Grade Three

In the years prior to grade three, students gained an understanding of place value and used methods based on place value to add and subtract within 1000. They developed fluency with addition and subtraction within 100 and laid a foundation for understanding multiplication based on equal groups and the array model. Students worked with standard units to measure length, and they described shapes (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE THREE

[Note: Sidebar]

Grade Three Critical Areas of Instruction

In grade three, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes (CCSSO 2010, Grade 3 Introduction).

Students also fluently add and subtraction within 1,000 and multiply and divide within 100. By the end of grade 3, students know all products of two one-digit numbers from memory.

Grade Three Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and linking major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence, and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Grade 3 Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting” and “Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Grade 3 Cluster-Level Emphases

Operations and Algebraic Thinking

- [m]: Represent and solve problems involving multiplication and division. (**3.OA.1-4▲**)
- [m]: Understand properties of multiplication and the relationship between multiplication and division. (**3.OA.5-6▲**)

- [m]: Multiply and divide within 100. (**3.OA.7▲**)
- [m]: Solve problems involving the four operations, and identify and explain patterns in arithmetic. (**3.OA.8-9▲**)

Number and Operations in Base Ten

- [a/s]: Use place value understanding and properties of operations to perform multi-digit arithmetic. (**3.NBT.1-3**)

Number and Operations—Fractions

- [m]: Develop understanding of fractions as numbers. (**3.NF.1-3▲**)

Measurement and Data

- [m]: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (**3.MD.1-2▲**)
- [a/s]: Represent and interpret data.[1] (**3.MD.3-4)**
- [m]: Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (**3.MD.5-7▲**)
- [a/s]: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (**3.MD.8**)

Geometry

- [a/s]: Reason with shapes and their attributes. [2] (**3.G.1-2**)

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[3] [m] (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

[1] *Students multiply and divide to solve problems using information presented in scaled bar graphs. Pictographs and scaled bar graphs are a visually appealing context for one- and two-step word problems.*

[2] *Work should be positioned in support of area measurement and understanding of fractions.*

[3] The ▲symbol will indicate standards in a Major Cluster in the narrative.

(Adapted from Smarter Balanced Assessment Consortia [Smarter Balanced], DRAFT Content Specifications 2012, Grade 3).

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their ability to make sense of mathematics. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for Grade 3 students. (Refer to pages 9–12 in the Overview of the Standards Chapters for a complete description of the MP standards.)

Standards for Mathematic Practice (MP)

Explanations and Examples for Grade Three

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In third grade, mathematically proficient students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Students may use concrete objects, pictures, or drawings to help them conceptualize and solve problems, such as "Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?" or "Describe another situation where there would be 5 groups of 3 or 5 × 3." Students may check their thinking by asking themselves, "Does this make sense?" Students listen to other students' strategies and are able to make connections between various methods for a given problem.
MP.2 Reason abstractly and	Students recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of

quantitatively.	quantities. For example, students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given 4 × ? = 40, they might think: • 4 groups of some number is the same as 40 • 4 times some number is the same as 40 • I know that 4 groups of 10 is 40 so the unknown number is 10 • The missing factor is 10 because 4 times10 equals 40. Teachers might ask, "How do you know" or "What is the relationship between the quantities?" to reinforce students' reasoning and understanding.
MP. 3 Construct viable arguments and critique the reasoning of others.	Students may construct arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions that the teacher facilities by asking questions such as "How did you get that?" and "Why is that true?" Students explain their thinking to others and respond to others' thinking. For example, after investigating patterns on the 100s chart, students might explain why the pattern makes sense.
MP.4 Model with mathematics.	Students represent problem situations in multiple ways using numbers, words (mathematical language), drawing pictures, and objects. They might also represent a problem by acting it out or by creating charts, lists, graphs, or equations. For example, students use various contexts and a variety of models (e.g., circles, squares, rectangles, fraction bars, and number lines) to represent and develop understanding of fractions. Students use models to represent both equations and story problems and can explain their thinking. They evaluate their results in the context of the situation and reflect on whether the results make sense. Students should be encouraged to answer questions, such as "What math drawing or diagram could you make and label to represent the problem?" or "What are some ways to represent the quantities?"
MP.5 Use appropriate tools strategically.	Mathematically proficient students consider the available tools (including drawings or estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper to find all the possible rectangles that have a given perimeter. They compile the possibilities into an organized list or a table and determine whether they have all the possible rectangles. Students should be encouraged to answer questions such as, "Why was it helpful to use…?"
MP.6 Attend to precision.	Students develop mathematical communication skills as they use clear and precise language in their discussions with others and in their own reasoning. They are careful to specify units of measure and to state the meaning of the symbols they choose. For instance, when calculating the area of a rectangle they record the answer in square units.
MP.7 Look for and make use of structure.	Students look closely to discover a pattern or structure. For instance, students use properties of operations (e.g., commutative and distributive properties) as strategies to multiply and divide. Teachers might ask, "What do you notice when…?" or "How do you know if something is a pattern?"
MP.8 Look for and express regularity in repeated reasoning.	Students notice repetitive actions in computations and they look for "shortcut" methods. For instance, students may use the distributive property as a strategy to work with products of numbers they do know to solve products they do not know. For example, to find the product of 7 × 8, students might decompose 7 into 5 and 2 and then multiply 5 × 8 and 2 × 8 to arrive at 40 + 16 or 56. Third grade students continually evaluate their work by asking themselves, "Does this make sense?" Students should be encouraged to answer questions, such as "What is happening

	in this situation?" or "What predictions or generalizations can this pattern support?"

(Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina Department of Public Instruction [N. Carolina] 2011)

Standards-based Learning at Grade Three

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (**▲)** indicates standards in the major clusters (refer to the Grade 3 Cluster-Level Emphases table on page 2).

Domain: Operations and Algebraic Thinking

In kindergarten through grade two, students focused on developing an understanding of addition and subtraction. Beginning in grade three, students will focus on concepts, skills, and problem solving for multiplication and division. Students develop multiplication strategies, they make a shift from additive to multiplicative reasoning, and they relate division to multiplication. Third grade students become fluent with these operations within 100. This work will continue in grades four and five, preparing the way for work with ratios and proportions in grades six and seven. (Adapted from the University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA 2011, and PARCC 2012)

Multiplication and division are new concepts in grade three, and meeting fluency is a major portion of students' work (see **3.OA.7▲**)**.** Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multi-digit whole numbers and to add, subtract, multiply, and divide with fractions.

After students understand the situational equal groups and array/area meanings of multiplication and division, reasoning about patterns in products (e.g., products involving factors of 5 or 9) can help students remember particular products and quotients (see **3.OA.9▲).**

There are many patterns involved in how well multiples of numbers fit or do not fit with the base-ten system. Examining and articulating these patterns is an important part of the mathematical work on multiplication and division. Practice — and if necessary, extra support — should continue all year for those who need it to attain fluency. This practice can begin with the easier multiplications and divisions while the pattern work is occurring with more difficult numbers. (Adapted from PARCC 2012) Relating and practicing multiplications and divisions involving the same number (e.g., the 4s) can be helpful.

Operations and Algebraic Thinking	3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[4]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = __ ÷ 3, 6 × 6 = ?.*

A critical area of instruction is to develop student understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models. (CCSSI 2010, Grade 3). Multiplication

[4] See Glossary, Table 2.

and division are new concepts in grade three. Initially students need opportunities to develop, discuss, and use efficient, accurate, and generalizable methods to compute. The goal is for students to use general written methods for multiplication and division, which are variations of the standard algorithms, and that students can explain and understand (e.g., using visual models or place value language). Reaching fluency with these operations requires students to use variations of the standard algorithms without visual models, and this could take much of the year for many students.

Students recognize multiplication as finding the total number of objects in a certain number of equal-sized groups **(3.OA.1▲).** Also, students recognize division in two different situations—*partitive (or fair-share) division,* which requires equal sharing (e.g., how many are in each group?), and *quotitive (or measurement division*)*,* which requires determining how many groups (e.g., how many groups can you make?) (**3.OA.2▲**). These two interpretations of division have important uses later when studying division of fractions, and both should be explored as representations of division. In grade three teachers should use the terms "number of shares" or "number of groups" with students rather than "partitive" or "quotitive."

Multiplication of Whole Numbers: Note that the standards define multiplication of whole numbers $a \times b$ as finding the total number of objects in a groups of b objects. **Example:** There are 3 bags of apples on the table. There are 4 apples in each bag. How many apples are there altogether?
Partitive ("Fair Share" or Group Size Unknown) Division: The number of groups/shares to be made is known, but the number of objects in (or size of) each group/share is unknown. **Example:** There are 12 apples on the counter. If you are sharing the apples equally among 3 bags, how many apples will go in each bag?
Quotitive (Measurement or Number of Groups Unknown) Division: The number of objects in (or size of) each group/share is known, but the number of groups/shares is unknown. **Example:** There are 12 apples on the counter. If you put 3 apples in each bag, how many bags will you fill?

Students are exposed to related terminology for multiplication (factor and product) and division (quotient, dividend, divisor, and factor). They use multiplication and division within 100 to solve word problems (**3.OA.3▲**) in situations involving equal groups, arrays and measurement quantities. Note that while “repeated addition” can be used as a strategy for finding whole number products in some cases, repeated addition should not be misconstrued as the meaning of multiplication. The intention of the standards in grade three is to move students beyond additive thinking to multiplicative thinking.

The three major common types of multiplication and division word problems are summarized in the following table.

	Unknown Product	**Group Size Unknown (Partitive or Fair Share Division)**	**Number of Groups Unknown (Quotitive or Measurement Division)**
	3 × 6 = ?	**3 × ? = 18 and 18 ÷ 3 = ?**	**? × 6 = 18 and 18 ÷ 6 =?**
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement Example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 plums to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, Area	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area Example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement Example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is three times as much as a blue hat costs. How much does a blue hat cost? *Measurement Example.* A rubber band is stretched to be 18 cm long and that is three times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement Example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

(CCSSI 2010, Glossary) [Table is also included in the Framework Glossary.]

In grade three, students focus on equal groups and array problems. Compare problems will be introduced in grade four. The more difficult problem structures include “Group Size Unknown” (3 × ? = 18 or 18 ÷ 3 = 6) or “Number of Groups Unknown” (? × 6 = 18, 18 ÷ 6 = 3). To solve problems, students determine the unknown whole number in a multiplication or division equation relating three whole numbers (**3.OA.4▲**). Students use numbers, words, pictures, physical objects, or equations to represent problems, explain their thinking, and show their work. (**MP.1, MP.2, MP.4, MP.5**)

Example: (Number of Groups Unknown):

Molly the zookeeper has 24 bananas to feed the monkeys. Each monkey needs to eat 4 bananas. How many monkeys can Molly feed?

Solution: (? × 4 = 24)

The student might simply draw on the remembered product 6 × 4 = 24 to say that the related quotient is 6. Alternatively, the student might draw on other known products—for example, if 5 × 4 = 20 is known, then since 20 + 4 = 24, the student can reason that one more group of 4 will give the desired factor (5 + 1 = 6). Or, knowing that 3 × 4 = 12 and 12 + 12 = 24, the student might reason that the desired factor is 3 + 3 = 6. Any of these methods (or others) might be supported by an abstract drawing that shows the equal groups in the situation.

Operations and Algebraic Thinking 3.OA

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[5] *Examples: If 6 × 4 = 24 is known, then 4 × 6 =24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associate property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

In grade three, students apply properties of operations as strategies to multiply and divide (**3.OA.5▲**). At third grade students do not need to use the formal terms for these properties. Students use increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn about the relationship between multiplication and division. (CCSSI 2010, Grade 3)

[Note: Sidebar]

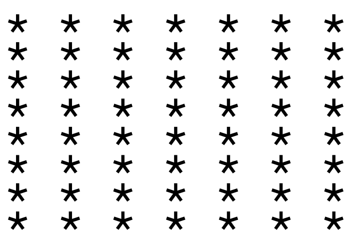
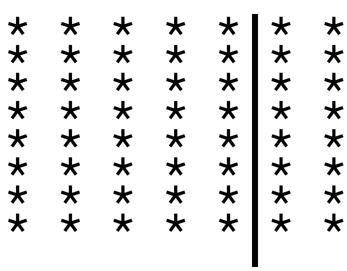
Focus, Coherence, and Rigor:

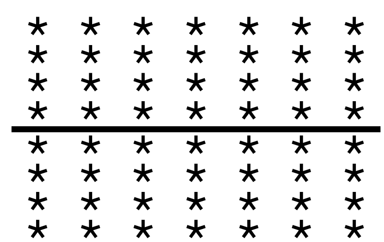
Arrays can be seen as an instance of equal-sized groups, where objects are arranged by rows and

[5] Students need not use formal terms for these properties.

columns, and they form a major transition to understanding multiplication as finding area (connection to standard (**3.MD.7 ▲**)). For example, students can analyze the structure of multiplication and division (**MP.7**) through their work with arrays (**MP.2**) and work towards precisely expressing their understanding of the connections between area and multiplication (**MP.6**).

The distributive property is the basis for the standard multiplication algorithm that students can use to fluently multiply multi-digit whole numbers, which appears in grade five. Third grade students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they do not know. (**MP.2, MP.7**)

<table>
<tr><td colspan="2">Example: Students can use the distributive property to discover new products of whole numbers (such as 7 × 8) based on products they can find more easily.</td></tr>
<tr>
<td>Strategy 1: By creating an array, I want to find how many total stars there are in 7 columns of 8 stars.

I see that I can arrange the 7 columns into a group of 5 rows and a group of 2 columns.

I know that the 5 × 8 array gives me 40 and the 2 × 8 array gives me 16. So altogether I have
5 × 8 + 2 × 8 = 40 + 16 = 56 stars.</td>
<td>Strategy 2: By creating an array, I want to find how many total stars there are in 7 rows of 8 stars.

I see that I can arrange the 8 up-down rows of stars into two groups of 4 rows.

I know that each new 4 × 7 array gives me 28 stars, and so altogether I have 4 × 7 + 4 × 7 = 28 + 28 = 56 stars.</td>
</tr>
</table>

(Adapted from Arizona 2012)

The connection between multiplication and division should be introduced early in the year. Students understand division as an unknown-factor problem (**3.OA.6▲**). *For example, find 15 ÷ 3 by finding the number that makes 15 when multiplied by 3.*

Multiplication and division are inverse operations and students use this inverse relationship to compute and check results. Below are some general strategies that can be used to develop multiplication and division facts in grade three.

Strategies for learning multiplication facts include:

Patterns

- Multiplication by zeros and ones
- Doubles (2s facts), Doubling twice (4s); Doubling three times (8s)
- Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
- Five facts (knowing the five facts are half of the tens facts)

General Strategies

- “Count bys” (counting groups of __ and knowing how many groups have been counted). For example, students count by twos keeping track of how many groups (to multiply) and when they reach the known product (to divide). Students gradually abbreviate the “count by” list and can start within it.
- Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
- The principle of “Turn-around facts” (based on the Commutative Property – knowing 2 x 7 is the same as 7 x 2 reduces the total number of facts to memorize)

Other Strategies

- Square numbers (e.g., 6 x 6)
- Nines (e.g., understanding this is 10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3, or knowing 9 times a number results in a tens place that is one below the number and that the two digits in the tens and ones place will add to 9 -- 9 x 6 is 5 in the tens place and 4 in the ones place, which equal a sum of 9).

Strategies for learning division facts include:

- Unknown factors. Students can state a division problem as a unknown factor problem (e.g., $24 \div 4 = ?$ becomes $4 \times ? = 24$). Knowing the related multiplication facts can help a student obtain the answer and vice versa, which is why studying multiplications and divisions involving a particular number can be helpful.
- Related facts (e.g., $6 \times 4 = 24$; $24 \div 6 = 4$; $24 \div 4 = 6$; $4 \times 6 = 24$). Students know $6 \times 4 = 24$ and $4 \times 6 = 24$; and $4 \times ? = 24$ and $6 \times ? = 24$ are related facts).

(Adapted from Arizona 2012)

Operations and Algebraic Thinking 3.OA

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows that 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

In grade three, students fluently multiply and divide within 100, using various strategies. (**3.OA.7▲**) These are new concepts in grade three and reaching fluency with these operations represents a major portion of students' work. By the end of grade three, students also know all products of two one-digit numbers from memory (**3.OA.7▲**). Organizing practice to focus most heavily on understood but not yet fluent products and unknown factors can speed learning and support fluency with multiplication and division facts. Practice and extra support should continue all year for those who need it to attain fluency.

[Note: Sidebar]

FLUENCY

In the standards for kindergarten through grade six there are individual content standards that set expectations for fluency in computation (e.g., "Fluently" multiply and divide within 100). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary.

The word "fluent" is used in the standards to mean "reasonably fast and accurate" and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (Adapted from Progressions K-5 CC and OA 2011 and PARCC 2012).

Operations and Algebraic Thinking 3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[6]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always*

[6] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

even, and explain why 4 times a number can be decomposed into two equal addends.

Students in third grade begin the step towards formal algebraic language by using a letter for the unknown quantity in expressions or equations when solving one and two-step word problems (**3.OA.8▲**). Students are not formally solving algebraic equations at this grade level. Students know to perform operations in the conventional order when there are not parentheses to specify a particular order (Order of Operations). Students use estimation during problem solving and then revisit their estimates to check for reasonableness.

Example 1: Chicken Coop. There are five nests in the chicken coop with 2 eggs in each nest. If the farmer wants 25 eggs, how many more eggs does she need?

Solution: Students might create a picture representation of this situation using a tape-like diagram:

2	2	2	2	2	*m*
25					

Students might solve this by seeing that when the 5 nests with 2 eggs are added up, they have 10 eggs. To make 25 eggs the farmer would need 25 – 10 = 15 more eggs. A simple equation that represents this situation could be $5 \times 2 + m = 25$, where m is how many more eggs the farmer needs.

Example 2: Soccer Club. The soccer club is going on a trip to the water park. The cost of attending the trip is $63. Included in that price is $13 for lunch and the cost of 2 wristbands, one for the morning and one for the afternoon. Both wristbands are the same price. Find the price of one of the wristbands. Write an equation that represents this situation.

Solution: Students might solve the problem by seeing that the cost of the two tickets must be $63 − $13 = $50.

w	*w*	$13
$63		

Therefore the cost of one of the wristbands must be $50 ÷ 2 = $25. Equations that represents this situation is $w + w + 13 = 63$ or $63 = w + w + 13$.

In grade three, students identify arithmetic patterns and explain them using properties of operations (**3.OA.9▲**). Students can investigate addition and multiplication tables in search of patterns (**MP.7**) and explain or discuss why these patterns make sense mathematically and how they are related to properties of operations (e.g., why is the multiplication table symmetric about its diagonal?) (**MP.3**) (Adapted from N. Carolina 2011 and the Kansas Association of Teachers of Mathematics [KATM] 3rd FlipBook 2012)

Domain: Number and Operations in Base Ten

Number and Operations in Base Ten	3.NBT
Use place value understanding and properties of operations to perform multi-digit arithmetic.[7]	
1. Use place value understanding to round whole numbers to the nearest 10 or 100.	
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g. 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	

In grade three, students are introduced to the concept of rounding whole numbers to the nearest 10 or 100 **(3.NBT.1)**, an important prerequisite for working with estimation problems. Students can use a number line or a hundreds chart as tools to support their work with rounding. They learn when and why to round numbers and extend their understanding of place value to include whole numbers with four digits.

Third grade students continue adding and subtracting within 1000 and achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction **(3.NBT.2)**.

Grade three students continue to add and subtract using methods they developed in grade two and their understanding of place value and the properties of operations Students in grade two were already adding and subtracting within 1000 (without the expectation of full fluency) and using at least one method that generalizes readily to larger numbers, so this is a relatively small and incremental expectation for third graders. Such methods will continue to be the focus in grade three so the extension at

[7] A range of algorithms may be used.

grade four to generalize these methods to larger numbers (up to 1,000,000) should also be relatively easy and rapid.

Third grade students also multiply one-digit whole numbers by multiples of 10 (**3.NBT.3**) in the range 10–90, using strategies based on place value and properties of operations (e.g., “I know 5 × 90 = 450 because 5 × 9 = 45 and so 5 × 90 should be ten times as much.”). Students also interpret 2 × 40 as 2 groups of 4 tens or 8 groups of ten. They understand 5 × 60 is 5 groups of 6 tens or 30 tens, and they know 30 tens is 300. After developing this understanding students begin to recognize the patterns in multiplying by multiples of 10 (Adapted from Arizona 2012). The skill of multiplying one-digit numbers by multiples of 10 can support later student learning of standard algorithms for multiplication of multi-digit numbers.

Domain: Number and Operations—Fractions

In grade three students develop an understanding of fractions as numbers, beginning with unit fractions by building on the idea of partitioning a whole into equal parts. Student proficiency with fractions is essential for success in more advanced mathematics such as percentages, ratios and proportions, and in algebra at later grades.

Number and Operations—Fractions[8] 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
    a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
    b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0.

[8] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

In grades one and two, students partitioned circles and rectangles into two, three, and four equal shares and used fraction language (e.g., halves, thirds, half of, a third of). In grade three, students begin to enlarge their concept of number by developing an understanding of fractions as numbers (Adapted from PARCC 2012).

Grade three students understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts and the fraction *a/b* as the quantity formed by *a* parts of size 1/*b*. **(3.NF.1▲).**

[Note: Sidebar]

Focus, Coherence, and Rigor:
When working with fractions, two main ideas should be emphasized:

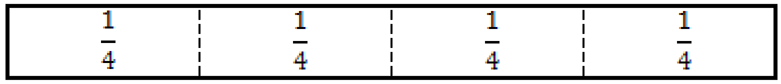
- Specifying the whole
- Explaining what is meant by "equal parts"

Student understanding of fractions hinges on understanding these ideas.

Students build on the idea of *partitioning* or dividing a whole into equal parts to understand fractions. Students start with unit fractions (fractions with numerator 1), which are formed by partitioning a whole into equal parts (the number of equal parts becomes the denominator) and taking one of those parts. An important goal is for students to see unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of the whole numbers. Students make the connection that just as every whole number is obtained by combining a sufficient number of 1s; every fraction is obtained by combining a sufficient number of unit fractions (Adapted from Progressions 3-5 NF 2012). They explore fractions first using concrete models such as fraction bars and geometric shapes, which will culminate in understanding fractions on the number line.

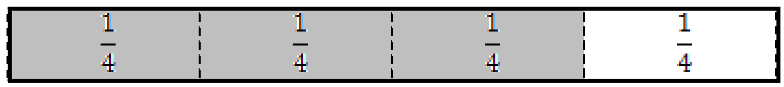
Examples:
Show the fraction $\frac{1}{4}$ by folding the piece of paper into equal parts.

"I know that when the number on the bottom is 4, I need to make four equal parts. By folding the paper in half once and then again, I get four parts and each part is equal. Each part is worth $\frac{1}{4}$."



Shade 3/4 using the fraction bar you created.

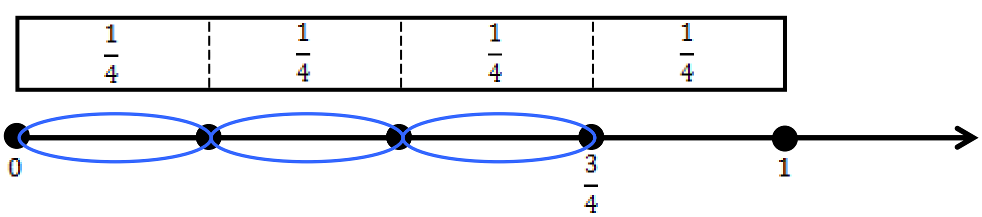
"My fraction bar shows fourths. The 3 tells me I need three of them, so I'll shade them. I could have shaded any three of them and I would still have 3/4."



Eventually, students represent fractions by dividing a number line from 0 to 1 into equal parts and recognize that each segmented part represents the same length **(MP.2, MP.4, MP.7)**. Stacking fraction bars and number lines can help students see how the unit length has been divided into equal parts. Important is that students "mark off" lengths of 1/*b* when locating fractions on the number line. Notice the difference between how the fraction bar and number line are labeled in the example shown below **(3.NF.2a-b)**.

Example (Representing Fractions on the Number Line): Use your fraction bar and the number line given to locate the fraction $\frac{3}{4}$. Explain how you know your mark is in the right place.

Solution: "When I use my fraction strip as a measuring tool, it shows me how to divide the unit interval into four equal parts (since the denominator is 4). Then I start from the mark that has '0' and I measure off three pieces of $\frac{1}{4}$ each. I circled the pieces to show that I marked three of them. This is how I know I have marked $\frac{3}{4}$."

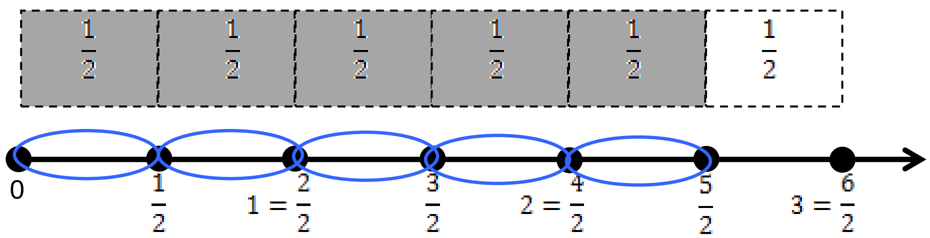


Third grade students need opportunities to place fractions on a number line and understand fractions as a related component of the ever-expanding number system. The number line reinforces the analogy between fractions and whole numbers. Just as

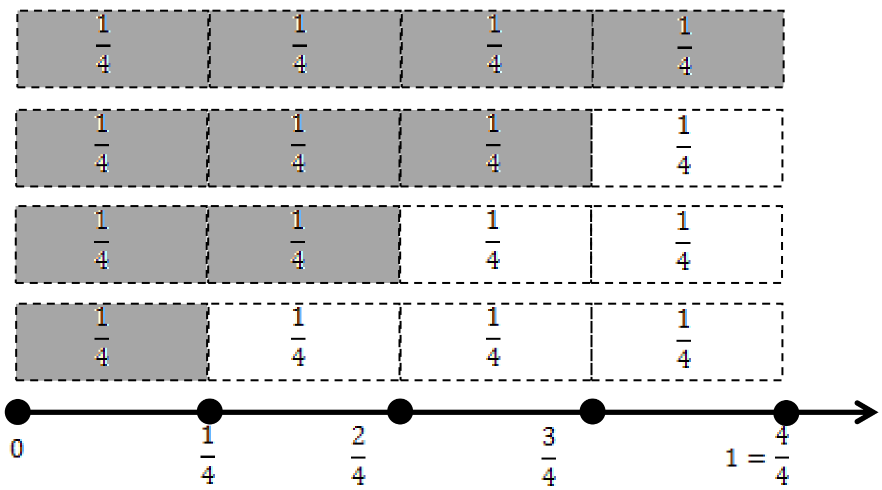
5 is the point on the number line reached by marking off 5 times the length of the unit interval from 0, so is $\frac{5}{3}$ the point obtained by marking off 5 times the length of a different interval as the basic unit of length, namely the interval from 0 to $\frac{1}{3}$.

Fractions Greater Than One. Note that the standards do not distinguish fractions greater than one as being “improper fractions.” Fractions greater than one, such as $\frac{5}{2}$, are simply numbers in themselves and are constructed in the same way as other fractions.

Thus, to construct $\frac{5}{2}$, we might use a fraction strip as a measuring tool to mark off lengths of $\frac{1}{2}$. Then we count five of those halves, to get $\frac{5}{2}$.



Students recognize that when examining fractions with common denominators, the wholes have been divided into the same number of equal parts, so the fraction with the larger numerator has the larger number of equal parts. Students develop an understanding of the numerator and denominator as they label each fractional part based on how far it is from zero to the endpoint. (**MP.7**)



Number and Operations—Fractions[9] **3.NF**

Develop understanding of fractions as numbers.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
    a. Understand two fractions as equivalent (equal) if they are the same size, or the same end point on a number line.
    b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
    c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 =6; locate 4/4 and 1 at the same point on a number line diagram.*
    d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a fraction model.

Students develop an understanding of fractions as they use visual models and a number line to represent, explain, and compare unit fractions, equivalent fractions (e.g., $\frac{1}{2} = \frac{2}{4}$), whole numbers as fractions (e.g., $3 = \frac{3}{1}$), and fractions with the same numerator (e.g., $\frac{4}{3}$ and $\frac{4}{6}$) or the same denominator (e.g., $\frac{4}{8}$ and $\frac{5}{8}$. **(NF.2-3▲**).

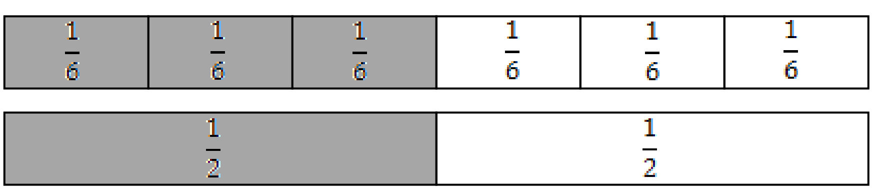
[9] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

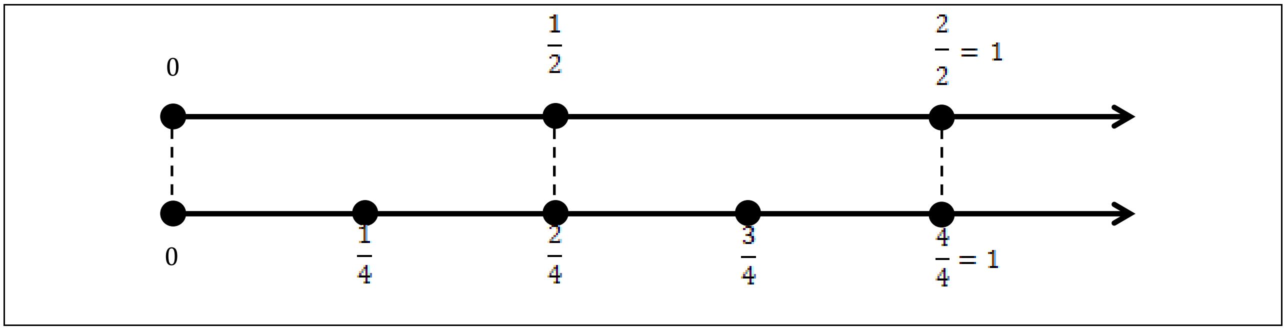
Students develop an understanding of order in terms of position on a number line. Given two fractions—thus two points on the number line—students understand that the one to the left is said to be smaller, and the one to the right is said to be larger (Adapted from Progressions 3-5 NF 2012).

Students learn that when comparing fractions they need to look at the size of the parts and the number of the parts. For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole of the same size is cut into 2 pieces.

To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts is different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces (Adapted from Arizona 2012 and KATM 3rd FlipBook 2012).

Students develop an understanding of equivalent fractions as they compare fractions using a variety of visual fraction models and justify their conclusions **(MP.3)**. Through opportunities to compare fraction models with the same whole divided into different numbers of pieces, students identify fractions that show the same amount or name the same number, and learn that they are equal (or equivalent).

Basic Fraction Equivalence Using Models
Using Fraction Bars: 
Using a number line:



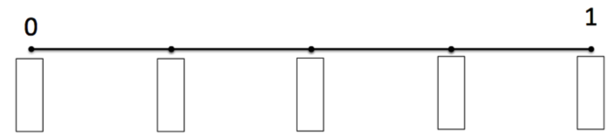
(Adapted from Progressions 3-5 NF 2012)

Some important concepts related to understanding fractions include:
• Fractional parts must be equal-sized • The number of equal parts tells how many make a whole • As the number of equal pieces in the whole increases, the size of the fractional pieces decreases. • The size of the fractional part is relative to the whole. • When a shape is divided into equal parts, the denominator represents the number of equal parts in the whole and the numerator of a fraction is the count of the demarcated congruent or equal parts in a whole (e.g., $\frac{3}{4}$ means that there are 3 one-fourths or 3 out of 4 equal parts). • Common benchmark numbers such as 0, $\frac{1}{2}$, $\frac{3}{4}$ and 1 can be used to determine if an unknown fraction is greater of smaller than a benchmark fraction.

(Adapted from Arizona 2012 and KATM 3rd FlipBook 2012)

The *Fractions Progression Module* provides an overview of understanding fractions and is available at http://www.illustrativemathematics.org/pages/fractions_progression (Illustrative Mathematics 2013).

Following is a sample classroom activity that connects the Standards for Mathematical Content and Standards for Mathematical Practice.

Standards	***Explanations and Examples***
3.NF.1: Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*. **3.NF.2**: Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off the lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line. **3.NF.3**: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. b. Recognize and generate simple equivalent fractions, (e.g. 1/2=2/4, 4/6=2/3). Explain why the fractions are equivalent, e.g. by using a visual fraction model.	**Task: *The Human Fraction Number Line Activity*.** In this activity, the teacher posts a long sheet of paper on a wall of the classroom to act as a number line, with 0 marked at one end and 1 marked at the other. Gathered around the wall, groups of students will be given cards with different sized fractions indicated on them (e.g. $(\frac{0}{4},\frac{1}{4},\frac{2}{4},\frac{3}{4},\frac{4}{4})$ and will be asked to locate themselves approximately along the number line. Depending on the size of the class and the length of the number line, fractions with denominators 2, 3, 4, 6, and 8 can be used. The teacher can ask students to explain to each other why their placements are correct or incorrect, emphasizing that the students with cards marked in fourths, say, have divided the number line into four equal parts. Furthermore, a student with the card *a*/*b* is standing in the correct place if they represent *a* lengths of size 1/*b* from 0 on the number line. As a follow-up activity, teachers can give students several unit number lines that are marked off into equal parts but that are unlabeled. Students are required to fill in the labels on the number lines. An example is shown here:  **Classroom Connections:** There are several big ideas included in this activity. One is that when talking about fractions as points on a number line, the whole is represented by the *length* or amount of distance from 0 to 1. By physical requiring students to line up in the correct places on the number line, the idea of partitioning this distance into equal parts is emphasized. In addition, other students can physically mark off the placement of fractions by starting from 0 and walking the required number of lengths 1/*b* from 0; for example, with students placed at the locations for sixths, another student can start at 0 and walk off a distance of 5/6. As an extension, teachers can have students mark off equivalent fraction distances, such as 1/2, 2/4, and 3/6, and can have a discussion as to why those fractions represent the same amount. **Connecting to the Standards for Mathematical Practice:** (MP.2) Students reason quantitatively as they determine why a placement was correct or incorrect and by assigning a fractional value to a distance. (MP.4) Students are using the number line model for fractions. While not an application of mathematics to a real-world situation in the sense of true modeling, it is an appropriate use of modeling for the grade level. (MP.8) Students see repeated reasoning in dividing up the number line into equal parts, though of various sizes, and form the basis for how they would place fifths, tenths, and other fractions.

Domain: Measurement and Data

Measurement and Data	3.MD

Solve problems involving measurement and estimation and intervals or time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[10] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measuring scale) to represent the problem.[11]

Students have experience telling and writing time from analog and digital clocks to the hour and half hour in grade one and in five minute intervals in grade two. In grade three students write time to the nearest minute and measure time intervals in minutes. Students solve word problems involving addition and subtraction of time intervals in minutes and represent these problems on a number line. (**3.MD.1▲**).

Students begin to understand the concept of continuous measurement quantities and they add, subtract, multiply or divide to solve one-step word problems involving such quantities. Multiple opportunities to weigh classroom objects and fill containers will help students develop a basic understanding of the size and weight of a liter, a gram, and a kilogram **(3. MD.2▲).**

[Note: Sidebar]

Focus, Coherence, and Rigor:
Students' understanding and work with measuring and estimating continuous measurement quantities, such as liquid volume and mass **(3. MD.2▲),** are an important context for fraction arithmetic in later grades.

Measurement and Data	3.MD

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.

[10] Excludes compound units such as cm^3 and finding the geometric volume of a container.
[11] Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2.)

Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.* 4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

In grade three, the most important development in data representation for categorical data is that students now draw picture graphs in which each picture represents more than one object, and they draw bar graphs in which the scale uses multiples so the height of a given bar in tick marks must be multiplied by the scale factor in order to yield the number of objects in the given category. These developments connect with the emphasis on multiplication in this grade (Adapted from Progressions K-5 MD, data part 2011).

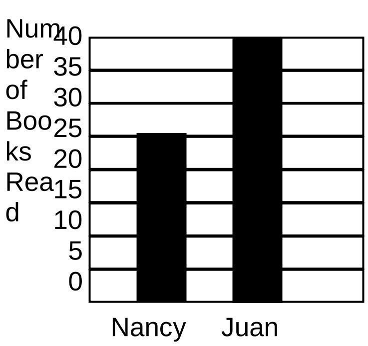
Students draw a scaled pictograph and a scaled bar graph to represent a data set and solve word problems **(3.MD.3)**.

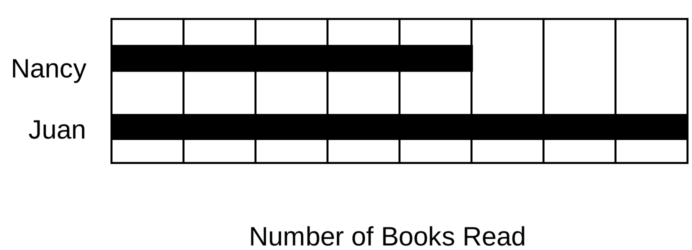
Examples:

Students might draw or reference a pictograph with symbols that represent multiple units.

Number of Books Read	
Nancy	□ □ □ □ □
Juan	□ □ □ □ □ □ □ □
□ 5 books	

Students might draw or reference bar graphs to solve related problems.





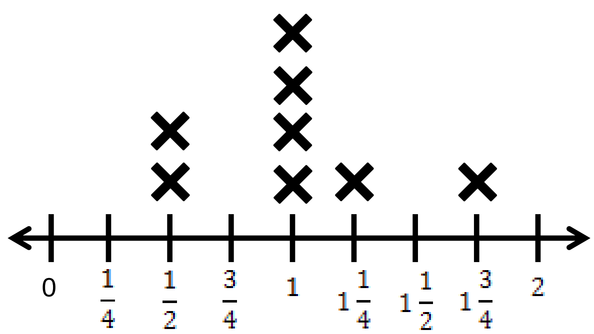
(Adapted from KATM 3rd FlipBook 2012).

[Note: Sidebar]

Focus, Coherence, and Rigor:
Pictographs and scaled bar graphs are a visually appealing context that support major work at the grade in the cluster “Represent and solve problems involving multiplication and division” as students solve multiplication and division word problems (**3.OA.3▲**).

Students use their knowledge of fractions and number lines to work with measurement data involving fractional measurement values. They generate data by measuring lengths using rulers marked with halves and fourths of an inch and create a line plot to display their findings **(3.MD.4)** (Adapted from Progressions K-5 MD, data part 2011). For example, students might use a line plot to display data.

Number of Objects Measured



(Adapted from N. Carolina 2011)

A critical area of instruction at grade three is for students to develop an understanding of the structure of rectangular arrays and of area measurement.

Measurement and Data 3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. A square with side length 1 unit, called a “unit square,” is said to have “one square unit” of area, and can be used to measure area.
    b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

7. Relate area to the operations of multiplication and addition.
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
    b. Multiply side lengths to fin areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
    c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
    d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

Students recognize area as an attribute of plane figures, and they develop an understanding of concepts of area measurement (**3.MD.5▲**). They discover a square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area and can be used to measure area. Students measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units) (**3.MD.6▲**). Students develop an understanding of using square units to measure area by using different sized square units, filling in an area with the same sized square units, and then counting the number of square units.

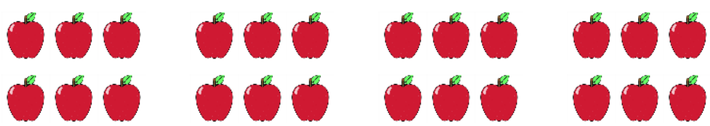
Students relate the concept of area to the operations of multiplication and addition and show that the area of a rectangle can be found by multiplying the side lengths (**3.MD.7▲**). Students make sense of these quantities as they learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle’s interior. For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are. (Adapted from Progressions K-5 MD, measurement part, 2012)

Students need opportunities to tile a rectangle with square units and then multiply the side lengths to show that they both give the area. For example, to find the area, a student could count the squares or multiply 4 × 3 = 12.

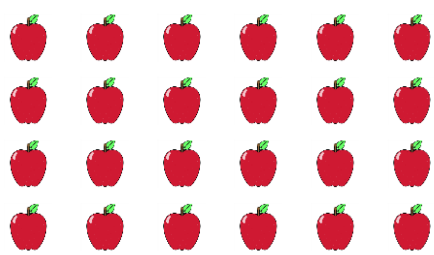
1	2	3	4
5	6	7	8
9	10	11	12

To aid in transitioning from counting unit squares to multiplying side lengths to find area, students should see the progression from multiplication as equal groups to multiplication as total number of objects in an array, and then see the area of a rectangle as an array of unit squares. For example,

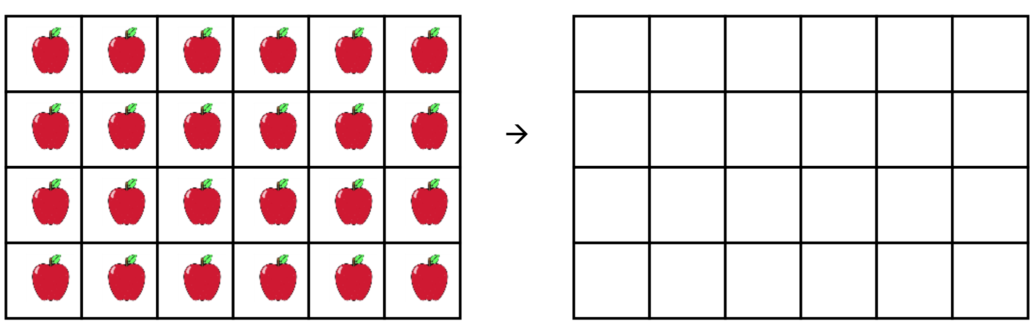
Students see multiplication as counting objects in equal groups, for example 4 × 6 as four groups of six apples:



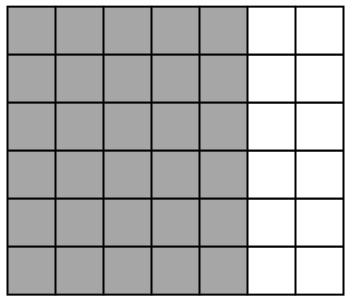
They see the objects arranged in arrays, as in a 4 × 6 array of the same apples:



They move to seeing how finding areas by counting unit squares is like counting an array of objects, where the objects are unit squares.



Students use area models to represent the distributive property in mathematical reasoning. For example, the area of a 6 × 7 figure can be determined by finding the area of a 6 × 5 and 6 × 2 and adding the two sums.

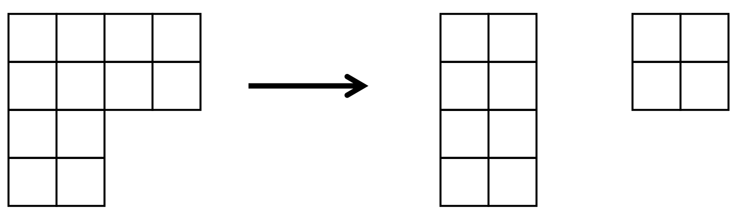


6 × 5 6 × 2

Students recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts.

The standards mention rectilinear figures. A *rectilinear figure* is a polygon with only right angles. Such figures can be decomposed into rectangles to find their areas.

Example:



Area of Rectilinear Figure? 4 × 2 2 × 2

By breaking the figure into two pieces, it becomes easier to see that the area of the figure is 8 + 4 = 12 square units.

(Adapted from N Carolina)

Students apply these techniques and understandings to solve real-world problems.

[Note: Sidebar]

Focus, Coherence, and Rigor:

The use of area models **(3.MD.7▲)** also supports multiplicative reasoning, a major focus in grade three in the domain “Operations and Algebraic Thinking” **(3.OA.1-9▲)**. Students must begin work with multiplication and division at or near the very start of the year to allow time for understanding and to develop fluency with these skills. Because area models for products are an important part of this process **(3.MD.7▲)**, work on concepts of area **(3.MD.5–6▲)** should likely begin at or near the start of the year as well (Adapted from PARCC 2012).)

Measurement and Data	3.MD

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding unknown side lengths, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

In grade three students solve real-world and mathematical problems involving perimeters of polygons **(3.MD.8)**. Students can develop an understanding of the concept of perimeter as they walk around the perimeter of a room, use rubber bands to represent the perimeter of a plane figure with whole number side lengths on a geoboard, or trace around a shape on an interactive whiteboard. They find the perimeter of objects, use addition to find perimeters, and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles. They explain their reasoning to others.

Given a perimeter and a length or width, students use objects or pictures to find the unknown length or width. They justify and communicate their solutions using words, diagrams, pictures, and numbers.

Domain: Geometry

Geometry	3.G

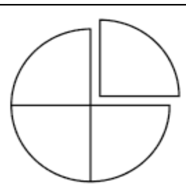
Reason with shapes and their attributes.

1. Understand that shape in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

A critical area of instruction at grade three is for students to describe and analyze two-dimensional shapes. Students compare common geometric shapes (e.g., rectangles and quadrilaterals) based on common attributes, such as four sides **(3.G.1).** In earlier grades, students informally reasoned about particular shapes through sorting and classifying based on geometric attributes. Students also built and drew shapes given the number of faces, number of angles, and number of sides. In grade three students describe properties of two-dimensional shapes in more precise ways using properties that are shared rather than the appearances of individual shapes. For example, students could start by identifying shapes with right angles, explain and discuss why the remaining shapes do not fit this category, and determine common characteristics of the remaining shapes.

Students relate their work with fractions to geometry as they partition shapes into parts with equal areas and represent each part as a unit fraction of the whole **(3.G.2)**.

Example:
The figure below was partitioned/divided into four equal parts. Each part is $\frac{1}{4}$ of the total area of the figure.


(Adapted from N. Carolina 2011)

[Note: Sidebar]

Focus, Coherence, and Rigor:

As students partition shapes into parts with equal areas **(3.G.2)**, they also reinforce concepts of area measurement and fractions that are part of the major work at the grade in the clusters “Geometric measurement: understand concept of area and relate area to multiplication and addition” (**3.MD. 5-7▲**) and “Develop understanding of fractions as numbers” (**3.NF▲).**

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, procedural skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way. Multiplication and division of whole numbers and fractions are an instructional focus in grades three through five.

To be prepared for grade four mathematics students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade three and have met the fluency expectations for the grade. For third graders, the expected fluencies are add and subtract within 1000 using strategies and algorithms (**3.NBT.2▲**), and multiply and divide within 100 using various strategies and know all products of two one-digit numbers from memory. (**3.OA.7▲)** These fluencies and the conceptual understandings that support them are foundational for work in later grades.

Of particular importance at grade four are concepts, skills, and understandings needed to represent and solve problems involving multiplication and division (**3.OA.1-4▲**); understand properties of multiplication and the relationship between multiplication and division (**3.OA.5-6▲**); multiply and divide within 100 (**3.OA.7▲**); solve problems involving the four operations and identify and explain patterns in arithmetic (**3.OA.8-9▲**); develop understanding of fractions as numbers (**3.NF.1-3▲**); solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects (**3.MD.1-2▲**); and geometric measurement: concepts of area and relating area to multiplication and to addition (**3.MD. 5-7▲**).

Multiplication and Division

By the end of grade three students develop both conceptual understanding and procedural skills of multiplication and division. Students are expected to fluently multiply and divide within 100 and to know from memory all of the products of two one-digit numbers (**3.OA.7▲**). Fluency in multiplication and division within 100 includes understanding and being able to apply strategies such as using mental math, understanding division as an unknown factor problem, applying the properties of operations, and identifying arithmetic patterns. Students also need to understand the relationship between multiplication and division and apply that understanding by using inverse operations to verify the reasonableness of their answers. Students with a firm grasp of grade-three multiplication and division can apply their knowledge to interpret, solve, and even compose simple word problems, including problems involving equal groups, arrays, and measurement quantities. Fluency in multiplication and division ensures that when students know from memory all of the products of two one-digit numbers they have an understanding of the two operations, and have not merely learned to produce answers through rote memorization.

Fractions

In grade three students are formally introduced to fractions as numbers, thus broadening their understanding of the number system. An understanding of fractions as composed of unit fractions is essential for students' ongoing work with the number system. Students must be able to place fractions on a number line and use the number line as a tool to compare fractions and recognize equivalent fractions. Students should be able to use other visual models to compare fractions. Student must also be able to express whole numbers as fractions and place them on a number line. It is essential for students to understand that the denominator determines the number of equally sized pieces that make up a whole and the numerator determines how many pieces of the whole are being referred to in the fraction.

Addition and Subtraction

By the end of grade three, students are expected to fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction (**3.NBT.2).** This fluency is both the culmination of work in previous grade levels and preparation for solving multistep word problems using all four operations beginning in grade four. Students should be able to use more than one strategy to add or subtract and should also be able to relate the strategies they use to a written method.

Grade 3 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 3

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as* 5 × 7.
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56÷8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \square \div 3$, $6 \times 6 = ?$.

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

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[1] See Glossary, Table 2.
[2] Students need not use formal terms for these properties.
[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten — 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] — 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
   b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data — 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

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[4]A range of algorithms may be used.
[5]Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.
[6]Excludes compound units such as cm^3 and finding the geometric volume of a container.
[7]Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
    a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
    b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
    a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
    b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
    c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
    d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Geometry	**3.G**

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*