiplication and division and became fluent with such operations. Students developed an understanding of fractions as built up from unit fractions (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE FOUR

[Note: Sidebar]

Grade Four Critical Areas of Instruction

In grade four instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. (CCSSO 2010, Grade 4 Introduction).

Students also work toward fluency in addition and subtraction within 1,000,000 using the standard algorithm.

Grade Four Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and linking major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Grade 4 Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting” and “Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Grade 4 Cluster-Level Emphases

Operations and Algebraic Thinking

- [m]: Use the four operations with whole numbers to solve problems. (**4.OA.1-3▲**)

- [a/s]: Gain familiarity with factors and multiples.[1] (**4.OA.4**)
- [a/s]: Generate and analyze patterns. (**4.OA.5**)

Number and Operations in Base Ten

- [m]: Generalize place value understanding for multi-digit whole numbers. (**4.NBT.1-3▲**)
- [m]: Use place value understanding and properties of operations to perform multi-digit arithmetic. (**4.NBT.4-6▲**)

Number and Operations—Fractions

- [m]: Extend understanding of fraction equivalence and ordering. (**4.NF.1-2▲**)
- [m]: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (**4.NF.3-4▲**)
- [m]: Understand decimal notation for fractions, and compare decimal fractions. (**4.NF.5-7▲**)

Measurement and Data

- [a/s]: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.[2] (**4.MD.1-2**)
- [a/s]: Represent and interpret data. (**4.MD.4**)
- [a/s]: Geometric measurement: understand concepts of angle and measure angles. (**4.MD.5-7**)

Geometry

- [a/s]: Draw and identify lines and angles, and classify shapes by properties of their lines and angles. (**4.G.1-3**)

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[3] [m] clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

[1] Supports students' work with multi-digit arithmetic as well as their work with fraction equivalence.
[2] Students use a line plot to display measurements in fractions of a unit and to solve problems involving addition and subtraction of fractions, connecting this work to the Number and Operations – Fractions clusters.
[3] The ▲symbol will indicate standards in a Major Cluster in the narrative.

(Adapted from Smarter Balanced Assessment Consortia [Smarter Balanced], DRAFT Content Specifications 2012)

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their ability to make sense of mathematics. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for grade four students. (Refer to pages 9–13 in the "Overview of the Standards Chapters" for a complete description of the MP standards.)

Standards for Mathematical Practice (MP)

Explanations and Examples for Grade Four

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade four students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students might use an equation strategy to solve the word problem. For example, students could solve the problem "Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?" with the equation $3 \times \$12 + \$15 = a$. Students may use visual models to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
MP.2 Reason abstractly and	Fourth graders recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They

quantitatively.	extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts. Students might use array or area drawings to demonstrate and explain 154 × 6, as 154 added six times, and so develop an understanding of the distributive property. For example, 154 × 6 = (100 + 50 + 4) × 6 = (100 × 6) + (50 × 6) + (4 × 6) = 600 + 300 + 24 = 924. Teachers might ask, “How do you know” or “What is the relationship of the quantities?” to reinforce students’ reasoning and understanding.
MP.3 Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?”, “Explain your thinking,” and “Why is that true?” They not only explain their own thinking, but listen to others’ explanations and ask questions. Students explain and defend their answers and solution strategies as they answer question that require an explanation.
MP.4 Model with mathematics.	Students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, and creating equations. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Students should be encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?” Fourth graders evaluate their results in the context of the situation and reflect on whether the results make sense. For example, a student may use an area/array rectangle model to solve the following problem by extending from multiplication to division: A fourth grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?
MP.5 Use appropriate tools strategically.	Students consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line or drawings of dimes and pennies to represent and compare decimals or protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units. Students should be encouraged to answer questions such as, “Why was it helpful to use…?”
MP.6 Attend to precision.	As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
MP.7 Look for and make use of structure.	Students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They generate number or shape patterns that follow a given rule. Teachers might ask, “What do you notice when…?” or “How do you know if something is a pattern?”

MP.8 Look for and express regularity in repeated reasoning.	In grade four students notice repetitive actions in computation to make generalizations. Students use models to explain calculations and understand how algorithms work. Students examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions. Students should be encouraged to answer questions, such as “What is happening in this situation?” or “What predictions or generalizations can this pattern support?”

(Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina Department of Public Instruction [N. Carolina] 2011)

Standards-based Learning at Grade Four

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (**▲)** indicates standards in the major clusters (refer to the Grade 4 Cluster-Level Emphases chart on page #3).

Domain: Operations and Algebraic Thinking

Previously in grade three, students focused on concepts, skills, and problem solving with single-digit multiplication and division (within 100). In grade four a critical area of instruction is developing understanding and fluency with multi-digit multiplication and developing understanding of division to find quotients involving multi-digit dividends.

Operations and Algebraic Thinking	4.OA
Use the four operations with whole numbers to solve problems. 1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1] 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies	

[1] See Glossary, Table 2.

including rounding.

In earlier grades students focused on addition and subtraction, and worked with additive comparison problems (e.g., what *amount* would be added to one quantity in order to result in the other: bigger quantity = smaller quantity + difference), in grade four students compare quantities multiplicatively for the first time.

In a multiplicative comparison problem, the underlying structure is that a *factor* multiplies one quantity to result in the other (e.g., b is n times as much as a, represented by $n \times a = b$, bigger quantity = n × smaller quantity). Students interpret a multiplication equation as a comparison and solve word problems involving multiplicative comparison (**4.OA.1-2▲**) and should be able to identify and verbalize all three quantities involved: which quantity is being multiplied (the smaller quantity), which number tells how many times, and which number is the product (the bigger quantity). Teachers should be aware that students often have difficulty with understanding the order and meaning of numbers in multiplicative comparison problems, and so special attention should be paid to understanding these types of problem situations (**MP.1**).

Example: Multiplicative Comparison Problems.
Unknown Product: "Sally is 5 years old. Her mother is 8 times as old as Sally is. How old is Sally's mother?" This problem takes the form $a \times b = ?$, where the factors are known but the product is unknown.
Unknown Factor (Group Size Unknown): "Sally's mother is 40 years old. That is 8 times as old as Sally is, How old is Sally?" This problem takes the form $a \times ? = p$, where the product is known, but the quantity being multiplied to become bigger, is unknown.
Unknown Factor 2 (Number of Groups Unknown): "Sally's mother is 40 years old. Sally is 5 years old. How many times older than Sally is this?" This problem takes the form $? \times b = p$, where the product is known but the multiplicative factor, which does the enlarging in this case, is unknown.

In grade four students solve three major common types of multiplication and division problems, which are summarized in the following table.

	Unknown Product	Group Size Unknown (Partitive Division)	Number of Groups Unknown (Measurement Division)
	3 × 6 = ?	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 =?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement Example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area Example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement Example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement Example.* A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement Example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

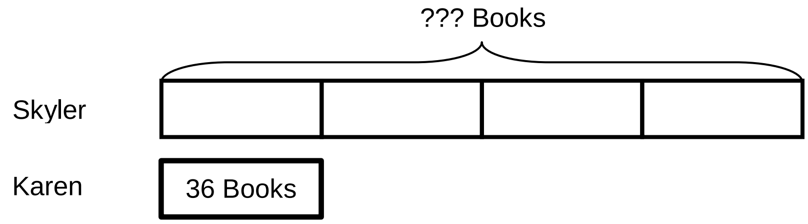
(CCSSI 2010, Glossary)[The table is also included in this Framework's Glossary.]

Students need many opportunities to solve contextual problems. A tape or bar diagram can help students visualize and solve multiplication and division word problems. Tape diagrams are useful for connecting what is happening in the problem with an equation that represents the problem.

Examples: Using Tape Diagrams to Represent Multiplication Compare Problems.

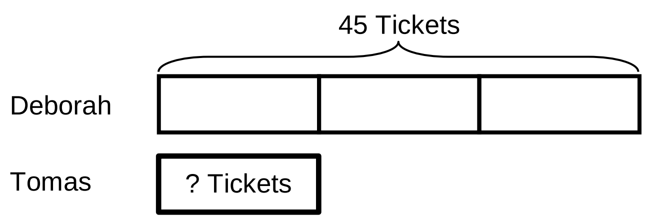
Unknown Product: "Skyler has 4 times as many books as Karen. If Karen has 36 books, how many books does Skyler have?"

Solution: If we represent the number of books that Karen has with a piece of tape, then the number of books Skyler has is represented by 4 pieces of tape of the same size. Students can represent this as $4 \times 36 = \square$.



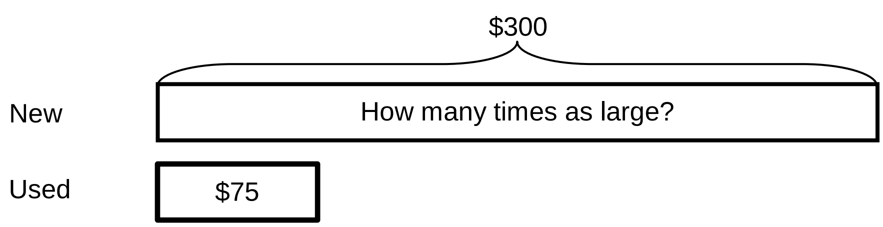
Unknown Factor (Group Size Unknown): "Deborah sold 45 tickets to the school play, which is 3 times as many as Tomas sold. How many tickets did Tomas sell?"

Solution: Here, the number of books Deborah has (the product) is known and is represented by 3 pieces of tape. The number of tickets Tomas sold would be represented by one piece of tape. This representation helps students see that the equations $3 \times \square = 45$ or $45 \div 3 = \square$ represent the problem.



Unknown Factor (Number of Groups Unknown): "A used bicycle costs $75 while a brand new one costs $300. How many times as much does the new bike cost compared to the old bike?"

Solution: Here, the student represents the cost of the used bike by a piece of tape, and decides how many pieces of this tape will make up the cost of the new bicycle. The representation leads to the equations $\square \times 75 = 300$ and $300 \div 75 = \square$.



Additionally, students solve multi-step word problems using the four operations, including problems in which remainders must be interpreted. (**4.OA.3▲**). Students use estimation to solve problems. They identify when estimation is appropriate, determine the level of accuracy needed to solve a problem and select the appropriate method of estimation. This gives rounding usefulness, rather than making rounding a separate topic that is covered arbitrarily.

Examples: Multi-step Word Problems and Strategies Called for in Standard 4.OA.3▲.
1. "There are 146 students going on a field trip. If each bus held 30 students, how many buses are needed?" *Solution:* Since 150 ÷ 30 = 5, it seems like there should be around 5 buses. When we try to divide 146 by 30, we get 4 groups with 26 leftover. This means that $146 = 4 \times 30 + 26$. There are 4 filled with 30 students, with a fifth bus holding only 26 students. (In this case, one more than the quotient is the answer.)
2. "Suppose that 250 pencils were distributed equally among 33 students for a geometry project. What is the largest number of pencils each student can receive?" *Solution:* Since 240 ÷ 30 = 8, it seems like each student should receive close to 8 pencils. When we divide 250 by 33, we get 7 with a remainder of 19. This means that $250 = 33 \times 7 + 19$. This tells us that each student can have 7 pencils with 19 leftover for the teacher to hold on to.
3. "Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each pack. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?" *Solution:* "First, I multiplied 3 packs by 6 bottles per pack which equals 18 bottles. Then I multiplied 6 packs by 6 bottles per pack which is 36 bottles. I know 18 plus 36 is around 50. Since we're trying to get to 300, we'll need about 250 more bottles**."**

As students compute and interpret multi-step problems with remainders (**4.OA.3▲**), they also reinforce important mathematical practices as they make sense of the problem and reason about how the context connects to the four operations (**MP.1, MP.2**).

Common Misconceptions.
• Teachers may try to help their students by telling them that multiplying a number two numbers in a multiplicative comparison situation always makes the product *bigger*. While this is true with whole numbers greater than 1, it is *not true* when the first factor is a fraction smaller than 1 (or when the first factor is negative), something students will encounter in later grades. Teachers should be careful to emphasize that multiplying by a number *greater than 1* results in a product larger than the original number (**4.OA.1-2▲**). • Students might be confused by the difference between six more than a number (additive) compared to six times a number (multiplicative). For example, using 18 and 6, a question could be "How much more is 18 than 6?" Thinking multiplicatively the answer is 3, however, thinking additively the answer is 12. (Adapted from KATM 4th FlipBook 2012). • It is common practice when dividing numbers to write $250 \div 33 = 7 \text{ R } 19$, for example. While this

notation has been used for quite some time, it obscures the relationship between the numbers in the problem, e.g., when students find fractional answers the correct equation becomes $250 \div 33 = 7\frac{19}{33}$. It is more accurate to write the answer in words, such as by saying, "when we divide 250 by 33, the quotient is 7 with 19 leftover," or to write the equation "$250 = 33 \times 7 + 19$," as in the example above. See standard **(4.NBT.6▲)**.

Operations and Algebraic Thinking 4.OA

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Students find all factor pairs for whole numbers in the range 1–100 **(4.OA.4).** They extend the idea of decomposition to multiplication and learn to use the term *multiple*. Any whole number is a multiple of each of its factors. For example, 21 is a multiple of 3 and a multiple of 7 because 21 = 3 × 7. A number can be multiplicatively decomposed into equal groups and expressed as a product of these two factors (called factor pairs). A *prime number* has only one and itself as factors. A *composite number* has two or more factor pairs. The number 1 is neither prime nor composite. To find all factor pairs for a given number, students need to search systematically, by checking if 2 is a factor, then 3, then 4, and so on, until they start to see a "reversal" in the pairs (e.g., after finding the pair 6 and 9 for 54, students will next find the reverse pair, 9 and 6). Knowing how to determine factors and multiples is the foundation for finding common multiples and factors in grade six (Adapted from The University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA 2011).

Common Misconceptions.

- Students may think the number 1 is a prime number or that all prime numbers are odd numbers (counterexample: 2 has only 2 factors—1 and 2).
- When listing multiples of numbers students may not list the number itself. Students should be reminded that the smallest multiple is the number itself.
- Students may think larger numbers have more factors.

Having students share all factor pairs and how they found them will help students avoid some of these misconceptions (Adapted from KATM 4th FlipBook 2012).

Focus, Coherence, Rigor:

The concepts and terms "prime" and "composite" are new at grade four. As students gain familiarity with factors and multiples (**4.OA.4**) they also reinforce and support major work at the grade, such as multi-digit arithmetic in the cluster "Use place value understanding and properties of operations to perform multi-digit arithmetic" (**4.NBT.4-6▲**) and fraction equivalence in the cluster "Extend understanding of fraction equivalence and ordering" (**4.NF.1-2▲**).

Operations and Algebraic Thinking 4.OA

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Understanding patterns is fundamental to algebraic thinking. In grade four students generate and analyze number and shape patterns that follow a given rule **(4.OA.5)**. Students begin by reasoning about patterns, connecting a rule for a given pattern with its sequence of numbers or shapes. A pattern is a sequence that repeats or evolves in a predictable process over and over. A rule dictates what that process will look like. Patterns that consist of repeated sequences of shapes or growing sequences of designs can be appropriate for the grade.

For example, students could examine a sequence of dot designs in which each design has 4 more dots than the previous one and then reason about how the dots are organized in the design to determine the total number of dots in the 100th design. **(MP.2, MP.4, MP.5, MP.7)** (Adapted from Progressions K-5 CC and OA 2011).

Following are examples of problems that can help students understand patterns:

"Double Plus One" available at http://illustrativemathematics.org/standards/k8# (Illustrative Mathematics 2013).

"Patterns that Grow" available at http://illuminations.nctm.org/LessonDetail.aspx?ID=U103 (National Council of Teachers of Mathematics [NCTM] Illuminations 2013).

Focus, Coherence, Rigor:

Numerical patterns (**4.OA.5**) allow students to reinforce facts and develop fluency with operations and support major work at the grade in the cluster "Use place value understanding and properties of operations to perform multi-digit arithmetic" (**4.NBT.4-6▲**).

Domain: Number and Operations in Base Ten

In grade four, students extend their work in the base-ten number system and generalize previous place value understanding to multi-digit whole numbers (less than or equal to 1,000,000).

Numbers and Operations in Base Ten	4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Students read, write, and compare numbers based on the meaning of the digits in each place (**4.NBT.1-2▲**). In the base-ten system, the value of each place is 10 times the value of the place to the immediate right. By reasoning that each unit in a place becomes one unit in the next left place (because it is multiplied by ten), students can come to see and understand that multiplying by 10 yields a product in which each digit of the multiplicand is shifted one place to the left (Adapted from Progressions K-5 NBT 2011).

Students need multiple opportunities to use real-world contexts to read and write multi-digit whole numbers. Student need to reason about the magnitude of digits in a number and analyze the relationships of number. They can build larger numbers by using graph paper with very small squares and labeling examples of each place with digits and words (e.g., ten thousand and 10,000).

To read and write numerals between 1,000 and 1,000,000, students need to understand the role of commas. Each sequence of three digits made by commas is read as hundreds, tens, and ones, followed by the name of the appropriate base-thousand unit (e.g., thousand, million). Layered place value cards such as those used in earlier grades can be put on a frame with the base-thousand units labeled below. Then cards forming hundreds, tens, and ones can be placed on each section and the name read off using the card values followed by the word “million”, then “thousand”, then the silent ones **(MP.2, MP.3, MP.8).**

Fourth-grade students build on the grade-three skill of rounding to the nearest 10 or 100 to round multi-digit numbers and to make reasonable estimates of numerical values. (**4.NBT.3▲**).

Example: Rounding Numbers in Context. (MP.4)

The population of Midtown, U.S.A., was last recorded to be 76,398. The city council wants to round the population to the nearest thousand for a business brochure. What number should they round the population to?

Solution: When students represent numbers stacked vertically, they can see the relationships between the numbers more clearly. Students might think: “I know the answer is either 76,000 or 77,000. If I write 76,000 below 76,398 and 77,000 above it, I can see that the midpoint is 76,500, which is *above* 76,398. This tells me they should round the population to 76,000.”

77,000
76,398
76,000

Numbers and Operations in Base Ten 4.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-

digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

At grade four, students become fluent with addition and subtraction with multi-digit whole numbers to 1,000,000 using standard algorithms (**4.NBT.4▲**). A central theme in multi-digit arithmetic is to encourage students to develop methods they understand, can explain, and can think about, rather than merely following a sequence of directions, rules or procedures they do not understand. In previous grades, students built a conceptual understanding of addition and subtraction with whole numbers as they applied multiple methods to compute and solve problems. The emphasis in grade four is on the power of the regular one-for-ten trades between adjacent places that let students extend a method they already know to many places. Because students in grades two and three have been using at least one method that will generalize to 1,000,000, this extension in grade four should not have to take a long time. Thus, students will also have sufficient time for the major new topics of multiplication and division (**4.NBT.5-6▲**).

[Note: Sidebar]

Fluency
In kindergarten through grade six there are individual content standards that set expectations for fluency with computations using the standard algorithm (e.g., "fluently" add and subtract multi-digit whole numbers using the standard algorithm (**4.NBT.4▲**)). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding (such as reasoning about quantities, the base-ten system, and properties of operations), thoughtful practice, and extra support where necessary.
The word "fluent" is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (Adapted from Progressions K-5 CC and OA 2011 and PARCC 2012).

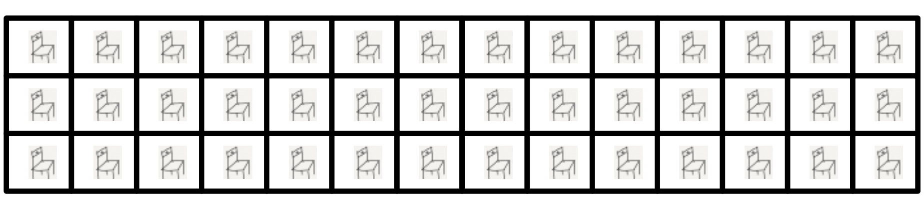
In grade four students extend multiplication and division to include whole numbers greater than 100. Students should use methods they understand and can explain to multiply and divide. The standards (**4.NBT.5-6▲**) call for students to use visual representations such as area and array models that students draw and connect to equations and written numerical work that supports student reasoning and explanation of methods. By reasoning repeatedly about the connections between math drawings and written numerical work, students can come to see multiplication and division algorithms as abbreviations or summaries of their reasoning about quantities.

After students have discussed how to show an equal groups situation or a multiplication compare situation with an area model, they can use area models for any multiplication situation. The rows represent the equal groups of objects or the larger compared quantity and students imagine that the objects in the situation lie in the squares and so form an array. Such array models become too difficult to draw, so students can make sketches of rectangles and then label the resulting product as the number of things or square units. When using area models to represent an actual area situation, the two factors are in length units (e.g., cm) while the product is in square units (e.g., cm^2).

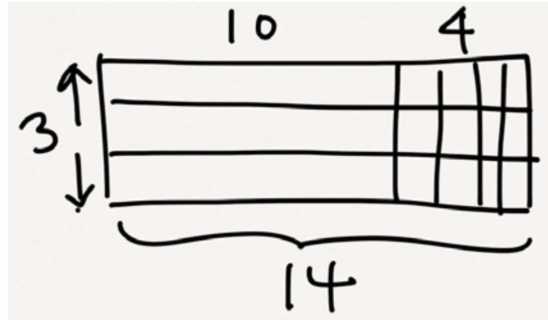
Example: Area Models and Strategies for Multi-digit Multiplication , Single Digit Multiplier (4.NBT.5▲)

"Chairs are being set up for a small play. There should be 3 rows of chairs and 14 chairs in each row. How many chairs will be needed?"

Solution: As in grade three, when students first made the connection between array models and the area model, students might start by drawing a sketch of the situation. They can then be reminded to see the chairs as if surrounded by unit squares and hence a model of a rectangular region. With base-ten blocks or math drawings **(MP.2, MP.5)**, students abstract the problem and see it being broken down into $3 \times (10 + 4)$.



Making a sketch like the one above becomes cumbersome, so students move toward representing such drawings abstractly, with rectangles, as shown to the right. This builds on the work begun in grade 3. Such diagrams help children see the distributive property: "3×14 can be written as $3 \times (10 + 4)$, and I can do the multiplications separately and add the results, $3 \times (10 + 4) = 3 \times 10 + 3 \times 4$. The answer is $30 + 12 = 42$, or 42 chairs."



In grade three students worked with multiplying single digit numbers by multiples of 10 **(3.NBT.3)**. This idea is extended in grade four, e.g., since $6 \times 7 = 42$, it must be true that:

- $6 \times 70 = 420$, since this is "six times seven tens," which is 42 tens,
- $6 \times 700 = 4200$, since this is "six times seven hundreds," which is 42 hundreds,
- $6 \times 7000 = 42{,}000$, since this is "six times seven thousands," which is 42 thousands,
- $60 \times 70 = 4200$, since this is "sixty times seven tens," which is 420 tens, or 4200.

Math drawings and base-ten blocks support the development of these *extended multiplication facts*. The ability to find products such as these is important when using variations of the standard algorithm for multi-digit multiplication, described below.

Examples: Developing Written Methods for Multi-Digit Multiplication. (4.NBT.5▲)

Left to right Right to left Right to left

Find the product: 6×729.

Solution: Sufficient practice with drawing rectangles (or constructing them with base-ten blocks) will help students understand

	729 = 700	+ 20	+ 9
6	6 × 700 = 6 groups of 7 hundreds = 42 hundreds = 4200	6 × 20 = 6 groups of 2 tens = 12 tens = 120	6 × 9 = 54

that the problem can be represented with a rectangle such as the one shown. The product is given by the total area: $6 \times 729 = 6 \times 700 + 6 \times 20 + 6 \times 9$. Understanding extended multiplication facts allows students to find the *partial products* quickly. Student can record the multiplication in several ways:

showing the partial products

729	
× 6	thinking:
4200	6 × 7 hundreds
120	6 × 2 tens
54	6 × 9
4374	

showing the partial products

729	
× 6	
54	6 × 9
120	6 × 2 tens
4200	6 × 7 hundreds
4374	

recording the "carries" below

729
× 6
1 5
4224
4374

Find the product: 27×65.

Solution: This time, a rectangle is drawn, and "like" base-ten units (e.g., tens and ones) are represented by sub-regions of the rectangle. Repeated use of the distributive property shows that:

$$27 \times 65 = (20 + 7) \times 65 = 20 \times 65 + 7 \times 65$$
$$= 20 \times (60 + 5) + 7 \times (60 + 5)$$
$$= 20 \times 60 + 20 \times 5 + 7 \times 60 + 7 \times 5.$$

	60	+ 5
20	20 × 60 = 2 tens times 6 tens= 12 hundreds = 1200	20 × 5 = 2 tens × 5= 10 tens= 100
+ 7	7 × 60 = 7 × 6 tens = 42 tens = 420	7 × 5 = 35

The product is again given by the total area:

$$1200 + 100 + 420 + 35 = 1755.$$

Below are two written methods for recording the steps of the multiplication.

Showing the partial products

65	
× 27	thinking:
35	7 × 5
420	7 × 6 tens
100	2 tens × 5
1200	2 tens × 6 tens
1755	

Recording the carries below for correct place value placement

65
× 27
4 3
25
1 1
20**0**
1755

Notice that the boldfaced **0** is included in the second method, indicating that we are multiplying not just by 2 in this row, but by 2 *tens*.

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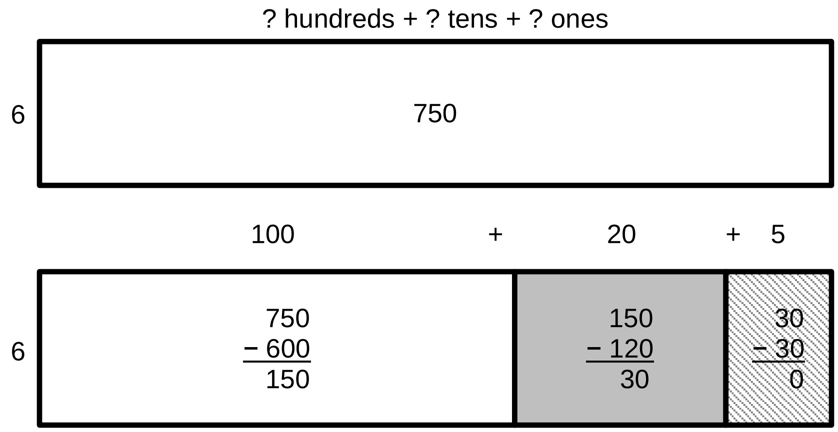
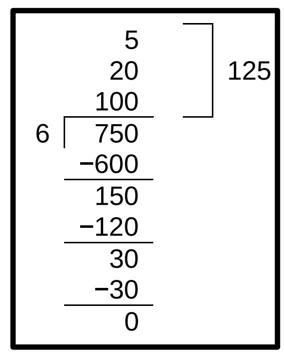
General methods for computing quotients of multi-digit numbers and one-digit numbers **(4.NBT.6▲)** rely on the same understandings as for multiplication, but these are cast in terms division. For example, students may see division problems as knowing the area of a rectangle but not one side length (the quotient) or as finding the size of a group when the number of groups is known (measurement division).

Example: Using the Area Model to Develop Division Strategies.

Find the quotient: 750 ÷ 6.



Solution: "Just like with multiplication, I can set this up as a rectangle, but with one side unknown since this is the same as $?? \times 6 = 750$. I find out what the number of hundreds would be for the unknown side length; that's 1 hundred or 100, since 100 × 6 = 600 and that's as large as I can go. Then, I have $750 - 600 = 150$ square units left, so I find the number of tens that are in the other side. That's 2 tens or 20, since 20 × 6 = 120. Last, there are $150 - 120 = 30$ square units left, so the number of ones on the other side must be 5 since $5 \times 6 = 30$."



One way students can record this is shown, wherein *partial quotients* are stacked atop one another, with 0s included to indicate place value and as a reminder of how students obtained the numbers. The full quotient is the sum of these stacked numbers.

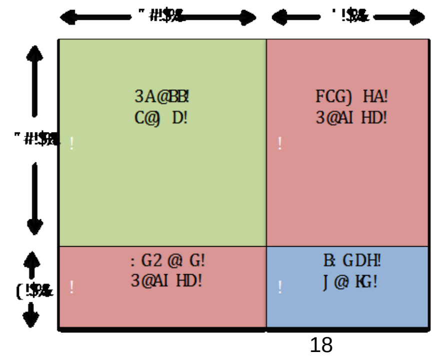
General methods for multi-digit division computation include decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This work will continue in grade five and culminate in fluency with the standard algorithm in grade six (Adapted from PARCC 2012).

In grade four students also find whole number quotients with remainders **(4.NBT.6▲).** When students experience finding remainders, they should learn the appropriate way to write the result. For instance, students divide and find that $195 \div 9 = 21$ with 6 leftover. This can be written as $195 = 21(9) + 6$. When put into a context, the latter equation makes sense. For instance, if 195 books are distributed equally among 9 classrooms, then each classroom gets 21 books with 6 books leftover. The equation $195 = 21(9) +$

6 is closely related to the equation $195 \div 9 = 21\frac{6}{9}$ which students will write in later grades. The notation $195 \div 9 = 21 \text{ R } 6$ is best avoided.

As students decompose numbers to solve multiplication problems they also reinforce important mathematical practices such as seeing and making use of structure (**MP.7**). As they illustrate and explain calculations they model (**MP.4**), use appropriate drawings as tools strategically (**MP.5**) and attend to precision (**MP.6**) using base-ten units.

Following is a sample problem that connects the Standards for Mathematical Content and the Standards for Mathematical Practice.

Standards	*Explanations and Examples*
4.NBT.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and properties of operations. Illustrate and explain the calculation using equations, rectangular arrays, and/or area models. 4.MD.3: Apply the area and perimeter formals for rectangles in real-world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*	**Sample Problem:** What are the areas of the four sections of Mr. Griffin's backyard? There is a grass lawn, a flower garden, a tomato garden, and a stone patio. What is the area of his entire backyard? How did you find your answer?  18 × 14 Area of Stone Patio→ 32 (4×8) Area of Tomato Garden→ 40 (4×10) Area of Flower Garden→ 80 (10×8) Area of Grass Lawn→100 (10×10) Area of Entire Backyard→ 252 (14×18) **Solution:** The areas of the four sections are 100 sq. ft., 80 sq. ft., 40 sq. ft., and 32 sq. ft. respectively. The area of the entire backyard is the sum of these areas, (100+80+40+32) sq. ft., or 252 sq. ft. This is the same as finding the product (18×14) sq. ft. **Classroom Connections:** The purpose of this task is to illuminate the connection between the area of a rectangle as representing the product of two numbers and the partial products algorithm for multiplying multi-digit numbers. In this algorithm, which is shown to the right, each digit of one number is multiplied by the each digit of the other number and the "partial products" are written down. The sum of these partial products is the product of the original numbers. Place value can be emphasized by specifically reminding students that if we multiply the two 10s together, since each represents one 10, their product is 100. Finally, the area model provides a visual justification for how the algorithm works. **Connecting to the Standards for Mathematical Practice:** (MP.1) Students make sense of the problem when they see that the measurements on the side and top of the diagram persist and yield the measurements of the smaller areas. (MP.2) Students reason abstractly as they represent the areas of the yard as multiplication problems to be solved. (MP.5) Students use appropriate tools strategically when they apply the formula for the area of a rectangle to solve the problem. They organize their work in a way that makes sense to them. (MP.7) Teachers can use this problem and similar problems to illustrate the distributive property of multiplication. In this case, we have that 18×14 = (10×14) + (8×14) = (10×10) + (10×4) + (8×10) + (8×4).

Domain: Number and Operations—Fractions[4]

Student proficiency with fractions is essential to success in algebra at later grades. In grade three students developed an understanding of fractions as built from unit fractions. A critical area of instruction in grade four is developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers.

Numbers and Operations—Fractions	4.NF

Extend understanding of fraction equivalence and ordering.

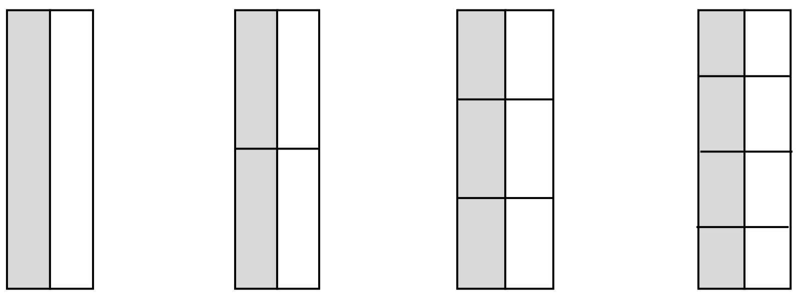
1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Grade four students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction (e.g., $\frac{a}{b} = \frac{n \times a}{n \times b}$, for $n \neq 0$). Students use visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size (**4.NF.1▲**). This property forms the basis for much of the work with fractions in fourth grade; including comparing, adding, and subtracting fractions and the introduction of finite decimals.

Students reason about and explain why fractions are equivalent using visual models. For example, the area models below all show fractions equivalent to $\frac{1}{2}$, and while in grade three students simply justified that all the models represent the same amount visually, in grade four students reason about *why* it is true that

[4] In grade four fractions include those with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

$\frac{1}{2} = \frac{2\times1}{2\times2} = \frac{3\times1}{3\times2} = \frac{4\times1}{4\times2}$, etc. They use reasoning such as: when a horizontal line is drawn through the center of the first model to obtain the second, both the number of equal parts and the number of those parts we are counting double ($2 \times 2 = 4$ in the denominator, $2 \times 1 = 2$ in the numerator, respectively), but even though there are more parts counted they are smaller parts. Students notice connections between the models and the fractions they represent in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions. Students also emphasize the inversely related changes: the number of unit fractions becomes larger, but the size of the unit fraction becomes smaller.



$\frac{1}{2}$ $\frac{2}{4} = \frac{2\times1}{2\times2}$ $\frac{3}{6} = \frac{3\times1}{3\times2}$ $\frac{4}{8} = \frac{4\times1}{4\times2}$

(Adapted from Arizona 2012)

Students should have repeated opportunities to use pictures such as these and the ones below to understand the general method for finding equivalent fractions. Of course, students may also come to see that the rule works both ways, for example:

$$\frac{28}{35} = \frac{7\times4}{7\times5} = \frac{4}{5}.$$

Teachers must be careful to not overemphasize this “simplifying” of fractions, as there is no mathematical reason for doing so, though depending on the problem context one form may be more desirable. In particular, teachers should avoid the use of the term “reducing” fractions for this process, as the value of the fraction itself is *not* being reduced. A more neutral term such as “renaming” (which hints to these fractions simply being different names for the same amount) allows for referring to this strategy without the potential for student misunderstanding.

[Note: Sidebar]

Focus, Coherence, Rigor:

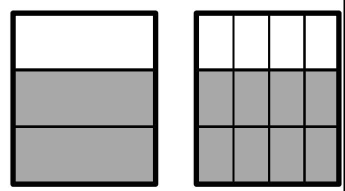
While it is true that one can justify that $\frac{a}{b} = \frac{n \times a}{n \times b}$ by arguing that:

$$\frac{n \times a}{n \times b} = \frac{n}{n} \times \frac{a}{b} = 1 \times \frac{a}{b} = \frac{a}{b}$$

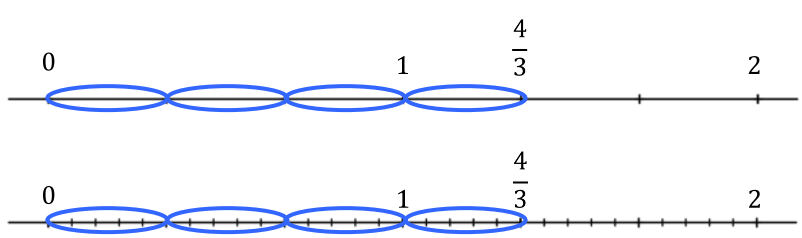
i.e., that we are simply multiplying by 1 in the form of $\frac{n}{n}$, since students *have not yet encountered* the general notion of fraction multiplication in fourth grade, this argument should be avoided in favor of developing an understanding with diagrams and reasoning about the size and number of parts that are created in this process. Students will learn the general rule that $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ in grade five.

Examples: Reasoning With Diagrams That $\frac{a}{b} = \frac{n \times a}{n \times b}$.

Using an Area Model: The whole is the rectangle, measured by its area. The picture on the left shows the area divided into three rectangles of equal area (thirds) with two of them shaded (2 pieces of size $\frac{1}{3}$), representing $\frac{2}{3}$. On the right, the vertical lines divide the parts (the thirds) into smaller parts. There are now 4×3 smaller rectangles of equal area, and the shaded area now comprises 4×2 of them, so it represents $\frac{4 \times 2}{4 \times 3}$.



Using a Number Line: The top number line shows $\frac{4}{3}$: it is 4 parts when the unit length is divided into three equal parts and then iterated. When each of the intervals of length $\frac{1}{3}$ is further divided into 5 equal parts, there are now 5×3 of these new equal parts in the unit interval. Since 4 of the $\frac{1}{3}$ parts were circled before, and each of these has been subdivided into 5 parts, there are now 5×4 of these new small parts. Therefore $\frac{4}{3} = \frac{5 \times 4}{5 \times 3} = \frac{20}{15}$.



(Above examples adapted from Progressions 3-5 NF 2012)

Creating equivalent fractions by dividing and shading squares or circles, and matching each fraction to its location on the number line can reinforce students' understanding of fractions. For example, see "Equivalent Fractions" available at http://illuminations.nctm.org/activitydetail.aspx?id=80 (NCTM Illuminations 2013).

Students apply their new understanding of equivalent fractions to compare two fractions with different numerators and different denominators (**4.NF.2▲**). They compare fractions using benchmark fractions, and by finding common denominators or common numerators. Students explain their reasoning and record their results using >, < and = symbols.

Examples: Comparing Fractions.
1. Students might compare fractions to benchmark fractions, e.g. comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8} = \frac{1}{2}$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6} = \frac{1}{2}$, it must be true that $\frac{3}{8} < \frac{2}{3}$.
2. Students compare $\frac{5}{8}$ and $\frac{7}{12}$ by writing them with a common denominator. They find that $\frac{5}{8} = \frac{5\times12}{8\times12} = \frac{60}{96}$ and $\frac{7}{12} = \frac{7\times8}{12\times8} = \frac{56}{96}$ and reason therefore that $\frac{5}{8} > \frac{7}{12}$. Notice that students do not need to find the smallest common denominator for two fractions; any one will work.
3. Students can also find a common numerator to compare $\frac{5}{8}$ and $\frac{7}{12}$. They find that $\frac{5}{8} = \frac{5\times7}{8\times7} = \frac{35}{56}$ and $\frac{7}{12} = \frac{7\times5}{12\times5} = \frac{35}{60}$. They then reason that since parts of size $\frac{1}{56}$ are larger than parts of size $\frac{1}{60}$ when the whole is the same, that $\frac{5}{8} > \frac{7}{12}$.

Numbers and Operations—Fractions 4.NF

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
    a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8*

+ 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

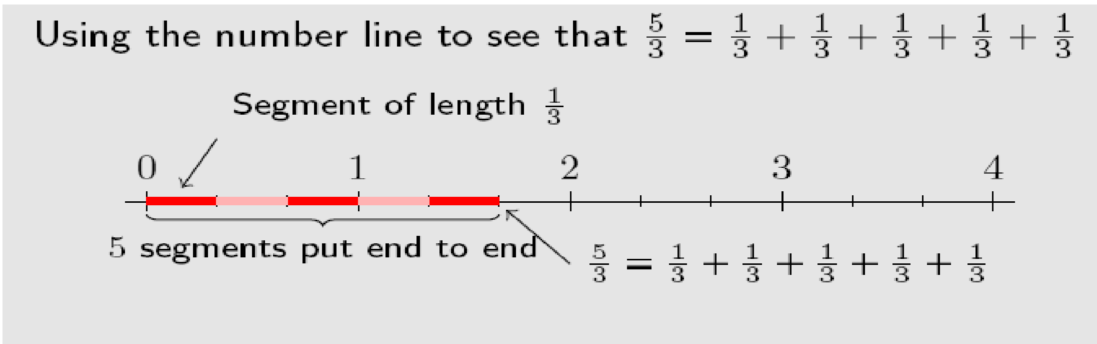
d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

In grade four students extend previous understanding of addition and subtraction of whole numbers to add and subtract fractions with like denominators (**4.NF.3▲**). They begin by understanding a fraction $\frac{a}{b}$ as a sum of the unit fractions $\frac{1}{b}$. In grade three, students learned that the fraction $\frac{a}{b}$ represented a parts when a whole is broken into b equal parts (i.e., parts of size $\frac{1}{b}$.) However, in grade four, students connect this understanding of a fraction with the operation of addition; for instance, they see now that if a whole is broken into 4 equal parts and 5 of them are taken, then this is represented by both $\frac{5}{4}$ and the expression $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ **(4.NF.3b▲)**. They experience composing fractions from and decomposing fractions into sums of unit fractions and non-unit fractions in this general way, e.g., by seeing $\frac{5}{4}$ also as

- $\frac{1}{4}+\frac{1}{4}+\frac{3}{4}$
- $\frac{2}{4}+\frac{3}{4}$
- $\frac{1}{4}+\frac{3}{4}+\frac{1}{4}$, etc.

Working with this standard supports student learning of **(4.NF.3a▲)** and **(4.NF.3d▲)** by writing and using unit fractions. It also helps students avoid the common misconception of adding two fractions by adding their numerators and denominators, e.g. erroneously writing $\frac{1}{2}+\frac{5}{6}=\frac{6}{8}$. Work with **(4.NF.3b▲)** helps students see that the unit fraction for the total is the same as the unit fractions being added and grouped into fractions made from that unit fraction. In general, the meaning of addition is the same for both fractions and whole numbers. Students understand addition as “putting together” like units and they visualize how fractions are built from unit fractions and that a fraction is a sum of unit fractions.

Students may use visual models to support this understanding, for example, showing that $\frac{5}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ by using a number line model. **(MP.1, MP.2, MP.4, MP.6, MP.7).**



(Source: Progressions 3-5 NF 2012)

Students add or subtract fractions with like denominators, including mixed numbers **(4.NF.3a, c▲)** and solve word problems involving fractions **(4.NF.3d▲).** They connect their understanding of any fraction as being composed of unit fractions to realize that, for example:

$$\frac{7}{5} + \frac{4}{5} = \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^{7} + \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^{4} = \overbrace{\frac{1}{5} + \cdots + \frac{1}{5}}^{7+4} = \frac{7+4}{5}.$$

This quickly allows students to develop a general principle that $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$. Using similar reasoning, students understand that $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$.

Students also compute sums of whole numbers and fractions, by realizing that any whole number can be written as an equivalent number of unit fractions of a given size, e.g. they find the sum $3 + \frac{7}{2}$ in the following way:

$$3 + \frac{7}{2} = \frac{6}{2} + \frac{7}{2} = \frac{13}{2}.$$

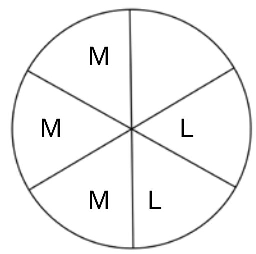
Understanding this method of adding a whole number and fraction allows students to accurately convert mixed numbers into fractions, e.g.:

$$4\frac{5}{8} = 4 + \frac{5}{8} = \frac{32}{8} + \frac{5}{8} = \frac{37}{8}.$$

Students should develop a firm understanding that a mixed number indicates the sum of a whole number and a fraction (i.e., $a\frac{b}{c} = a + \frac{b}{c}$), and should learn a method for converting them to fractions that is connected to the meaning of fractions such as the one above, rather than typical rote methods.

Examples: Reasoning With Addition and Subtraction of Fractions. (4.NF.3a-d▲).

1. Mary and Lacey share a pizza. Mary ate $\frac{3}{6}$ of the pizza and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat altogether? Use the picture of a pizza to explain your answer.

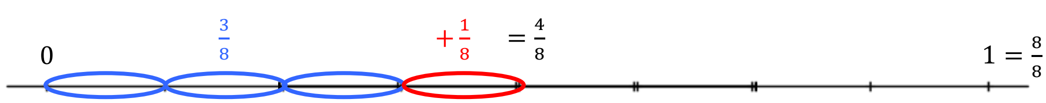


Solution: "I labeled three sixths for Mary and two sixths for Lacey. I can see that altogether they've eaten $\frac{5}{6}$ of the pizza. Also, I know that

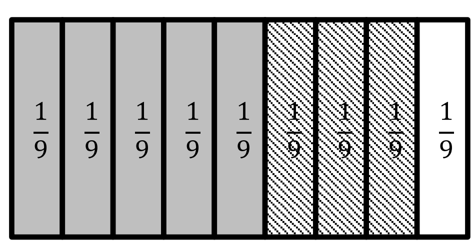
$$\frac{3}{6} + \frac{2}{6} = \frac{2+3}{6} = \frac{5}{6}."$$

2. Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Is it enough to complete the packaging?

Solution: "I know I need to find $5\frac{3}{8} + 3\frac{1}{8}$ to find out how much they have altogether. I know that altogether they have $3 + 5 = 8$ feet of ribbon plus the other $\frac{1}{8} + \frac{3}{8}$ feet of ribbon. Altogether this is $8\frac{4}{8}$ feet of ribbon, which means they have enough ribbon to do their packaging. They even have $\frac{1}{8}$ feet of ribbon left."



3. Elena, Matthew, and Kevin painted a wall. Elena painted $\frac{5}{9}$ of the wall and Matthew painted $\frac{3}{9}$ of the wall. Kevin paints the rest. How much of the wall does Kevin paint? Use the picture to help find your answer.



Solution: "I can show in the picture that Elena and Matthew painted $\frac{8}{9}$ altogether by shading what Elena and Matthew painted. The remaining that Kevin paints is $\frac{1}{9}$. I can write this as $1 - \frac{8}{9} = \frac{1}{9}$, or even $1 - \frac{5}{9} - \frac{3}{9} = \frac{1}{9}$." (New York State Education Department [NYSED] 2012).

Numbers and Operations—Fractions 4.NF

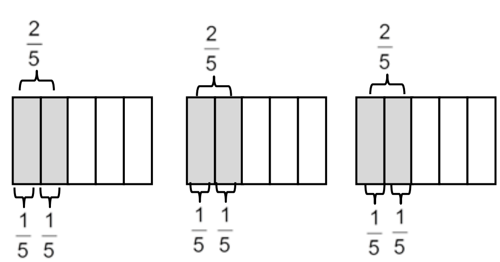
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Previously in grade three, students learned that 3 × 7 can be represented as the total number of objects in 3 groups of 7 objects, and that they could find this by finding the sum 7 + 7 + 7. Grade four students apply this concept to fractions, understanding a fraction $\frac{a}{b}$ as a multiple of $\frac{1}{b}$ (**4.NF.4a▲**). Intimately connected with standard **(4.NF.3)**, students make the shift to seeing $\frac{5}{3}$ as $5 \times \frac{1}{3}$, for example by seeing:

$$\frac{5}{3} = \overbrace{\frac{1}{3} + \cdots + \frac{1}{3}}^{5\text{ times}} = 5 \times \frac{1}{3}$$

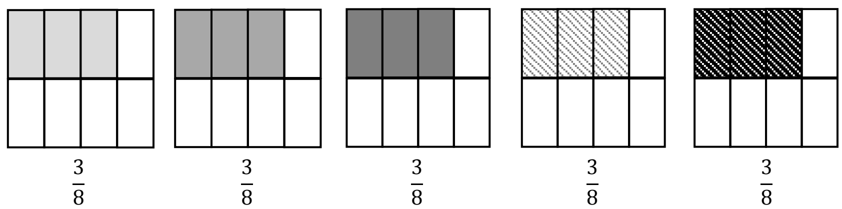
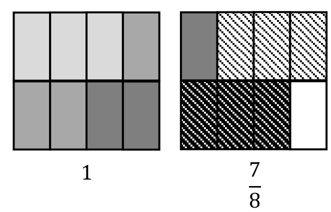
Students then extend this understanding to make meaning of the product of a whole number and a fraction (**4.NF.4b▲**), for example, by seeing $3 \times \frac{2}{5}$ as:



$$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5}.$$

(Progressions 3-5 NF 2012)

Students are presented with opportunities to work with problems involving multiplication of a fraction by a whole number in context to relate situations, models, and corresponding equations (**4.NF.4c▲**).

Example: Multiplying a Fraction by a Whole Number (4.NF.4c▲).
Each person at a dinner party eats $\frac{3}{8}$ of a pound of pasta. There are 5 people at the party. How many pounds of pasta are needed? Pasta comes in 1-lb boxes. How many boxes should be bought? *Solution*: If five rectangles are drawn, with $\frac{3}{8}$ of a pound shaded in each rectangle, then students see that they are finding $5 \times \frac{3}{8} = \frac{15}{8}$.  The separate eighths can be collected together to illustrate that altogether $1\frac{7}{8}$ pounds of pasta will be needed for the party. This means that 2 boxes should be bought. 

(Adapted from Arizona 2012 and N. Carolina 2011)

Numbers and Operations—Fractions 4.NF
Understand decimal notation for fractions, and compare decimal fractions. 5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* 6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* 7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

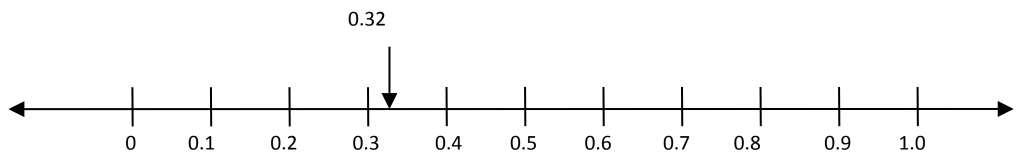
[4] Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

In fourth grade students develop an understanding of decimal notation for fractions and compare *decimal fractions* (fractions with denominator 10 or 100). This work lays the foundation for performing operations with decimal numbers in grade five. Students learn to add decimal fractions by converting them to fractions with the same denominator (**4.NF.5▲)**. For example, students express $\frac{3}{10}$ as $\frac{30}{100}$ before they add $\frac{30}{100}+\frac{4}{100}=\frac{34}{100}$. Students can use base ten blocks, graph paper, and other place value models to explore the relationship between fractions with denominators of 10 and 100 (Adapted from Progressions 3-5 NF 2012).

In grade four, students first use decimal notation for fractions with denominators 10 or 100 (**4.NF.6▲**), understanding that the number of digits to the right of the decimal point indicates the number of zeros in the denominator. Students make connections between fractions with denominators of 10 and 100 and place value. They read and write decimal fractions; for example, students say 0.32 as "thirty-two hundredths" and learn to flexibly write this as both 0.32 and $\frac{32}{100}$.

> **Focus, Coherence, Rigor.**
> Teachers are urged to consistently use place value based language when naming decimals to reinforce student understanding, i.e., by saying "four tenths" when referring to 0.4, as opposed to "point four", and by saying "sixty eight hundredths" when referring to 0.68, as opposed to "point sixty eight" or "point six eight."

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. Students reason that $\frac{32}{100}$ is a little more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$, so it would be placed on the number line near that value. **(MP.2, MP.4, MP.5, MP.7)**



Students compare two decimals to hundredths by reasoning about their size (**4.NF.7▲**).They relate their understanding of the place value system for whole numbers to fractional parts represented as decimals. Students compare decimals using the meaning of a decimal as a fraction, making sure to compare fractions with the same denominator and that the "wholes" are the same.

Common misconceptions:

- Students sometimes treat decimals as whole numbers when making comparisons of two decimals, ignoring place value. For example, they think that $0.2 < 0.07$ simply because $2 < 7$.
- Students sometimes think the longer the decimal number the greater the value. For example they think that 0.03 is greater than 0.3.

Domain: Measurement and Data

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Students will need ample opportunities to become familiar with new units of measure. In prior years, work with units was limited to units such as pounds, ounces, grams, kilograms, and liters, and students did not convert measurements. Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. For example:

kg	g
1	1000
2	2000
3	3000

ft	in
1	12
2	24
3	36

lb	oz
1	16
2	32
3	48

Students in grade four begin using the four operations to solve word problems involving measurement quantities such as liquid volume, mass, and time (**4.MD.2**), including problems involving simple fractions or decimals.

Examples: Word Problems Involving Measures (4.MD.2).

1. **Division/fractions**: Susan has 2 feet of ribbon. She wants to give her ribbon to her 3 best friends so each friend gets the same amount. How much ribbon will each friend get? Students may record their solutions using fractions or inches.
   Solution: The answer would be $\frac{2}{3}$ of a foot or 8 inches. Students are able to express the answer in inches because they understand that $\frac{1}{3}$ of a foot is 4 inches and $\frac{2}{3}$ of a foot is 2 groups of $\frac{1}{3}$.
2. **Addition:** Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?
   Solution: Students know that every 60 minutes make an hour. We know she ran one hour which is 60 minutes. She also ran 15 + 25 +40 = 80 minutes more, which makes 140 total minutes.
3. **Multiplication:** Mario and his two brothers are selling lemonade. Mario brought one and a half liters, Javier brought 2 liters, and Ernesto brought 450 milliliters. How many total milliliters of lemonade did the boys have?
   Solution: Students know that 1 liter is 1000 ml, so that Mario bought 1000 + 500 = 1500 ml, and Javier bought 2 × 1000 = 2000 ml. This means altogether they had 1500 + 2000 + 450 = 3950 ml.

(Adapted from Arizona 2012)

Focus, Coherence, Rigor:

In grade four students use the four operations to solve word problems involving measurement quantities such as liquid volume, mass and time (**4.MD.1-2**). Measurement provides a context for solving problems using the four operations and connects to and supports major work at the grade in the cluster "Use the four operations with whole number to solve problems" (**4.OA.1-3▲**) and clusters in the domain "Number and operations—Fractions" (**4.NF.1-4▲**). For example, students use whole-number multiplication to express measurements given in a larger unit in terms of a smaller unit and students solve word problems involving addition and subtraction of fractions or multiplication of a fraction by a whole number (Adapted from PARCC 2012).

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Students developed an understanding of area and perimeter in third grade by using visual models. In grade four students are expected to use formulas to calculate area and perimeter of rectangles; however, they still need to understand and be able to communicate their understanding of why the formulas work. It is still important for students to make length units or square units inside a small rectangle to keep the distinction fresh and visual, and some students may still need to write the lengths of all four sides before finding the perimeter. Students know that answers for the area formula $(\ell \times w)$ will be in square units, and that answers for the perimeter formula ($2\ell + 2w$, or $2[\ell + w]$) will be in linear units (Adapted from Arizona 2012).

Example: Area and Perimeter of Rectangles. (MP.2, MP.4)

Sally wants to build a pen for her dog Callie. Her parents give her $200 to buy the fencing material, but they want Sally to design the pen. Her parents suggest that she consider different plans. Her parents also remind her that Callie needs as much room as possible to run and play, and that the pen can be placed anywhere in the yard and the wall of the house could be used as one side of the pen. Sally decides to buy fencing material that costs $8.50 per foot. She will also need at least one three foot wide gate for the pen that costs $15.

- Design a pen for Callie. Experiment with different pen designs and consider the advice from Sally's parents. Sally's house can also be any configuration.

Write a letter to Sally with your various diagrams and calculations. Explain why certain designs are better for Callie (Adapted from CMC Margaret DeArmond).

Measurement and Data 4.MD

Represent and interpret data.

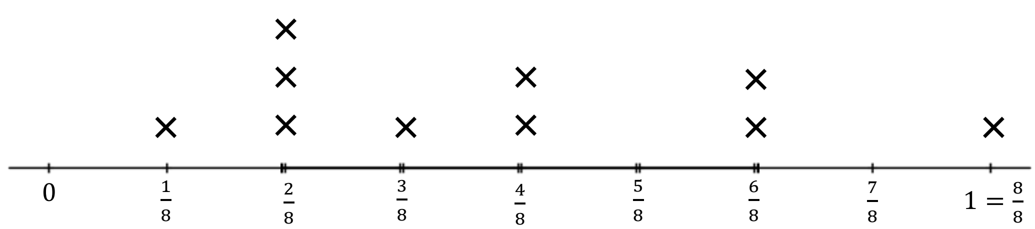
4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

As students work with data in kindergarten through grade five, they build foundations for the study of statistics and probability in grades six and beyond, and they strengthen and apply what they are learning in arithmetic.

In grade four students make a line plot to display a data set of measurements in fractions of a unit $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$ and they solve problems involving addition and subtraction of fractions by using information presented in line plots (**4.MD.4**).

Example: Interpreting Line Plots.

Ten students measure objects in their desk to the nearest $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}\right)$ inch. They record their results on the line plot below (in inches).



Possible related questions:

- How many objects measured $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ inch?
- If you put the objects end to end, what would the total length be?
- If five $\frac{1}{8}$-inch pencils are placed end to end, what would be the total length?

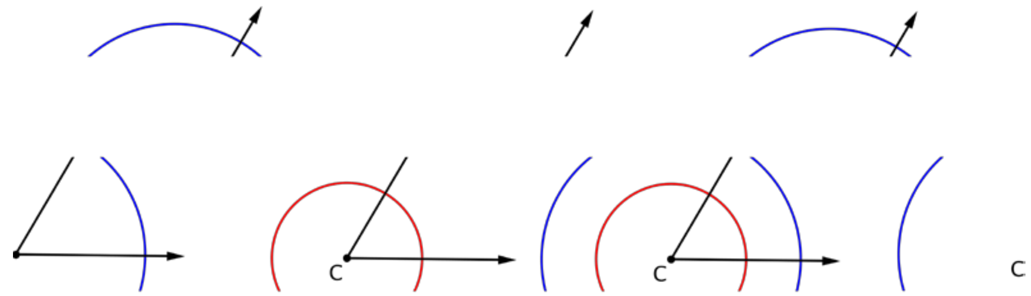
(Adapted from Arizona 2012)

Measurement and Data 4.MD

Geometric measurement: understand concepts of angle and measure angles.

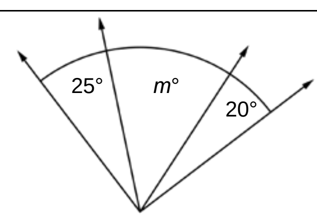
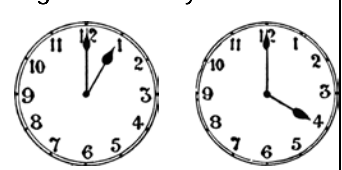
5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
    a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
    b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Students in grade four learn that angles are geometric shapes formed by two rays that share a common endpoint. They understand angle measure as being that portion of a circular arc that is formed by the angle when a circle is centered at their shared vertex. The diagram helps students see that an angle is determined by the arc it creates relative to the size of the entire circle, evidenced by the picture showing two angles of the same measure though their circles are not the same. However, the pie-shaped pieces formed by each angle are different-sized; this shows that angle measure is not defined in terms of these areas.



The angle in each case is 60°, since it measures an arc that is $\frac{1}{6}$ the total circumference of the circle in both the blue and red circles. However, the pie-slices that the angle forms have different areas.

Before students begin measuring angles with protractors **(4.MD.6)**, they need to have some experience with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. Students use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular). Students recognize angle measure as additive and use this to solve addition and subtractions problems to find unknown angles on a diagram.

Examples: Angle Measure is Additive.
1. If the two rays are perpendicular (see 4.G.1), what is the value of m? *Solution:* "Since perpendicular lines make an angle that measures 90°, I know that $25 + m + 20 = 90$, this means that $m = 90 - 45 = 45$." 
2. Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30°. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4? *Solution:* "This looks like it is four times as much, so it is 4 × 30° = 120°." 

(Adapted from Arizona 2012)

Focus, Coherence, and Rigor:

Students' work with concepts of angle measures (**4.MD.5a and 7**) also connects to and supports adding fractions, which is major work at the grade in the cluster "Building fractions from unit fractions by applying and extending previous understandings of operations on whole numbers" (**4.NF.3-4▲**). For example, a one degree measure is a fraction of an entire rotation and adding angle measures together is the same as adding fractions with a denominator of 360.

Before students solve word problems involving unknown angle measures (**4.MD.7**), they need to understand concepts of angle measure (**4.MD.5**) and, presumably, gain some experience measuring angles (**4.MD.6**). Students also need some familiarity with the geometric terms that are used to define angles as geometric shapes (**4.G.1**) (Adapted from PARCC 2012).

Domain: Geometry

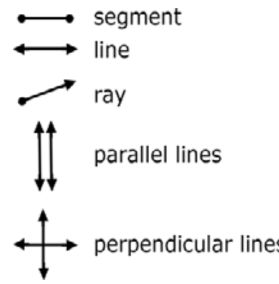
A critical area of instruction in grade four is for students to understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA**
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade four is the first time students are exposed to rays, angles, and perpendicular and parallel lines **(4.G.1)**. In addition, students classify figures based on the presence and absence of parallel or perpendicular lines and angles (**4.G.2**). It is helpful if examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. For example a wall chart with the images shown could be displayed in the classroom.

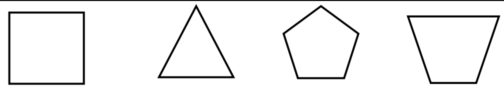


Students need to see all of these in different orientations. Students could draw these in different orientations and decide if all of the drawings are correct. Also they need to see the range of angles that are acute and obtuse.

Two-dimensional figures may be classified using different characteristics, such as the presence of parallel or perpendicular lines or by angle measurement. Students may use transparencies with lines drawn on them to arrange two lines in different ways to determine that the two lines might intersect in one point or may never intersect, thereby understanding the notion of parallel lines. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

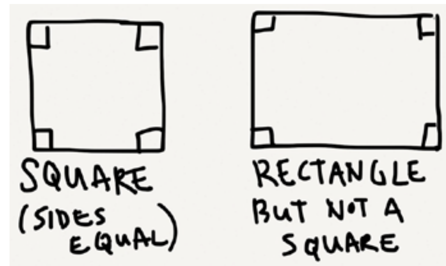
Students' prior experience with drawing and identifying right, acute, and obtuse angles helps them classify two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles. Right triangles (triangles with one right angle) can be a category for classification, with subcategories, e.g., an isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

Examples: Classifying Shapes According to Attributes. (MP.3)
1. Identify which of these shapes have perpendicular or parallel sides and justify your selection.



2. Explain why a square is considered a rectangle but a rectangle isn't necessarily a square.

Solution: "I know that rectangles are four-sided shapes that have four right angles. This makes any square a rectangle since a square has four sides and four right angles also. *But*, a square is a *special* kind of rectangle. What I mean is that you can have a rectangle that has its sides not all equal, and then it isn't a square. I drew examples to show what I mean."



Finally, students recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts.

(Adapted from Arizona 2012)

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, procedural skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way. Multiplication and division of whole numbers and fractions are an instructional focus in grades three through five.

To be prepared for grade five mathematics, students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade four and have met the fluency expectations for the grade. For fourth graders, the expected fluencies are to add and subtract multi-digit whole numbers using the standard algorithm within 1,000,000 (**4.NBT.4▲**).

These fluencies and the conceptual understandings that support them are foundational for work in later grades.

Of particular importance at grade four are concepts, skills, and understandings needed to use the four operations with whole numbers to solve problems (**4.OA.1-3▲**); generalize place value understanding for multi-digit whole numbers (**4.NBT.1-3▲**); use place value understanding and properties of operations to perform multi-digit arithmetic (**4.NBT.4-6▲**); extend understanding of fraction equivalence and ordering (**4.NF.1-2▲**); build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers (**4.NF.3-4▲**); and understand decimal notation for fractions, and compare decimal fractions (**4.NF.5-7▲**).

Fractions

Fraction equivalence is an important theme within the standards. Understanding fraction equivalence is necessary to extend arithmetic from whole numbers to fractions and decimals. Students need to understand fraction equivalence and that $\frac{a}{b} = \frac{n \times a}{n \times b}$. They should be able to represent equivalent common fractions and apply this understanding to compare fractions and express their relationships using the symbols, >, <, or =. Students understand how to represent and read proper fractions, improper fractions, and mixed numbers in multiple ways.

Grade four students should understand addition and subtraction with fractions having like denominators. This understanding represents a multi-grade progression as students add and subtract fractions here in grade four with like denominators by thinking of adding or subtracting so many unit fractions. Students should be able to solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem. Students should understand how to add and subtract proper fractions, improper fractions, and mixed numbers with like denominators.

Students extend their developing understanding of multiplication to multiply a fraction by a whole number. To support their understanding students should understand a fraction as the numerator times the unit fraction with the same denominator. Students should be able to rewrite fractions as multiples of the unit fraction of the same denominator, multiply a fraction by a whole number using a visual model, and use equations to represent problems involving the multiplication of a fraction by a whole number by multiplying the whole number times the numerator.

Four operations with whole numbers

By the end of grade four, students should fluently add and subtract multi-digit whole numbers to 1,000,000 using the standard algorithm. Students should also be able to use the four operations to solve multi-step word problems with whole number remainders.

In grade four students develop their understanding and skills with multiplication and division. Students combine their understanding of the meanings and properties of multiplication and division with their understanding of base-ten units to begin to multiply and divide multi-digit numbers. Grade four students should know how to express the product of two multi-digit numbers as another multi-digit number. They also should know how to find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. Representing multiplication and division using a rectangular area model helps students visualize these operations. This work will develop further in grade five and culminate in fluency with the standard algorithms in grade six.

Grade 4 Overview

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 4

Operations and Algebraic Thinking — 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] — 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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[1] See Glossary, Table 2.
[2] Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, $n \times (a/b) = (n \times a)/b$.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*
6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using **the number line or another** visual model. **CA**

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[3]Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

[4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:
    a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.
    b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.
6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.
7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry 4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. **(Two dimensional shapes should include special triangles, e.g., equilateral, isosceles, scalene, and special quadrilaterals, e.g., rhombus, square, rectangle, parallelogram, trapezoid.) CA**
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.