Grade One

In kindergarten students learned to count in order, count to find out "how many," and to add and subtract with small sets of numbers in different kinds of situations. They also developed fluency with addition and subtraction within five. They saw teen numbers as composed of ten ones and more ones. Kindergarten students identified and described geometric shapes and created and composed shapes (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE ONE

[Note: Sidebar]

Grade One Critical Areas of Instruction

In grade one instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of and composing and decomposing geometric shapes (CCSSO 2010, Grade 1Introduction).

Students also work towards fluency in addition and subtraction with whole numbers within 10.

Grade One Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and linking major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence, and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Grade 1 Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting” and “Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Grade 1 Cluster-Level Emphases

Operations and Algebraic Thinking

- [m]: Represent and solve problems involving addition and subtraction. **(1.OA.1-2▲)**
- [m]: Understand and apply properties of operations and the relationship between addition and subtraction. **(1.OA.3-4▲)**

- [m]: Add and subtract within 20. **(1.OA.5-6▲)**
- [m]: Work with addition and subtraction equations. **(1.OA.7-8▲)**

Number and Operations in Base Ten

- [m]: Extend the counting sequence. **(1.NBT.1▲)**
- [m]: Understand place value. **(1.NBT.2-3▲)**
- [m]: Use place value understanding and properties of operations to add and subtract. **(1.NBT.4-6▲)**

Measurement and Data

- [m]: Measure lengths indirectly and by iterating length units. **(1.MD.1-2▲)**
- [a/s]: Tell and write time. **(1.MD.3)**
- [a/s]: Represent and interpret data. **(1.MD.4)**

Geometry

- [a/s]: Reason with shapes and their attributes**. (1.G.1-3)**

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[1] [m] (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

(Adapted from Achieve the Core 2012)

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their

[1] The ▲symbol will indicate standards in a Major Cluster in the narrative.

ability to make sense of mathematics. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for grade one students. (Refer to pages 9–12 in the Overview of the Standards Chapters for a complete description of the MP standards.)

Standards for Mathematic Practice (MP)

Explanations and Examples for Grade One

Standards for Mathematical Practice	**Explanation and Examples**
MP.1 Make sense of problems and persevere in solving them.	In first grade, students realize that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Younger students may use concrete objects or math drawings to help them conceptualize and solve problems. They may check their thinking by asking themselves, “Does this make sense?” They are willing to try other approaches.
MP.2 Reason abstractly and quantitatively.	Younger students recognize that a number represents a specific quantity. They connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. In first grade students make sense of quantities and relationships while solving tasks. They represent situations by decontextualizing tasks into numbers and symbols. For example, “There are 14 children on the playground and some children go line up. If there are 8 children still playing, how many children lined up?” Students translate the situation into the situation equation: $14 - ? = 8$, and then into the related equation $8 + ? = 14$ and solve the task. Students also contextualize situations during the problem solving process. For example, students refer to the context of the task to determine they need to subtract 8 from 14 because the total number of children on the playground is the total number less the 8 that are still playing. Teachers might ask, “How do you know” or “What is the relationship of the quantities?” to reinforce students’ reasoning and understanding. Students might also reason about ways to partition two-dimensional geometric figures into halves and fourths.

MP.3 Construct viable arguments and critique the reasoning of others.	First graders construct arguments using concrete referents, such as objects, pictures, drawings, and actions. They practice mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” or “Explain your thinking,” and “Why is that true?” They explain their own thinking and listen to the explanations of others. For example, “There are 9 books on the shelf. If you put some more books on the shelf and there are now 15 books on the shelf, how many books did you put on the shelf?” Students might use a variety of strategies to solve the task and then share and discuss their problem solving strategies with their classmates.
MP.4 Model with mathematics	In early grades, students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, and creating equations. Students need opportunities to connect the different representations and explain the connections. They should be able to use any of these representations as needed. First grade students model real-life mathematical situations with an equation and check to make sure equations accurately match the problem context. Students use concrete models and pictorial representations while solving tasks and also write an equation to model problem situations. For example to solve the problem, “There are 11 bananas on the counter. If you eat 4 bananas, how many are left?” students could write the equation 11 – 4 = 7. Students should be encouraged to answer questions, such as “What math drawing or diagram could you make and label to represent the problem?” or “What are some ways to represent the quantities?”
MP.5 Use appropriate tools strategically.	Students begin to consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, first graders decide it might be best to use colored chips to model an addition problem. Students use tools such as counters, place value (base ten) blocks, hundreds number boards, concrete geometric shapes (e.g., pattern blocks, 3-dimensional solids), and virtual representations to support conceptual understanding and mathematical thinking. Students determine which tools are appropriate to use. For example, when solving 12 + 8 = __, students might explain why place value blocks are appropriate to use to solve the problem. Students should be encouraged to answer questions such as, “Why was it helpful to use…?”
MP.6 Attend to precision.	As young children begin to develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and when they explain their own reasoning. In grade one, students use precise communication, calculation, and measurement skills. Students are able to describe their solutions strategies to mathematical tasks using grade-level appropriate vocabulary, precise explanations, and mathematical reasoning. When students measure objects iteratively (repetitively), they check to make sure there are no gaps or overlaps. Students regularly check their work to ensure the accuracy and reasonableness of solutions.
MP.7 Look for and make use of structure.	First grade students look for patterns and structures in the number system and other areas of mathematics. While solving addition problems, students begin to recognize the commutative property, for example 7 + 4 = 11, and 4 + 7 = 11. While decomposing two-digit numbers, students realize that any two-digit number can be broken up into tens and ones, e.g. 35 = 30 + 5, 76 = 70 + 6. Grade one students make use of structure when they work with subtraction as an unknown addend problem, such as 13 – 7 = __ can be written as 7+ __ = 13 and can be thought of as how much more do I need to add to 7 to get to 13?

MP.8 Look for and express regularity in repeated reasoning.	In the early grades, students notice repetitive actions in counting and computation. When children have multiple opportunities to add and subtract “ten” and multiples of “ten” they notice the pattern and gain a better understanding of place value. Students continually check their work by asking themselves, “Does this make sense?” Grade one students begin to look for regularity in problem structures when solving mathematical tasks. For example, students add three one-digit numbers by using strategies such as “make a ten” or doubles. Students recognize when and how to use strategies to solve similar problems. For example, when evaluating 8 + 7 + 2, a student may say, “I know that 8 and 2 equals 10, then I add 7 to get to 17. It helps if I can make a 10 out of two numbers when I start.” Students use repeated reasoning while solving a task with multiple correct answers. For example, solve the problem, “There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?” Students use repeated reasoning to find pairs of numbers that add up to 12 (e.g., the 12 crayons could include 6 of each color (6 + 6 = 12), 7 of one color and 5 of another (7 + 5 = 12), etc.) Students should be encouraged to answer questions, such as “What is happening in this situation?” or “What predictions or generalizations can this pattern support?”

(Adapted from Arizona Department of Education [Arizona] 2010 and North Carolina [N. Carolina] Department of Public Instruction 2012)

Standards-based Learning at Grade One

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills from previous grades and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical practice (**MP**) , and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (**▲)** indicates standards in the major clusters (refer to the Grade 1 Cluster-Level Emphases table on page 2).

Domain: Operations and Algebraic Thinking

In kindergarten students added and subtracted small numbers and developed fluency with these operations with whole numbers within 5. A critical area of instruction in grade one is for students to develop an understanding of and strategies for addition and subtraction within 20. Second grade students also become fluent with these operations within 10.

First grade students represent word problems (e.g., using objects, drawings and equations) and relate strategies to a written method to solve addition and subtraction word problems within 20 **(1.OA.1-2▲).** Students extend their prior work in three major and interrelated ways:

- Representing and solving a new type of problem situation (*compare* problems);
- Representing and solving the subtypes for all unknowns in all three types (see Table 1 “Common addition and subtraction situations by grade level” on page 12);
- Using Level 2 and 3 methods (see description in margin) to extend addition and subtraction problem solving beyond 10, to problems within 20.

[Note: Sidebar]

To solve word problems, students learn to apply various computational methods. Kindergarten students generally use Level 1 methods and Level 2 and 3 methods are used in grades one and two.

Methods used for solving single-digit addition and subtraction problems

Level 1: Direct Modeling by Counting All or Taking Away
Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2: Counting On
Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3: Convert to an Easier Problem
Decompose an addend and compose a part with another addend.

Refer to Appendix F for additional information about methods used for solving single-digit addition and subtraction problems.

(Adapted from the University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA (pg. 12) 2011).

Operations and Algebraic Thinking	1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

In kindergarten students worked with the following types of addition and subtraction situations: Add To with result unknown; Take From with result unknown; and Put Together/Take Apart with total unknown. In first grade students are introduced to a new type of addition and subtraction problem— Compare problems. First graders also extend this work to new unknown numbers in the problem types familiar from kindergarten.

First grade students add and subtract within 20 **(1.OA.1-2▲)** to solve the types of problems shown in the following two tables (**MP.1, 2, 3, 4, 5, 6**). A major goal for grade one is the use of Level 2 methods for addition (find the total) and subtraction (find the unknown addend). Level 2 methods represent a new challenge for students, since when "counting on", an addend is already embedded in the total to be found; it is the named starting number of the "counting on" sequence. The new problem subtypes that students work with support the development of this strategy. In particular, Compare problems that are solved with tape diagrams can serve as a visual support for the "counting on" strategy, and are helpful as students move away from representing all

[2] See Glossary, Table 1.

objects in a problem and represent objects solely with numbers. Compare problems are new to grade one and are therefore presented after the more familiar problem types. (Adapted from Progressions, K-5 CC and OA 2011, pg.16.)

Grade One Addition and Subtraction Problem Types (Non-Compare Problems). [Note: ***** *Indicates problem types grade one students solve but may not master until grade two.*]
Add To (with change unknown). "Bill had 5 toy robots. His mom gave him some more. Now he has 9 robots. How many toy robots did his mom give him?"
In this problem the starting quantity is provided (5 robots), a second quantity is added to that amount (some robots) and the result quantity is given (9 robots). This question type is more algebraic and challenging than the "result unknown" problems and can be modeled by a situational equation $5 + \square = 9$, which can be solved by counting on from 5 to 9. [Refer to 1.OA.6 for a discussion of various addition and subtraction strategies students use to solve problems].
Take From (with change unknown). Andrea had 8 stickers. She gave some stickers away. Now she has 2 stickers. How many stickers did she give away?
This question can be modeled by a situational equation $8 - \square = 2$ or a solution equation $8 – 2 = \square$. Both the Take From and Add To questions involve actions.
Add to (with start unknown)*. Some children were playing in the playground. 5 more children joined them. Then there were 12 children. How many children were playing before?"
This problem can be represented by $\square + 5 = 12$. The "start unknown" problems are difficult for students to solve because the initial quantity is unknown and therefore cannot be represented. Children need to see both addends as making the total and then some children can solve this by $5 + \square = 12$.
Take From (with start unknown)*. Some children were lining up for lunch. 4 children left and then there were 6 children still waiting in line. How many children were there before?
This problem can be modeled by $\square – 4 = 6$. Similar to the previous Add To (start unknown) problem, the Take From problems with the start unknown require a high level of conceptual understanding. Children need to see both addends as making the total and then some children can solve this by $4 + 6 = \square$.
Put Together/Take Apart (with addend unknown). Roger puts 10 apples in a fruit basket. 4 are red and the rest are green. How many are green?"
There is no direct or implied action. The problem involves a set and its subsets. It can be modeled by $10 – 4 = \square$ or $4 + \square = 10$. This type of problem provides students with opportunities to understand addends hiding inside a total and also to relate subtraction and an unknown-addend problem.

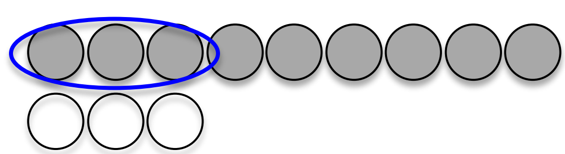
Initially, addition and subtraction problems include numbers that are small enough that students can make math drawings to solve problems that include all the objects. Students also use the addition symbol (+) to represent Add To and Put Together situations, the subtraction symbol (−) to represent Take From and Take Apart situations, and the equal sign (=) to represent a relationship regarding equality between one side of the equation and the other.

Compare problems are introduced in first grade. In a compare situation, two quantities are compared to find “How many more” or “How many less.” One reason compare problems are more advanced than the other two major problem types is that in compare problems one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation, by showing the “extra” that when added to the smaller unknown makes the total equal to the bigger unknown, or by finding this quantity embedded within the bigger unknown.

The language of these problems can also be difficult for students. For example, “Julie has three more apples than Lucy,” states that both (1) Julie has more apples and (2) the difference is three. Many students “hear” the part of the sentence about who has more, but do not initially hear the part about how many more. Students need experience hearing and saying a separate sentence for each of the two parts to help them comprehend and say the one-sentence form.

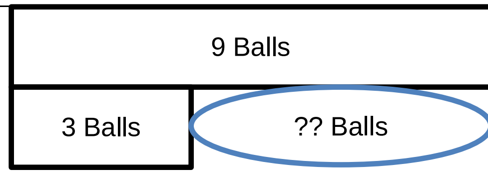
Example: Compare Problems. Abel has 9 balls. Susan has 3 balls. How many more balls does Abel have than Susan?

Students experience using objects to represent the two sets of balls and comparing.

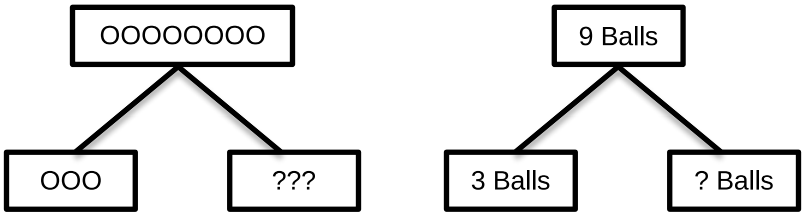


Teachers can also ask the related question, “How many fewer balls does Susan have than Abel?”

Students also experience using “comparison bars.” Rather than representing the actual objects with manipulatives or drawings, they use the numbers in the problem to represent the quantities.



Finally, students also work with "number bond" diagrams, such as shown below. They can have drawings that represent quantities or only the numbers represented in the problem.



While most adults know to solve compare problems with subtraction, students often represent compare problems as missing addend problems (e.g. representing the previous Abel and Susan example as 3 + ☐= 9). Student methods such as these should be explored, and the connection between addition and subtraction made explicit. (Adapted from Progressions, K-5 CC and OA (pg. 13) 2011).

Grade One Addition and Subtraction Compare Problem Types. [Note: * *Indicates problem types grade one students solve but may not master until grade two.*]
Compare (with difference unknown). Pat has 9 peaches. Lynda has 4 peaches. How many more peaches does Lynda have than Pat?"
Compare problems involve relationships between quantities. While most adults might use subtraction to solve this type of Compare problem (9 – 4 = □), students will often solve this problem as an unknown addend problem (4 + □ = 9) or by using a counting up or matching strategy. In all mathematical problem solving, what matters is the explanation a student gives to relate a representation to a context and not the representation separated from its context.
Compare (with bigger unknown—"more" version). Theo has 7 action figures. Rosa has 2 more action figures than Theo. How many action figures does Rosa have?"
This problem can be modeled by 7 + 2 = □.
Compare (with bigger unknown— "fewer," misleading language version) *. Lucy has 8 apples. She has 2 fewer apples than Marcus. How many apples does Marcus have?"

This problem can be modeled as 8 + 2 = □. The misleading language form “fewer” may lead students to choose the wrong operation.
Compare (with smaller unknown— “fewer” version). Bill has 8 stamps. Lisa has 2 fewer stamps than Bill. How many stamps does Lisa have?”
This problem can be modeled as 8 – 2 = □.
Compare (with smaller unknown— “more” misleading language version)*. David has 7 more bunnies than Keisha. David has 8 bunnies. How many bunnies does Keisha have?”
This problem can be modeled by 8 – 7 = □. The misleading language form “more” may lead students to choose the wrong operation.

As mentioned previously, the language and conceptual demands of compare problems are challenging for students in grade one. Some students may also have difficulty in general with the conceptual demands of “start unknown” problems. Grade one students should have an opportunity to solve and discuss such problems, but proficiency with these most difficult subtypes should wait until grade two. Refer to the unshaded problems in the following table.

TABLE 1. Common addition and subtraction situations by grade level.

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

Put Together/ Take Apart	**Total Unknown** Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	**Addend Unknown** Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	**Both Addends Unknown**[127] Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
Compare	**Difference Unknown** (“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	**Bigger Unknown** (Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	**Smaller Unknown** (Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[Note: Kindergarten students solve problem types in dark shading; grade one and two students solve problems of all sub-types. Unshaded problems are the most difficult, grade one students work with these problems but do not master them until grade two.]

(CCSSI 2010)

Literature can be incorporated into problem-solving with young students. Many literature books include mathematical ideas and concepts. Books that contain problem situations involving addition and subtraction with numbers 0 to 20 would be appropriate for grade one students (KATM 1st FlipBook 2012).

Focus, Coherence, and Rigor:
Problems that provide opportunities for students to explanation their thinking and use objects and drawings to represent word problems **(1.OA.▲)** also support the mathematical practices such as make sense of problems (**MP.1**), reason quantitatively to make sense of quantities and their relationships in problems (**MP.2**), and justify conclusions (**MP.3**).

Common Misconceptions.

- Some students misunderstand the meaning of the equal sign. The equal sign means "is the same as," but many primary students think the equal sign means "the answer is coming up" to the right of the equal sign. When students see only examples of number sentences with the operation to the left of the equal sign and the answer to the right, they overgeneralize the meaning of the equal sign, which creates this misconception. First graders should see equations written multiple ways, for example 5 + 7 = 12 and 12 = 5 + 7. The Put Together/Take Apart Both (with addends unknown) problems—such as "Robbie puts 12 balls in a basket. 4 are white and the rest are black. How many are black?"—are particularly helpful for eliciting equations such as 12 = 5 + 7. These equations can begin in kindergarten with small numbers (5 = 4 + 1) and they should be used throughout grade one for such problems.
- Many students assume key words or phrases in a problem suggest the same operation every time. For example, students might assume the word "left" always means subtract to find a solution. To help students avoid this misconception include problems in which key words represent different operations. For example, Joe took 8 stickers he no longer wanted and gave them to Anna. Now Joe has 11 stickers "left". How many stickers did Joe have to begin with? Facilitate students' understanding of scenarios represented in word problems. Students should analyze word problems (**MP.1, MP.2**) and not rely on key words.

Students can collaborate in small groups to develop problem solving strategies. Grade one students use a variety of strategies and models, such as drawings, words, and equations with symbols for the unknown numbers, to find the solutions. Students explain, write, and reflect on their problem solving strategies. **(MP.1, MP.2, MP.3, MP.4, MP.6)** For example, each student could write or draw a problem in which three whole things are to be combined. Students might exchange their problems with other students, solve them individually, and then discuss their models and solution strategies. The students work together to solve each problem using a different strategy. The level of difficulty for these problems can also be differentiated by using smaller numbers (up to 10) or larger numbers (up to 20).

Operations and Algebraic Thinking	1.OA
Understand and apply properties of operations and the relationship between addition and	

subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*
4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

First grade students build their understanding of the relationship between addition and subtraction. Instruction should include opportunities for students to investigate, identify and then apply a pattern or structure in mathematics. For example, pose a string of addition and subtraction problems involving the same three numbers chosen from the numbers 0 to 20 (e.g., 4 + 6 = 10 and 6 + 4 = 10, 10 – 6 = 4 and 10 – 4 = 6). These are related facts—a set of three numbers that can be expressed with an addition or subtraction equation. Related facts help develop an understanding of the relationship between addition and subtraction and the commutative and associative properties.

Students apply properties of operations as strategies to add and subtract **(1.OA.3▲)**. Although it is not necessary for grade one students to learn the names of the properties, students need to understand the important ideas of the following properties:

- Identity property of addition (e.g., 6 = 6 + 0). "Adding 0 to a number results in the same number."
- Identity property of subtraction (e.g., 9 – 0 = 9). "Subtracting 0 from a number results in the same number."
- Commutative property of addition (e.g., 4 + 5 = 5 + 4). "The order in which you add numbers doesn't matter."
- Associative property of addition (e.g., 3 + (9 + 1) = (3 + 9) +1 = 12 + 1 =13). "When adding more than two numbers, it doesn't matter which numbers you add together first."

Example.
Students build a tower of 8 green cubes and 3 yellow cubes, and another tower of 3 yellow and 8 green

[3] Students need not use formal terms for these properties.

cubes to show that order does not change the result in the operation of addition. Students can also use cubes of 3 different colors to demonstrate that (2 + 6) + 4 is equivalent to 2 + (6 + 4) and then to prove 2 + (6 + 4) = 2 + 10.

[Note; Sidebar]

Focus, Coherence, and Rigor

Students apply the commutative and associative properties as strategies to solve addition problems **(1.OA.3▲)** (these properties do not apply to subtraction). They use mathematical tools, such as cubes and counters, and visual models (e.g., drawings and a 100 chart) to model and explain their thinking. Students can share, discuss, and compare their strategies as a class. (**MP.2, MP.7, and MP.8**)

Students understand subtraction as an unknown-addend problem. **(1.OA.4▲).** Word problems such as Put Together/Take Apart (with addend unknown) afford students a context to see subtraction as the opposite of addition by finding an unknown addend. Understanding subtraction as an unknown-addend addition problem is one of the essential understandings students will need in middle school to extend arithmetic to negative rational numbers (Adapted from Arizona 2010 and Progressions, K-5 CC and OA 2011).

Common Misconceptions.

Students may assume that the commutative property applies to subtraction. After students have discovered and applied the commutative property of addition, ask them to investigate whether this property works for subtraction. Have students share and discuss their reasoning and guide them to conclude that the commutative property does not apply to subtraction (Adapted from KATM 1st FlipBook 2012).

This can be challenging because students might think they can switch the addends in subtraction equations because of their work with related fact equations using the commutative property for addition, but students need to understand they cannot switch the total and an addend.

Operations and Algebraic Thinking 1.OA

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between

addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Primary students come to understand addition and subtraction as they connect counting and number sequence to these operations **(1.OA.5▲)**. First grade students connect “counting on” and “counting back” to addition and subtraction. For example, students count on (3) from 4 to solve the addition problem 4 + 3 = 7. Similarly students count back (3) from 7, to solve the subtraction problem 7 – 3 = 4. The “counting all” strategy requires students to count an entire set. The counting on and counting back strategies occur when students are able to hold the start number in their head and count on from that number. Students generally have difficulty knowing where to begin their count when counting backward, so it is much better to restate the subtraction as an unknown addend and solve by counting on: “7 – 3 means 3 + ? = 7, so 4, 5, 6, 7, I counted on 4 more to get to 7, so 4.” Solving subtraction problems by “counting on” helps to reinforce the view that subtraction problems are missing addend problems, important for later student understanding of operations with rational numbers.

Students will use different strategies to solve problems if given the time and space to do so. Teachers should explore the various methods that arise as all relate to student understanding of general properties of operations.

Example: There are crayons in a box. There are 4 green crayons, 5 blue crayons, and 6 red crayons. How many crayons are there total? Explain to others how you found your answer.

Student 1 (Adding with a Ten Frame and Counters):

I put 4 counters on a Ten Frame for the green crayons, Then, I put 5 different color counters on the Ten Frame for the blue crayons. Then, I put another 6 color counters out for the red crayons. Only one of the crayons fit, so I had 5 left over, one ten-frame and five leftover makes 15 crayons. (**MP.2, MP.3, MP.5**) **(1.OA.2▲)**

Student 2 (Making tens):

I know that 4 and 6 equal 10, so the green and red equal 10 crayons. Then, I added the 5 blue crayons to get 15 total crayons. (**MP.2, MP.6**) **(1.OA.3▲)**

Student 3 (Counting on)

I counted on from 6, first counting on 5 to get 11 and then counting on 4 to get 15. I used my fingers to keep track of the 5 and the 4. But now I see that I know that 5 and 4 make 9, so I could have counted on 6 from 9. So there were 15 total crayons. (**MP.1, MP.2**) **(1.OA.6▲)**

First grade students use various strategies to add and subtract within 20 **(1.OA.6▲)**. Students need ample opportunities to model operations using various strategies and explain their thinking. (**MP.2, MP.7, and MP.8**)

Example: 8 + 7 = ___	
Student 1: ***Making 10 and Decomposing a Number*** I know that 8 plus 2 is 10, so I decomposed (broke) the 7 up into a 2 and a 5. First I added 8 and 2 to get 10, and then added the 5 to get 15. 8 + 7 = (8 + 2) + 5 = 10 + 5 = 15	**Student 2:** ***Creating an Easier Problem with Known Sums*** I know 8 is 7 + 1. I also know that 7 and 7 equal 14 and then I added 1 more to get 15. 8 + 7 = (7 + 7) + 1 = 15
Example: 14 – 6 = ___	
Student 1: ***Decomposing the Number You Subtract*** I know that 14 minus 4 is 10 so I broke the 6 up into a 4 and a 2. 14 minus 4 is 10. Then I take away 2 more to get 8. 14 – 6 = (14 – 4) – 2 = 10 – 2 = 8	**Student 2:** ***Relationship between Addition and Subtraction*** I know that 6 plus 8 is 14, so that means that 14 minus 6 is 8. 6 + 8 = 14 so 14 – 6 = 8. I could make a ten if I didn't know: 6 + 4 is ten and 4 more is 14, and 4 and 4 is 8.

(Adapted from Arizona 2010 and Georgia Department of Education [Georgia] 2011)

Students begin to develop algebraic understanding when they create equivalent expressions to solve a problem (such as when they write a situation equation and then write a solution equation from that) or use addition or subtraction combinations they know to solve more difficult problems.

[Note: Sidebar]

FLUENCY
In the standards for kindergarten through grade six there are individual content standards that set expectations for fluency in computation ((e.g., "fluency" for addition and subtraction within 10 **(1.OA.6▲**).

Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary.

The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (Adapted from Progressions K-5 CC and OA 2011 and PARCC 2012).

Some strategies to help students develop understanding and fluency with addition and subtraction include the use of ten frames or ten-based drawings, comparison bars, and number bonds. The use of visuals (e.g., hundreds charts and base ten representations) can also support fluency and number sense.

Students continue to develop meanings for addition and subtraction as they encounter problem situations in kindergarten through grade two. They expand their ability to represent problems, and they use increasingly sophisticated computation methods to find answers. In each grade the situations, representations, and methods should foster growth from one grade to the next.

Operations and Algebraic Thinking 1.OA

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*

Students need to understand the meaning of the equal sign **(1.OA.7▲)** and know that the quantity on one side of the equal sign must be the same quantity on the other side of the equal sign. Interchanging the language of “equal to” and “is the same as” as well as “not equal to” and “is not the same as” will help students grasp the meaning of the equal sign.

To avoid common pitfalls such as the equal sign meaning "to do something" or the equal sign meaning "the answer is," students should be able to:

- Express their understanding of the meaning of the equal sign
- Realize sentences other than $a + b = c$ are true (e.g., $a = a$, $c = a + b$, $a = a + 0$, $a + b = b + a$)
- Know the equal sign represents a relationship between two equal quantities
- Compare expressions without calculating. For example, 3 + 4 = 3 + 3 + 2. Student says, "I know this statement is false because there is a 3 on both sides of the equal sign, but the right side has 3 + 2, and that makes 5. 5 is more than 4, so the two sides can't be equal."

True/False Statements for Developing Understanding of the Equal Sign.	
7 = 8 – 1	9 + 3 = 10
8 = 8	5 + 3 = 10 – 2
1 + 1 + 3 =7	3 + 4 + 5 = 3 + 5 + 4
4 + 3 = 3 + 4	3 + 4 + 5 = 7 + 5
6 – 1 = 1 – 6	13 = 10 + 4
12 + 2 – 2 = 12	10 + 9 + 1 = 19

Initially, students develop an understanding of the meaning of equality using models. Students can justify their answers, make conjectures (e.g., if you start with zero and add a number and then subtract that same number, you always get zero), and use estimation to support their understanding of equality (Adapted from Arizona 2010 and KATM 1st FlipBook 2012).

Domain: Number and Operations in Base Ten Domain

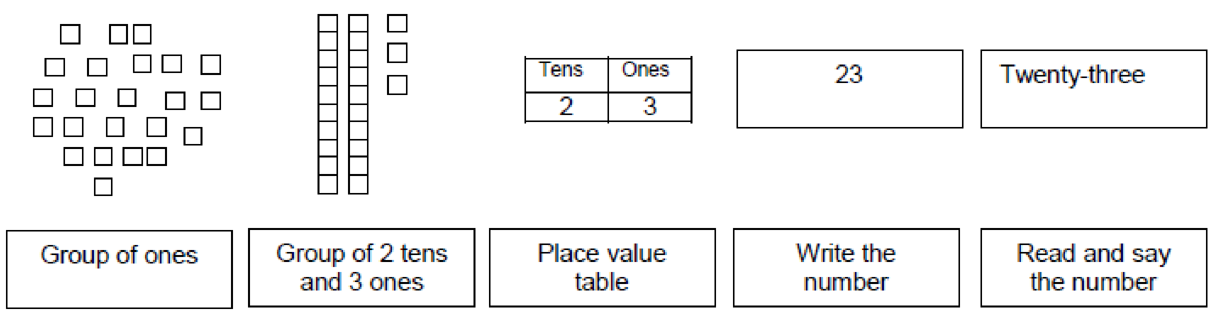
In kindergarten students developed an important foundation for understanding the base-ten system as they viewed "teen" numbers as comprised of ten ones and some more ones. A critical area of instruction in grade one is to extend students' place value understanding to view ten ones as a unit called a “ten” and two-digit numbers as amounts of tens and ones (Progressions, K-5 NBT 2011).

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

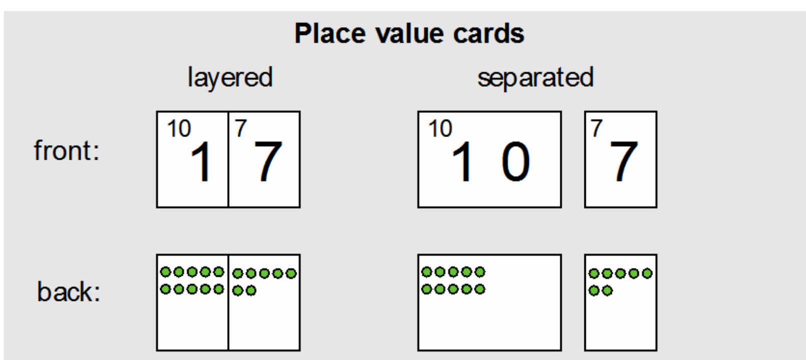
1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral

First grade students extend reading and writing numerals beyond 20 to 120 (**1.NBT.1▲**). Students use objects, words, and/or symbols to express their understanding of numbers. For a given numeral students count out the given number of objects, identify the quantity that each digit represents, and write and read the numeral. (**MP.2, MP.7, MP.8**) For example:



(Ohio Department of Education [Ohio] 2011)

Seeing different representations can help students develop an understanding of numbers. Posting the number words in the classroom helps students read and write them. Extending hundreds



Children can use layered place value cards to see the 10 “hiding” inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings.

charts to 120 and displaying them in the classroom can help students connect place value to the number symbols and with words for numbers 1 to 120. Students may need extra support with decade and century numbers when they orally count to 120. These transitions will be signaled by a 9 and require new rules to generate the next set of numbers. Students need experience counting from different starting points (e.g., start at 83; count to 120).

[Note: Sidebar]

Notice the power of the vertical hundreds chart: You can see all of the 9 tens in the numbers 91 to 99.

Part of a numeral list

91	101	111
92	102	112
93	103	113
94	104	114
95	105	115
96	106	116
97	107	117
98	108	118
99	109	119
100	110	120

In the classroom, a list of the numerals from 1 to 120 can be shown in columns of 10 to help highlight the base-ten structure. The numbers 101, . . . , 120 may be especially difficult for children to write.

(Progressions, K-5 NBT 2011)

Number and Operations in Base Ten 1.NBT

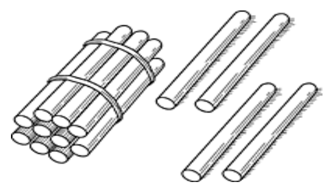
Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a “ten.”
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

First graders learn that the two digits of a two-digit number represent amounts of tens and ones (e.g., 67 represents 6 tens and 7 ones) (**1.NBT.2▲**).

Understanding the concept of 10 is fundamental to young students' mathematical development. This is the foundation of the place value system. In kindergarten students thought of a group of ten cubes as ten individual cubes. In first grade students understand ten cubes as a bundle—one bundle of ten (**1.NBT.2a▲**). Students can demonstrate this concept by counting 10 objects and "bundling" them into one group of ten. (**MP.2, MP.6, MP.7, MP.8**)

Students count between 10 and 20 objects and can make a bundle of 10 with or without some left over (this can help students write teen numbers (**1.NBT.2b▲**). They can continue counting any number of objects up to 99, making bundles of 10s with or without leftovers (**1.NBT.2c▲**). For example, a student represents the number 14 as one bundle (one group of ten) with four left over.



Students can also use models to express larger numbers as bundles of tens and 0 ones or some leftover ones. Students explain their thinking in different ways. For example:

> For the number 42, do you have enough to make four tens? Would you have any left? If so, how many would you have left?
>
> Student 1: I filled 4 ten frames to make four tens and had two counters left over. I had enough to make 4 tens with some leftover. The number 42 has 4 tens and 2 ones.
>
> Student 2: I counted out 42 place value cubes. I traded each group of 10 cubes for a ten-rod (stick). I now have 4 ten-rods and 2 cubes left over. So the number 42 has 4 tens and 2 ones (Adapted from Arizona 2010).

Students learn to read 53 as “fifty-three” as well as five tens and three ones. However, some number words require extra attention at first grade because of their irregularities. Students learn that the decade words (e.g., twenty, thirty, forty, etc.) indicate 2 tens, 3 tens, 4 tens, etc. They also realize many decade number words sound much like teen number words. For example, “fourteen” and “forty” sound very similar, as do “fifteen” and “fifty,” and so on to “nineteen” and “ninety.” Students learn the number words from 13 to 19 give the number of ones before the number of tens. Students also frequently make count errors such as “twenty-nine, twenty-*ten*, twenty-*eleven*, twenty-*twelve*” (Progressions, K-5 NBT 2011). Because of these complexities, it can be helpful for students to use regular tens words as well as English words: “53 is five tens 3 ones and also fifty-three.”

Following is an example of connecting the Standards for Mathematical Practice with the Standards for Mathematical Content.

404 **Connecting to the Standards for Mathematical Practice–Grade One**

Standard(s) Addressed	***Example(s) and Explanations***
1.OA.6: Add and subtract within 20. Demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums. **1.NBT.2:** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following special cases: a. 10 can be thought of as a bundle of ten ones—called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, refer to one, two, three, four, five, six, seven, eight, or nine tens. *Extension* **1.NBT.3:** Compare the two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	**Task:** The teacher has a spinner with the digits 0-9 on it. Each student has a collection of base-10 block units and rods (or "sticks"). The object of the task is for students to represent numbers that the teacher spins with their base-10 blocks, add the resulting numbers, and then represent the sum using the base-10 blocks, exchanging ten units for a rod when appropriate. For example, the teacher may first spin a 6, and asks students to represent six on the left side of their desk (or a provided mat). Then the teacher spins an 8, and students represent an 8 on the other side of their desk or mat. The teacher then instructs students to add the number of units together. Students will most likely combine the two piles and count the resulting number of units, 14. The teacher should then encourage exchanging out ten units for a rod, to emphasize that the number 14 is written as it is to represent 1 ten and 4 ones (that is, "1 rod and 4 units"). This can be repeated for several turns so that students represent larger numbers, continuing to add more and bundle. Extensions include: teachers can use spinners with different numbers on them (say 0-19) and students can represent the numbers and compare them; teachers can ask students to subtract the smaller number from the larger number; teachers can use a spinner with 0-9 and students can count out the indicated number of rods and name the number (e.g. teacher spins a 6, students take out 6 rods, and record and name the resulting number, 60); the first spin could represent the number of units while the second spin represents the number of sticks. **Classroom Connections:** A firm foundation in understanding the base-10 structure of the number system is essential for student success with operations, decimals, proportional reasoning, and later algebra. Experiences such as these give students ample practice in representing and explaining why numbers are written the way they are. Students can begin to associate mental images of why numbers have the value that they do (e.g. why the number 20 is different from and larger than the number 2). **Connecting to the Standards for Mathematical Practice** **MP.2:** Students reason abstractly and quantitatively as they move between the written representation of numbers and the base-10 block representation of numbers. **MP.5:** Students develop an understanding of the use of base-10 blocks that will lay a foundation for using them to develop and understand algorithms for operations. **MP.7:** Students begin to see that the numbers 0-9 can be represented with units only and that while the same is true for larger numbers, they can use bundles of ten units to represent them in a more organized way. This leads to the recording of numbers in the way that we do (e.g. 12 = 10 + 2, 1 stick and 2 units).

Grade one students use base-ten understanding to recognize that the digit in the tens place is more important than the digit in the ones place for determining the size of a two-digit number. **(1.NBT.3▲)** Students use models that represent two sets of numbers to compare numbers. Students attend to the number of tens, then, if necessary, to the number of ones. Students may also use math drawings of tens and ones and spoken or written words to compare two numbers. Comparative language includes but is not limited to more than, less than, greater than, most, greatest, least, same as, equal to, and not equal to (**MP.2, MP.6, MP.7, MP.8**) (Adapted from Arizona 2010, KATM 1st FlipBook 2012, and N. Carolina 2013).

Number and Operations in Base Ten	1.NBT

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Students develop understandings and strategies to add within 100 using visual models to support understanding. **(1.NBT.4▲)** In grade one students focus on developing, discussing, and using efficient, accurate, and generalizable methods to add within 100 and they subtract multiples of 10. Students might also use strategies they invent that are not generalizable.

[Note: Sidebar]

Focus, Coherence, and Rigor:

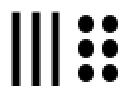
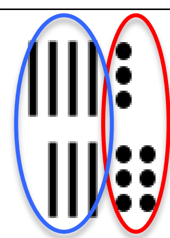
Grade one students develop understanding of addition and subtraction within 20 using various strategies **(1.OA.6▲)** and they generalize their methods to add within 100 using concrete models and drawings **(1.NBT.4▲).** Reasoning about strategies and selecting appropriate strategies is critical to developing conceptual understanding of addition and subtraction in all situations (**MP.1, MP.2,MP.3**) (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

Students should be exposed to problems both in and out of context and presented in horizontal and vertical forms. Students solve problems using language associated with proper place value, and they explain and justify their mathematical thinking. (**MP.2, MP.6, MP.7, MP.8**)

Students use various strategies and models for addition. Students relate the strategy to a written method and explain the reasoning used. **(MP. 2, MP.7, and MP.8)**

Examples: Models, Written Methods, and Other Addition Strategies.

1. Find 43 + 36. Students may total up the tens and then the ones. Place value blocks or other counters support understanding the recording of the written method:

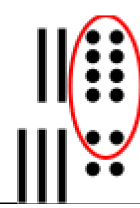
43	36	43 + 36 = (40 + 30) + (3 + 6) = 70 + 9 = 79
		

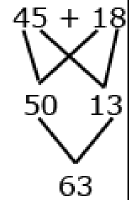
Students circle like units in the drawings and represent the results numerically.

2. Find

$$\begin{array}{r} 28 \\ +\ 34 \\ \hline \end{array}$$

Student thinks: Counting the ones I get 10, plus 2 more. I mark the ten with a little one. Adding the tens I had gives me 2 tens plus 3 tens which is 5 tens. Finally, 5 tens plus 1 more ten is 6 tens, or 60, and 2 more makes 62.



	28 + 34 1 52 62
3. Add 45 + 18 Student thinks: 4 tens and 1 ten is 5 tens, which is 50. To add the ones, I can make a ten by thinking of 5 as 3 + 2, then the 2 combines with the 8 to make 1 ten. So now I have 6 tens altogether, 60, and 3 ones left, so the total is 63.	
4. Add 29 + 14. Student thinks: Since 29 is 1 away from 30, I'll just think of it as 30. Since 30 + 14 = 44, I know that the answer is 1 too many, so the answer is 43.	

(Adapted from Arizona 2010, KATM 1st FlipBook 2012, and N. Carolina 2013)

First grade students engage in mental calculations, such as mentally finding 10 more or 10 less than a given two-digit number without counting by ones **(1.NBT.5▲)**. Drawings and place value cards such as those shown (refer to Example 1 in the previous table) can illustrate connections between place value and written numbers. Prior use of models such as connecting cubes, base ten blocks, and 100s charts helps facilitate this understanding. It also helps students see the pattern involved when adding or subtracting 10. For example:

- 10 more than 43 is 53 because 53 is one more ten than 43
- 10 less than 43 is 33 because 33 is one ten less than 43

Students may use interactive or electronic versions of models (base ten blocks, 100s charts, etc.) to develop conceptual understanding (Adapted from Arizona 2010, KATM 1st FlipBook 2012, and N. Carolina 2013).

First grade students need opportunities to represent numbers that are multiples of 10 (e.g., 90) with models or drawings and to subtract multiples of 10 (e.g., 20) using these representations or strategies based on place value **(1.NBT.5▲)**. These opportunities help develop fluency with addition and subtraction facts and reinforce counting up and counting back by 10s. As with single-digit numbers, counting back is difficult, so forward methods of counting on by tens should be emphasized instead of counting back.

Domain: Measurement and Data

A critical area of instruction in grade one is for students to develop an understanding of linear measurement and that we measure lengths by iterating length units.

Measurement and Data	1.MD
Measure lengths indirectly and by iterating length units. 1. Order three objects by length; compare the lengths of two objects indirectly by using a third object. 2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*	

In first grade students order three objects by length and compare the lengths of two objects indirectly by using a third object (**1MD.1▲**). Students indirectly compare the length of two objects by comparing each to a benchmark object of intermediate length. This concept is referred to as *transitivity*.

To compare objects, students learn that length is measured from one end point to another end point. They measure objects to determine which of two objects is longer, by physically aligning the objects. Based on length, students might describe objects as taller, shorter, longer, or higher. If students use less precise words such as bigger or smaller to describe a comparison, they should be encouraged to further explain what they mean. (**MP.6, MP.7**) If objects have more than one measurable length, students also need to identify the length(s) they are measuring. For example, both the length and the width of an object are measurements of lengths.

Examples: Comparing Lengths.

Direct Comparisons: Students can order three items by length, for example:

- Three students by height,
- Pencils, crayons, and/or markers by length,
- Towers built with cubes, ordered from shortest to tallest,
- Three students draw line segments, and then order the segments from shortest to longest.

Indirect Comparisons: Students make clay “snakes”. Given a tower of cubes, each student compares his/her snake to the tower. Then students make statements such as, “My snake is longer than the cube tower, and your snake is shorter than the cube tower. So, my snake is longer than your snake.”

(Adapted from Arizona 2010, Georgia 2011, and KATM 1st FlipBook 2012).

In standard **(1.MD.2▲)**, students gain their first experience with measuring length as the iteration of a smaller, uniform length, called a *length unit*. Students are learning here that measuring the length of an object requires placing same-size length units next to each other along the length without gaps of overlaps. The CCSS-M Progressions on Geometric Measurement recommends beginning with actual standard units (e.g. 1-inch cubes of centimeter cubes, referred to as “length-units”) to measure length, as well as even introducing rulers. Employing non-standard units (e.g. paperclips), while a popular approach, “*before students understand the concepts, procedures, and usefulness of measurement,* may actually deter student’s development.” (Adapted from Progressions Geometric Measurement 2012, pg. 9).

In order to fully understand the subtlety of using nonstandard units, students need to understand relationships between units of measure, a concept that will not appear in the curriculum until much later.

Standard **(1.MD.2▲)** limits measurement to whole numbers of length, though not all objects will measure to an exact whole unit. Students will need to adjust their answers for this fact. For example, if a pencil actually measures six to seven centimeter cubes long, the students could state the pencil is "about" centimeter cubes long, choosing the closer number.

Focus, Coherence, and Rigor:

As students measure objects **(1.MD.1-2▲)** they also reinforce counting skills and understandings that are part of the major work at grade one in the domain "Number and Operations in Base Ten"

Measurement and Data 1.MD

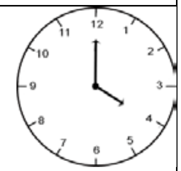
Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

First grade students understand several concepts related to telling time **(1.MD.3)**, such as:

- Within a day, the hour hand goes around a clock twice (the hand moves only in one direction). We start a day with both hands pointing up.
- When the hour hand of a clock points exactly to a number, the time is exactly on the hour
- Time on the hour is written in the same manner as it appears on a digital clock
- The hour hand on a clock moves as time passes, so when it is half way between two numbers it is at the half hour
- There are 60 minutes in one hour; so at halfway between an hour, 30 minutes have passed
- Half hour is written with "30" after the colon

Students need experiences exploring the idea that when the time is at the half-hour the hour hand is between numbers and not on a number. Further, the hour is the number before where the hour hand is because the hour hand has gone past that number. For example, in the clock below, the time is 8:30. The hour hand is between the 8 and 9, but the hour is 8 since it is not yet on the 9.

Telling Time.	
"The minute hand is halfway between 8 o'clock and 9 o'clock. It is 8:30." 	"It is 4 o'clock because the hour hand points to 4." 

The idea that 30 minutes is "halfway" is a difficult concept for students because they have to choose the hour that has passed. Understanding that two 30s make 60 is easy if students make drawings of tens or think about 3 tens and 6 tens. Students can also discuss the half visually on a clock when they work on standard **(1.G.3)**, finding half of a circle.

Measurement and Data	1.MD

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Students can use graphs and charts to organize and represent data **(1.MD.3)** about things in their life (e.g., favorite color, pets, shoe type, etc.).

Representing Data (MP.2, MP.4, and MP.5)

Tally Chart	**Picture Chart**
	

Charts may be constructed by groups of students as well as by individual students. These activities will help prepare students for work in grade two when they draw picture graphs and bar graphs (Adapted from Arizona 2010, Georgia 2011, and KATM 1st FlipBook 2012).

When students collect, represent, and interpret data, they reinforce number sense and counting skills. When students ask and answer questions about information in charts or graphs, they sort and compare data. Students use addition and subtraction and comparative language and symbols to interpret graphs and charts. (**MP.2, MP.3, MP.4, MP.5 and MP.6**)

Focus, Coherence, Rigor: Work in the cluster “Represent and interpret data” has students organize, represent, and interpret data with up to three categories (**1.MD.4**). This work can also connect to student work with geometric shapes (**1.G.1**) as students collect and sort different shapes and pose and answer related questions, such as, “How many triangles are in the collection?”, “How many rectangles are there?”, "How many triangles and rectangles are there?", “Which category has the most items?”, “How many more?”, “Which category has the least?” and “How many less?” Students’ work with data can also supports major work in the cluster “Represent and solve problems involving addition and subtraction” as students solve problems involving addition and subtraction with three whole numbers (**1.OA.1-2▲**).

Domain: Geometry

A critical area of instruction is for grade one students to reason about attributes of and composing and decomposing geometric shapes.

Geometry	1.G

Reason with shapes and their attributes.

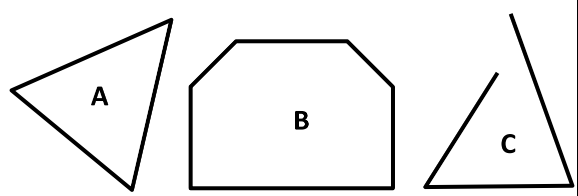
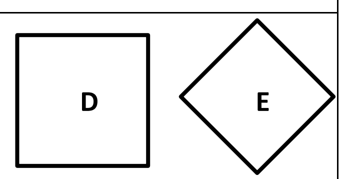
1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right

circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]

3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Grade one students describe and classify shapes by geometric attributes, and they explain why a shape belongs to a given category (e.g., squares, triangles, circles, rectangles, rhombuses, hexagons, and trapezoids). Students differentiate between defining attributes (e.g., "hexagons have six straight sides") and non-defining attributes (e.g., color, overall size, orientation) **(1.G.1)** (**MP.1, MP.3, MP.4, MP.7**) (Adapted from Progressions, K-6 G 2012).

An *attribute* refers to any characteristic of a shape. Students learn to use attribute language to describe two-dimensional shapes (e.g., number of sides, number of vertices/points, straight sides, closed figure). A student might describe a triangle as "right side up" or "red", but students learn these are not defining attributes because they are not relevant to whether a shape is a triangle or not.

Examples: Using Attributes to Name Shapes	
Which figure is a triangle? How do you know? "I know that shape A has three sides and the shape is closed up, so it is a triangle. Shape B has too many sides and shape C has an opening, so it's not closed."	
Are both the figures shown squares? Explain how you know. "I know that a square has 4 sides and they are all equal length. Even though figure E has a point facing down it is still a square."	

[4] Students do not need to learn formal names such as "right rectangular prism."

Students are exposed to both regular and irregular shapes. In first grade students use attribute language to describe why the following shapes are not triangles.

Students need opportunities to use appropriate language to describe a given three-dimensional shape (e.g., number of faces, number of vertices/points, and number of edges). For example, a cylinder is a three-dimensional shape that has two circular faces connected by a curved surface (which is not considered a face), but a first grade student might say, “It looks like a can.” Teachers can support learning by defining and using appropriate mathematical terms.

Students need opportunities to compare and contrast two-and three-dimensional figures using defining attributes. For example:

- Students list two things that are the same and two things that are different between a triangle and a cube.
- Given a circle and a sphere, students identify the sphere as three-dimensional and both shapes as round.

(Adapted from Arizona 2010)

The ability to describe, use, and visualize the effect of composing and decomposing shapes is an important mathematical skill **(1.G.2)**. It is not only relevant to geometry, but also to children’s ability to compose and decompose numbers.

Students may use pattern blocks, plastic shapes, tangrams, or computer environments to make new shapes. The teacher can provide students with cutouts of shapes and ask them to combine them to make a particular shape. Composing with squares and rectangles and with pairs of right triangles that make squares and rectangles is especially important for future geometric thinking.

Students need experiences with different sized circles and rectangles to recognize that when they cut something into two equal pieces, each piece will equal one half of its original whole **(1.G.3)**. Children should recognize that the halves of two different wholes are not necessarily the same size. Also they should reason that decomposing equal shares into more equal shares results in smaller equal shares.

Focus, Coherence, and Rigor:

As grade one students partition circles and rectangles into two and four equal shares and use related language (halves, fourths and quarters) (**1.G.3**), they build understanding of part-whole relationships and are introduced to fractional language. Fraction notation will first be introduced in grade three.

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In kindergarten through grade two students focus on addition and subtraction, and measurement using whole numbers. To be prepared for grade two mathematics students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of grade one and have met the fluency expectations for the grade. For first graders, the expected fluencies are to add and subtract within 10 (**1.OA.6▲**). These fluencies and the conceptual understandings that support them are foundational for work in later grades

Of particular importance for students to attain in grade one are the concepts, skills and understandings necessary to represent and solve problems involving addition and subtraction **(1.OA.1-2▲)**; understand and apply properties of operations and the relationship between addition and subtraction **(1.OA.3-4▲)**; add and subtract within 20 **(1.OA.5-6▲)**; work with addition and subtraction equations **(1.OA.7-8▲)**; extend the counting sequence **(1.NBT.1▲)**; understand place value and use place value understanding and properties of operations to add and subtract **(1.NBT.2-6▲)**; and measure lengths indirectly and by iterating length units (**1.MD.1-2▲**).

Place Value

By the end of grade one, students are expected to count to 120 (starting from any number), compare whole numbers (at least to 100), and read and write numerals in the same range. Students need to think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Counting to 120 and reading and representing these numbers with numerals will prepare students to count, read, and write numbers within 1,000 in grade two.

Addition and Subtraction

By the end of grade one, students are expected to add and subtract within 20 and demonstrate fluency with these operations within 10 **(1.OA.6▲).** Students can represent and solve word problems involving add-to, take-from, put-together, take-apart, and compare situations including addend unknown situations. They know how to apply properties of addition (associative and commutative) and strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems. Students use methods to add (within 100), subtract multiples of 10 (using various strategies), and mentally find 10 more or 10 less without counting. Students understand how to solve addition and subtraction equations.

Addition and subtraction are major instructional foci for kindergarten through grade two. Students who have met the grade one standards for addition and subtraction will be prepared to meet the second grade standards to add and subtract within 1,000 (using concrete models, drawings and strategies); fluently add and subtract within 100 (using various strategies) and within 20 (using mental strategies); and know from memory all sums of two one-digit numbers.

Measure Lengths

By the end of grade one, students are expected to order three objects by length (using non-standard units). Students indirectly measure objects by comparing the length of two objects by using a third object as a measuring tool. Mastering grade one measurement standards will prepare students to measure and estimate lengths (in standard units) as required in the grade two standards.

Grade 1 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 1

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*
4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*
8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = □ – 3, 6 + 6 = □.*

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a "ten."
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

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[2] See Glossary, Table 1.
[3] Students need not use formal terms for these properties.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]
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