Kindergarten

Students who are exposed to important mathematical concepts, such as numbers and their order, in quality pre-school programs will be ready for kindergarten mathematics and better prepared for later learning.

WHAT STUDENTS LEARN IN KINDERGARTEN

[Note: Sidebar]

Kindergarten Critical Areas of Instruction

In kindergarten instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects, and (2) describing shapes and space. More learning time in kindergarten should be devoted to numbers rather than to other topics.

Students also work toward fluency in addition and subtraction with whole numbers within 5.

Kindergarten Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and linking major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence and rigor will be indicated throughout the chapter.

Not all of the content in a given grade is emphasized equally in the standards. Cluster headings can be viewed as the most effective way to communicate the **focus** and **coherence** of the standards. Some clusters of standards require a greater instructional emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the later demands of college and career readiness.

The following Kindergarten Cluster-Level Emphases chart highlights the content emphases in the standards at the cluster level for this grade. The bulk of instructional time should be given to “Major” clusters and the standards within them. However, standards in the “Supporting” and “Additional” clusters should not be neglected. To do so will result in gaps in students’ learning, including skills and understandings they may need in later grades. Instruction should reinforce topics in major clusters by utilizing topics in the supporting and additional clusters. Instruction should include problems and activities that support natural connections between clusters.

Teachers and administrators alike should note that the standards are not topics to be checked off a list during isolated units of instruction, but rather content to be developed throughout the school year through rich instructional experiences and presented in a coherent manner (Adapted from the Partnership for Assessment of Readiness for College and Careers [PARCC] 2012).

[**Note**: The Emphases chart should be a graphic inserted in the grade level section. The explanation “key” needs to accompany it.]

Kindergarten Cluster-Level Emphases

Counting and Cardinality

- [m]: Know number names and the count sequence. (**K.CC.1-3▲**)
- [m]: Count to tell the number of objects. (**K.CC.4-5▲**)
- [m]: Compare numbers. (**K.CC.6-7▲**)

Operations and Algebraic Thinking

- [m]: Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. (**K.OA.1-5▲**)

Number and Operations in Base Ten

- [m]: Work with numbers 11–19 to gain foundations for place value. (**K.NBT.1▲**)

Measurement and Data

- [a/s]: Describe and compare measurable attributes. (**K.MD.1-2**)
- [a/s]: Classify objects and count the number of objects in categories. (**K.MD.3**)

Geometry

- [a/s]: Identify and describe shapes. (**K.G.1-3**)
- [a/s]: Analyze, compare, create, and compose shapes. (**K.G.4-6**)

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[1] [m] (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade **Supporting [s]** clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis. *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

(Adapted from Achieve the Core 2012)

Connecting Mathematical Practices and Content

The Standards for Mathematical Practice (MP) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject that makes use of their ability to make sense of mathematics. The MP standards represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the MP standards remains the same at all grades, the way these standards look as students engage with and master new and more advanced

[1] The ▲symbol will indicate standards in a Major Cluster in the narrative.

mathematical ideas does change. Below are some examples of how the MP standards may be integrated into tasks appropriate for Kindergarten students. (Refer to pages 9–12in the Overview of the Standards Chapters for a complete description of the MP standards.)

Standards for Mathematical Practice (MP)
Explanations and Examples for Kindergarten

Standards for Mathematical Practice	**Explanation and Examples**
MP.1 Make sense of problems and persevere in solving them.	In kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Real-life experiences should be used to support students' ability to connect mathematics to the world. To help students connect the language of mathematics to their everyday life ask students questions such as "How many students are absent" or have them gather enough blocks for the students at their table. Younger students may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" or they may try another strategy.
MP.2 Reason abstractly and quantitatively.	Younger students begin to recognize that a number represents a specific quantity and to connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. For example, a student may write the numeral "11" to represent an amount of objects counted, select the correct number card "17" to follow "16" on a vertical calendar with days arranged in tens, or build a pile of counters depending on the number drawn. In addition, kindergarten students begin to draw pictures, manipulate objects, or use diagrams or charts to express quantitative ideas. Students need to be encouraged to answer questions, such as, "How do you know", which reinforces their reasoning and understanding and helps student develop mathematical language.
MP.3 Construct viable arguments and critique the reasoning of others	Younger students construct arguments using actions and concrete materials, such as objects, pictures, and drawings. They begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. They begin to develop the ability to reason and analyze situations as they consider questions such as, "Are you sure...?", "Do you think that would happen all the time...?", and "I wonder why...?"
MP.4 Model with	In early grades students begin to represent problem situations in multiple ways

mathematics	(e.g., using numbers, words or mathematical language, objects, acting out, making a chart or list, drawing pictures, or creating equations). For example, a student may use cubes or tiles to show the different number pairs for 5, or place three objects on a ten frame and then determine how many more are needed to "make a ten." Students rely on manipulatives (or other visual and concrete representations) while solving tasks and record an answer with a drawing or equation.
MP.5 Use appropriate tools strategically	Younger students begin to consider the available tools when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergarteners may decide to use linking cubes to represent two quantities and then compare the two representations side-by-side or later, make math drawings of the quantities. Students decide which tools may be helpful to use depending on the problem or task and explain why they use specific mathematical tools.
MP.6 Attend to precision	Kindergarten students begin to develop precise communication skills, calculations, and measurements. Students describe their own actions, strategies; and reasoning using grade-level appropriate vocabulary. Opportunities to work with pictorial representations and concrete objects can help students develop understanding and descriptive vocabulary. For example, students analyze and compare two- and three-dimensional shapes and they sort objects based on appearance. While measuring objects iteratively (repetitively), students check to make sure that there are no gaps or overlaps. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions. Students should be encouraged to answer questions such as, "How do you know your answer is reasonable?"
MP.7 Look for and make use of structure	Younger students begin to discern a pattern or structure in the number system. For instance, students recognize that 3 + 2 = 5 and 2 + 3 = 5. Students notice patterns in counting strategies to build fluency in facts to five. Teachers might ask, "What do you notice when…?" Students may use various strategies to attain fluency such as counting on, counting all, and taking away.
MP.8 Look for and express regularity in repeated reasoning	In the early grades, students notice repetitive actions in counting, computations, and mathematical tasks. For example, the next number in a counting sequence is one more when counting by ones and ten more when counting by tens (or one more group of ten). Students should be encouraged to answer questions such as "What would happen if…? In the task "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?" Kindergarten students realize 8 crayons could include 4 of each color (8 = 4 + 4), 5 of one color and 3 of another (8 = 5 + 3), etc. For each solution, students repeatedly engage in the process of finding two numbers to join together to equal 8.

(Adapted from Arizona Department of Education [Arizona] 2010 and North Carolina Department of Public Instruction [N. Carolina] 2013)

Standards-based Learning at Kindergarten

The following narrative is organized by the domains in the Standards for Mathematical Content and highlights some necessary foundational skills and provides exemplars to explain the content standards, highlight connections to the various Standards for Mathematical Practice (**MP**) standards, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. A triangle symbol (▲) indicates standards in the major clusters (refer to the Kindergarten Cluster-Level Emphases table on page 2).

Domain: Counting and Cardinality

A critical area of instruction in kindergarten is representing, relating, and operating on whole numbers, initially with sets of objects.

Counting and Cardinality	K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Several learning progressions originate in knowing number names and the count sequence. One of the first major concepts in a student's mathematical development is *cardinality*. Cardinality can be explained as knowing that the number word said tells the quantity you have and that the number you end on when counting represents the entire amount counted. The big idea is that numbers mean *amount*, and no matter how you arrange and rearrange the items, the amount is the same. Students can generally say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects. (Adapted from the University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA 2011 and Georgia Department of Education [Georgia] 2011)

Kindergarten students are introduced to the counting sequence **(K.CC.1-2▲)**. When counting orally by ones, students begin to understand that the next number in the sequence is one more. Similarly, when counting by tens, the next number in the sequence is “ten more”.

Examples of counting sequences for forward counting to 100 by ones
The ones (1-10), the teens (10, 11, 12, 13, 14, 15, 16, 17, 18, 19), and "crossing the decade" (15, 16, 17, 18, 19, 20, 21, 22, 23, 24, or similarly 26-34, 35-44 etc.). Students often have trouble with counting forward sequences that cross from one family into the next family, such as when crossing the decade. Focusing on short counting sequences can be helpful.

(Adapted from the Kansas Association of Teachers of Mathematics [KATM] K FlipBook 2012).

Initially students might think of counting as a string of words, but gradually they transition to using counting as a tool to describe amounts in their world. Counting can be reinforced throughout the day.

Example:

- Count the number of chairs of the students who are absent.
- Count the number of stairs, shoes, etc.
- Counting groups of ten such as “fingers in the classroom” (ten fingers per student). (**MP.6, MP. 7, MP. 8**)

Kindergarten students also count forward—beginning from a given number—instead of starting at 1. Counting forward (or “counting on”) can be confusing for young students, because it conflicts with the initial learned strategy of counting from the beginning. Activities or games that require students to add on to a previous count to reach a goal number can encourage development of this concept (Adapted from KATM K FlipBook 2012).

Kindergarten students learn to write numbers from 0 to 20 (**K.CC.3▲)** and represent a number of objects with a written numeral 0-20 (numerals as symbols for quantities).

They understand 0 represents a count of no objects. Students need multiple opportunities to count objects and recognize that a number represents a specific quantity. As this understanding develops, students begin to read and write numerals. The emphasis should first be on quantity and then on connecting quantities to the written symbols.

Example: A Learning Sequence for Understanding Numbers.
A specific learning sequence might include: 1. Count up to 20 objects in many settings and situations over several weeks. 2. Start to recognize, identify, and read the written numerals, and match the numerals to given sets of objects. 3. Write the numerals to represent counted objects.

As students connect quantities and written numerals they also develop mathematical practices such as using precise vocabulary (**MP.6**) and noticing patterns in counting strategies (**MP.7**) (Adapted from Arizona 2010).

Common Misconceptions:

- Some students might not see zero as a number. Ask students to write 0 and say “zero” to represent the number of items left when all items have been taken away. Avoid using the word none to represent this situation.
- Teen numbers can also be confusing for young students. To help avoid confusion, these numbers should be taught as a bundle of ten ones and some extra ones. This approach supports a foundation for understanding both the place value concept and symbols that represent each teen number. Layered place value cards, described below, can help students understand the difficult teen numbers.

Adapted from (KATM K FlipBook 2012).

Counting and Cardinality K.CC

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

   c. Understand that each successive number name refers to a quantity that is one larger.

5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

In kindergarten students develop an understanding of the relationship between numbers and quantities and connect counting to cardinality. (**K.CC.4▲**). Learning to count is a complex mental and physical activity that requires relating objects distributed in space (or in time) to words said over time. Kindergarten students use their understanding of the relationship between numbers and quantities to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated concepts.

[Note: Sidebar]

The Five Major Principles of the Development of Students' Understanding of "How to Count" and "What to Count"

1. **One-to-One Correspondence Principle**: Students assign one, and only one, distinct counting word to each of the items to be counted. To follow this principle, students partition and re-partition the collection of objects to be counted into two categories: those that have been allocated a number name and those that have not. Students model numbers with objects and each object is assigned a unique number-name, based on one-to-one correspondence between each object and the number-name. If an item is not assigned a number name or is assigned more than one number name, the resulting count will be incorrect. (Refer to standard **K.CC.4a**)
2. **Standard-Order (of number-names) Principle**: Students recite a number-name list in a fixed order (e.g. students count "one, two, three" for a collection of three objects). In other words, students can rote-count. (Refer to standard **K.CC.4a**)
3. **Cardinal Principle**: Students understand that the last number-name used for the final object in a collection represents the number of items in that collection. This rule connects counting with "how many." (Refer to standard **K.CC.4b**)
4. **Order-Irrelevance Principle**: Students understand that the order in which objects are counted has no effect on the total number of objects and the quantity of a group of objects remains constant even when the objects are rearranged. (Refer to standard **K.CC.4b**)
5. **Abstraction Principle**: Students realize that the above four principles of counting apply to any collection of objects, whether tangible (e.g., marbles or blocks) or not (e.g., sounds or actions). Also objects can have similar attributes (e.g., yellow squares or red marbles) or different attributes (e.g., buttons of different colors, or toys of different types or sizes). (Refer to standard **K.CC.4**)

Many and varied opportunities for students to manipulate concrete objects or visual representations (e.g., dot cards, tens frames) and connect number-names with their quantities can help students master the concept of counting. (Adapted from N. Carolina 2013).

As students learn to count a group of objects, they pair each word said with one object (**K.CC.4a▲)**. This is usually facilitated by an indicating act (such as touching, pointing to objects, or moving them) that keeps each word said paired to one and only one object (one-to-one correspondence principle). Students learn that the last number named tells the number of objects counted (cardinality principle) and the number of objects is the same regardless of their arrangement or the order in which they were counted (order-irrelevance principle). They also understand that each successive number name refers to a quantity that is one larger. (**K.CC.4.b-c▲)** (Adapted from Progressions, K-5 CC and OA 2011).

To develop their understanding of the relationship between numbers and quantities, students might count objects, placing one more object in the group at a time.

Example:
Using cubes, students count an existing group, and then they place another cube in the set and continue counting. Students continue placing one more cube in the set at a time and then identify the new total number of cubes. Students see that the counting sequence results in a quantity that is one larger each time one more cube is placed in the group. Students may need to re-count from one, but the goal is for students to count on from the existing number of cubes—a conceptual start for the grade one skill of counting to 120, starting at any number less than 120.

To count accurately, students rely on:

- Knowing patterns and arbitrary parts of the number word sequence
- Assigning one number word to one object (one-to one-correspondence)
- Keeping track of objects that have already been counted

(Adapted from Arizona 2010 and Georgia 2011)

Students answer questions such as “How many are there?” by counting objects in a set and understanding that the last number stated represents the total amount of objects (cardinality) (**K.CC.5▲).** Over time students realize the same set counted several different times will be the same amount each time. Counting objects arranged in a line is easiest; with more practice, students learn to count objects in more difficult arrangements, such as rectangular arrays, circles, and scattered configurations.

Scattered arrangements are the most challenging for students, thus kindergarten students only count up to 10 objects if arranged this way. Given a number from 1-20, kindergarten students also count out that many objects. This is also more difficult for students than simply counting the total number of objects, because they need to remember the number of objects to be counted out as they count. (Adapted from Progressions, K-5 CC and OA 2011, and N. Carolina 2013).

Examples of Counting Strategies
Students might use various counting strategies depending on how objects are arranged, such as: • Move objects as they count each; • Line-up objects to count; • Touch objects in a scattered arrangement as they count each; and • Count objects in a scattered arrangement by visually scanning each object without touching.

(Adapted from KATM K FlipBook 2012).

Focus, Coherence, and Rigor:

As students use various counting strategies when they participate in counting activities they reinforce their understanding of the relationship between numbers and quantities and support mathematical practices such as modeling with mathematics (**MP.4**), the use of precise language (**MP.6**), and repeated reasoning to find a solution (**MP.8**).

Students come to quickly perceive the number of items in small groups—such as recognizing dot arrangements in different patterns, without counting the objects. This is a fundamental skill in the development of students' understanding of numbers called *perceptual subitizing*. Perceptual subitizing develops into *conceptual subitizing*—recognizing a collection of objects as a composite of subparts and as a whole (e.g., seeing a five-dot domino and thinking 1 and 4 or seeing a set with two subsets of 2 and saying 4) (Adapted from Progressions, K-5 CC and OA 2011). Particularly important is the 5-group pattern in which one row of 5 circles has 1, 2, 3, 4, or 5 dots below to show 6, 7, 8, 9, and 10. These rows are separated more than the individual dots to ensure seeing the group of 5 and the extra dots.

Subitizing supports the development of addition and subtraction strategies such as counting on and composing and decomposing of numbers. Students need practice to develop competency in perceptual subitizing.

Example :
The teacher might place different amounts of beans on a mat (beginning with amounts of 4 or less) and then ask students to say how many they see. As students become proficient, dot cards can also be utilized to develop fluency. For example, the teacher can show a large dot card to students, and students then take the number counters they think they need to cover the dots on the card. Then one child places his/her counters on the dots while the rest of the class counts and checks. Eventually, the teacher briefly shows one large dot card and puts it down quickly. Then students try to recognize the number of dots without counting.

Counting and Cardinality	K.CC
Compare numbers. 6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[2] 7. Compare two numbers between 1 and 10 presented as written numerals.	

In kindergarten, students compare the number of objects in one group (with up to ten objects) to the number of objects in another group (**K.CC.6▲**). Students need a strong sense of the relationship between quantities and numerals to accurately compare groups and answer related questions. They can use matching strategies or counting

[2] Includes groups with up to ten objects.

strategies to determine whether one group is greater than, less than, or equal to the number of objects in another group.

Example: (MP.1, MP.2)		
Student 1 I lined up one square to one triangle. Since there is one extra triangle, there are more triangles than squares. □□□□□□□□ ΔΔΔΔΔΔΔΔΔ	**Student 2** I counted the squares and I got 8. Then I counted the triangles and got 9. Since 9 is bigger than 8, there are more triangles than squares.	**Student 3** I put them in a pile. I then took away objects. Every time I took a square. I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.

Matching and Counting Strategies for Comparing Groups of Objects.
• Matching: Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects. • Counting: Students count the objects in each set and then identify which set has more, less, or an equal number of objects. • Observation: Students may use observation to compare two quantities (e.g., by looking at two sets of objects, they may be able to tell which set has more or less without counting). • Benchmark Numbers: Introduce the use of 0, 5 and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers. Benchmarks of 5 and 10 are especially useful with the 5-group patterns.

An important level of understanding is reached when students can compare two numbers from 1 to10 represented as written numerals, without counting (**K.CC.7▲**). Students demonstrate their understanding of numbers when they can justify their answer (**MP.3**).

Example:
Students might justify an answer (e.g., 7 is greater than 5) by demonstrating a one to-one match, counting again, or other similar approaches that makes sense to explain or verify the answer. Teachers can ask probing questions, such as “How do you know?” to elicit student thinking and reasoning (**MP.3, MP.8**) (Adapted from KATM K FlipBook 2012).

Focus, Coherence, and Rigor:
Comparing numbers and groups in kindergarten will progress to comparing addition and subtraction situations in first grade (e.g., “which is more” or “which is less” will progress to “how many more” or “how many less”).

Domain: Operations and Algebraic Thinking

Kindergarten students are introduced to addition and subtraction with small numbers, and they work toward fluency with these operations for numbers within five.

Operations and Algebraic Thinking	K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[3], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

Kindergarten students develop their understanding of addition and subtraction through making sense of word problems (**MP.1, MP.2**). Students experience a variety of addition situations that involve putting together and adding to and a variety of subtraction situations that involve taking apart and taking from (**K.OA.1-2▲**). Students use objects (such as two-color counters, clothespins on hangers, connecting cubes, five-frames and stickers), fingers, mental images, drawing, sounds, acting out situations, and verbal explanations to represent these operations (**MP. 1, MP.2, MP.4, MP.5**) (Adapted from KATM K FlipBook 2012).

[3] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards)

Students use both mathematical and non-mathematical language to explain their interpretation of a problem and the solution. Initially, students work with numbers within five, which helps them move from perceptual subitizing to conceptual subitizing, in which they say the addends and the total (e.g., two and one make three). Students will generally use fingers to keep track of addends and parts of addends and should develop rapid visual and kinesthetic recognition of numbers up to 5 on their fingers. Eventually, students will expand their work in addition and subtraction from within 5 to within 10.

Students are introduced to expressions and equations using appropriate symbols which include "+," "–," and "=". Teachers can write expressions (e.g., 3 – 1) or equations (e.g., 3 – 1 = □, or 3 = 1 + 2) that represent operations and problems with a real-world context to reinforce students' understanding of these concepts. Teachers should emphasize that an equal sign (=) means "is the same as." Students should see these equations and be encouraged to write them; however, they are not required to write equations. In kindergarten the use of formal vocabulary for both addition and subtraction (such as minuend, subtrahend and addend) is not necessary. For English learners phonological words (i.e., sum vs. some, whole vs. hole) can be challenging. Therefore for all learners it is better to use the word "total" instead of "sum" in Kindergarten and Grade one. Using the word "partners" instead of "addends" is also a helpful conceptual support for children in these grades. To support English learners these words should be explicitly taught as they are introduced (Adapted from Progressions, K-5 CC and OA 2011).

Focus, Coherence, and Rigor:

When students represent addition and subtraction this also supports mathematical practices as they use objects or pictures to represent quantities (**K.OA.1▲**), reason quantitatively to make sense of quantities and develop a clear representation of the problem (**MP.2**), "mathematize" a real-world situation (**MP.4**), and use tools appropriately to model the problem (**MP.5**). Math drawings also facilitate student reflection and discussion and help young students justify answers (**MP.3**).

Word problems with real-life applications provide students with a context to develop their understanding of addition and subtraction (**K.OA.2▲**). In kindergarten, students learn addition is *putting together* and *adding to,* and subtraction is *taking apart* and *taking from*. Kindergarteners use objects or math drawings (with simple shapes like circles) to model word problems (Adapted from Arizona 2010).

The main addition and subtraction situations in kindergarten are the dark shaded problem types in the following table. Students add and subtract within 10 to solve these problems types.

TABLE 1. Common addition and subtraction situations.

	Result Unknown	**Change Unknown**	**Start Unknown**
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
. **Take from**	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown127**
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[Note: Kindergarten students solve problem types in dark shading; grade one and two students solve problems of all sub-types. Un-shaded problems are the most difficult, grade one students work with these problems but do not master them until grade two.]
(CCSSI Glossary 2010)

To solve word problems, students learn to apply various computational methods, which are summarized in the table in the margin. Kindergarten students generally use Level 1 methods, moving on to Level 2 and 3 methods in later grades.

[Note: Sidebar]

To solve word problems, students learn to apply various computational methods. Kindergarten students generally use Level 1 methods and Level 2 and 3 methods are used in grades one and two.

Methods used for solving single-digit addition and subtraction problems

Level 1: Direct Modeling by Counting All or Taking Away
Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2: Counting On
Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3: Convert to an Easier Problem
Decompose an addend and compose a part with another addend.

Refer to Appendix F for additional information about methods used for solving single-digit addition and subtraction problems.

(Adapted from the University of Arizona Progressions Documents for the Common Core Math Standards [Progressions], K-5 CC and OA (pg. 12) 2011).

Students learn that a set of objects can be broken into two sets in multiple ways (e.g., a set of 5 objects can be separated into two sets—3 and 2, 4 and 1) (**K.OA.3▲**). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set. Students should have numerous experiences with decomposing sets of objects and recording with pictures and numbers, and the teacher makes connections between the drawings and symbols: 5 = 4 + 1, 5 = 3 + 2, 5 = 2 + 3, 5 = 1 + 4, and 5 = 5 + 0, showing the total on the left and the two addends on the right. Students can find patterns in all of the decompositions of a given number and eventually summarize these patterns for several numbers. Experience with decomposing also emphasizes the meaning of the equal sign (=) as "is the same as".

Students may use objects such as cubes, two-color counters, or square tiles to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, and 5 and 0. Students may also use drawings to show different number pairs for a given number. (**MP.1, MP.2, and MP.4**)

Example: Decomposing 5.

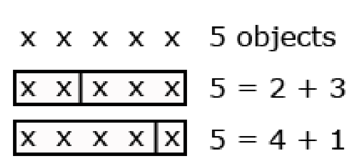
Students may draw 5 objects, showing how to decompose in several ways.

They may write equations involving 5 and its decompositions, such as:

5 = 4 + 1

3 + 2 = 5

2 + 3 = 4 + 1



The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. *The Mathematics Framework* has not been edited for publication.

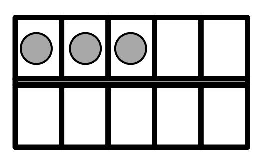
Students can systematically list all the possible number partners for a given number. For example, all number partners for 5 (0 + 5, 1 + 4, 2 + 3, 3 + 2, 4 + 1, and 5 + 0) and describe the pattern in the addends, e.g., each number is one less or one more than the previous addend (Adapted from KATM K FlipBook 2012).

Working with equations with one number on the left and an operation on the right (5 = 2 + 3) to record groups of 5 things decomposed as groups of 2 and 3 things (**K.OA.3▲**) helps students understand that equations indicate quantities on both sides of the equal sign have the same value (**MP.7).** Developing a clear understanding of the meaning of mathematical symbols allows students to develop precision in their communication about mathematics (**MP.6**). The equation can also be reversed so that an operation is on the left and the number is on the right (2 + 3 = 5). Such equations model Adding To situations. (Adapted from Progressions, K CC and K-5 OA 2011)

Number pairs that total ten are foundational for students' ability to work fluently within base-ten numbers and operations. In kindergarten, students find the number that makes 10 when added to the given number for any number from 1 to 9. Students use objects or drawings, and record their answers with a drawing or equation (**K.OA.4▲**). Students use different models, such as ten-frames, cubes, and two-color counters to help them visualize these number pairs for ten. (**MP.1**, **MP. 2, MP. 4**)

Examples: Tools and Strategies for Making Ten.

A student places three objects on a ten frame and then determines how many more are needed to "make a ten." Students may use electronic versions of ten frames to develop this skill. (**MP.5**)



A student snaps ten cubes together to make a "train."

- Student breaks the train into two parts. S/he identifies how many are in each part and records the associated equation (10 = ___ + ___).
- Student breaks the train into two parts. S/he counts how many are in one part and determines how many are in the other part without directly counting that part. Then s/he records the associated equation (if the counted part has 4 cubes, the equation would be 10 = 4 + ___).

- Student covers up part of the train, without counting the covered part. S/he counts the cubes that are showing and determines how many are covered up. Then s/he records the associated equation (if the counted part has 7 cubes, the equation would be 10 = 7 + ___).(**MP.8**)
- The student tosses ten two-color counters on the table and records how many of each color are facing up. (**MP. 8**) (Adapted from KATM K FlipBook 2012).

Later in the year, students solve addition and subtraction equations for numbers within 5, for example, 2 +1 = □ or 3 − 1 = □ while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with “Add To and Take From” situations enable students to begin to fluently add and subtract within 5 (**K.OA.5▲**).

[Note: Sidebar]

Fluency

In the standards for kindergarten through grade six there are individual content standards that set expectations for fluency in computation (e.g., “fluently” add and subtraction within five, **K.OA.5▲**). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary.

The word “fluent” is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently and appropriately. Developing fluency in each grade can involve a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies (Adapted from Progressions, K-5 CC and OA 2011).

Strategies kindergarten students may use to attain fluency with addition and subtraction within five include:

- Visualizing the small numbers involved.
- Counting on (e.g., for 3 + 2, students will state, “3,” and then count on two more, “4, 5,” and state the solution is “5”)
- Counting back (e.g., for 4 – 1, students will state, “4,” and then count back one: “3.” and state the solution is “3”)

- Counting up to subtract (e.g., for 5 – 3 , students will say, “3,” and then count up until they get to 5, keeping track of how many they counted up, stating that the solution is “2”)
- Using doubles (e.g., for 2 + 3, students may say, “I know that 2 + 2 is 4, and 1 more is 5”)
- Using commutative property (e.g., students may say, “I know that 2 + 1 = 3, so 1 + 2 = 3”)
- Using fact families (e.g., students may say, “I know that 2 + 3 = 5, so 5 – 3 = 2”)

Exemplar demonstrating conceptual understanding, application, and connection to the mathematical practices, available at http://www.illustrativemathematics.org/.
Shake and Spill Students use 5 two-color counters (e.g., red on one side and yellow on the other) and a cup (optional). The students put the counters in the cup, shake it, and spill them onto a table. The students determine how many of each color is showing and record the sum using drawings or equations. The students should "shake and spill" several times to show different pairs of numbers that sum to 5.

(Illustrative Mathematics 2013).

Domain: Number and Operations in Base Ten

Number and Operations in Base Ten	K.NBT
Work with numbers 11–19 to gain foundations for place value. 1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	

Kindergarten teachers help their students lay the foundation for understanding the base-ten system by drawing special attention to the number 10. Students compose and decompose numbers from 11 to 19 into ten ones and some further ones. Students use objects or drawings and record each composition or decomposition with a drawing or equation (e.g., 18 = 10 + 8) (**K.NBT.1▲**).

Students describe, explore, and explain how the counting numbers from 11 through 19 are composed of ten ones and some more ones. For example, when focusing on the number “14,” students count out fourteen objects using one-to-one correspondence and then use those objects to compose one group of ten ones and four additional ones. Students connect the representation to the symbol “14” and recognize the written pattern in these numbers; that the numbers start with 1 (represents one ten) and end with the number of additional ones (**MP.1, MP.2, MP.4, MP.5, MP.6, MP.7, MP.8**) (Adapted from Progressions, K-5 NBT 2011).

Students may have difficulty with ten as a singular word that means 10 things. For many students, the understanding that a group of 10 things can be replaced by a single word and that they both represent 10 is confusing. Students learn that this set of numbers (11-19) does not follow a consistent pattern in the verbal counting sequence. For example:

- Eleven and twelve are special number words.
- “Teen” means one “ten” plus ones.
- The verbal counting sequence for teen numbers is backwards—we say the ones digit before the tens digit. For example, 27 reads tens to ones (twenty-seven), but 17 reads ones to tens (seven-teen). Therefore, it is helpful for children to say regular tens words such as “one ten, seven ones,” as well as saying the English words. This clarifies the pattern for them. They can say the counting sequence in English and in regular tens words while pointed to a chart that shows objects arranged in tens and ones.

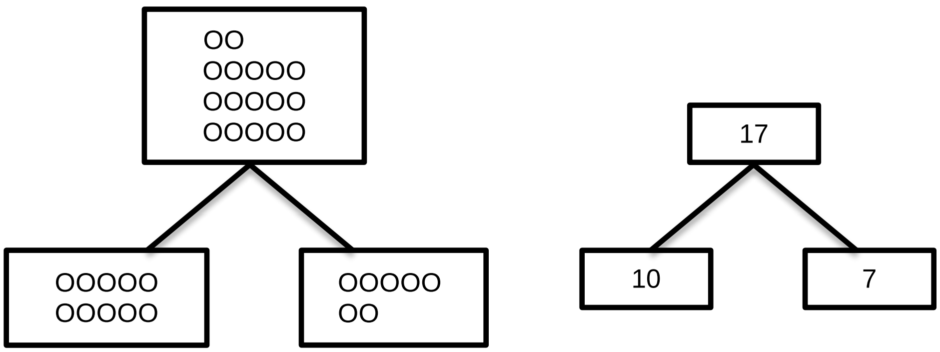
(Adapted from Arizona 2010)

To develop student understanding of written teen numbers, students read numbers as well as describe quantities. For example, for the number 17, students read “seventeen”, decompose the number as “one group of ten ones and seven additional ones,” and record their understanding as 17 = 10 + 7 or using math drawings. Kindergarten students should see addition and subtraction equations. Student writing of equations in kindergarten is encouraged, but it is not required (Adapted from KATM K FlipBook 2012).

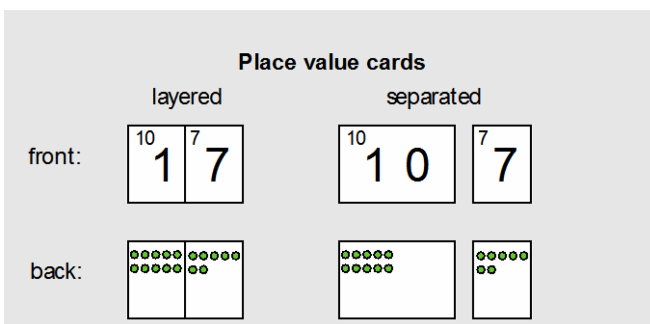
Math drawings and other methods can help develop students' place value understanding with "teen" numbers.

Examples: Understanding Teen Numbers.

Using Ten-Frames and Number-Bond Diagrams



Using Layered Place Value Cards



Children can use layered place value cards to see the 10 "hiding" inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings.

(Progressions, K-5 NBT 2011)

Domain: Measurement and Data

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Students recognize and distinguish measurable attributes (e.g., length, area, volume) from non-measurable attributes (e.g., big or bigger) (**K.MD.1**). Initially, many students will not be able to differentiate between these two types of attributes. Students will say one object is "bigger" than another without clarifying that it is longer, greater in area or volume, and so forth.

For students to accurately describe attributes such as length and weight, they need multiple opportunities to informally explore these attributes. Teachers encourage students' conversations to extend from describing objects as "big," or "small," as well as "long," "tall," or "high," to naming, discussing, and demonstrating with gestures the appropriate attribute (e.g., length, area, volume, or weight).

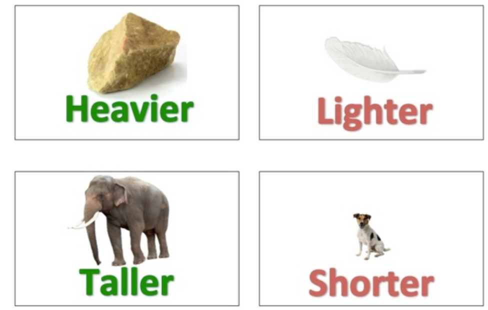
For example, a student might describe the measurable attributes of an empty can or milk carton by talking about how tall, wide, and heavy the can is, or how much liquid will fit inside the container. These are all measurable attributes. By contrast non-measurable attributes include designs, words, colors, or pictures on the can. As students discuss these situations and compare objects using different attributes, they learn to distinguish, label, and describe several measureable attributes of a single object. (**MP.4, MP.5, MP.6, MP.7**)

Students directly compare two objects with a measurable attribute in common, to see which object has "more/less of" the attribute, and describe the difference (**K.MD.2**). For example, students directly compare the heights of two children and describe one child as taller/shorter. Language plays an important role in this standard as students describe the similarities and differences of measurable attributes of objects (e.g., shorter than, taller than, lighter than, the same as, etc.). (**MP.2, MP.4, MP.6, MP.7**)

When making direct comparisons for length, students must attend to the “starting point” of each object (e.g., the ends need to be lined up at the same point) or students need to compensate when the starting points are not lined up. Students develop an understanding of conservation of length (if an object is moved, its length does not change), an important concept when comparing the lengths of two objects (Adapted from Arizona 2010 and Progressions K-5 Geometric Measurement 2012).

With practice, students become increasingly competent at direct comparison—comparing the amount of an attribute in two objects without measurement. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: Which box will hold more? Which box will hold the least? Will they hold the same amount? How could you find out? Students can decide to fill one box with dried beans then pour the beans into the other box to determine the answers to these questions (Adapted from KATM K FlipBook 2012).

Following is a sample problem that highlights connections between the Standards for Mathematical Content and the Standards for Mathematical Practice.

Standards	***Explanations and Examples***
K.MD.2: Directly compare two objects with a measurable attribute in common, to see which object has "more of/less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*	**Task:** ***The Comparison Game***: For this game, students have packs of 4 pairs of comparison cards, each pair corresponding to the following comparisons: heavier/lighter, taller/shorter, holds more/holds less, longer/shorter. In addition, each card pair has sample pictures on them that indicate the comparison, and furthermore, the words may be color-coded to aid students who cannot yet read the words on the cards (examples are shown in Figure 1). At the front of the room, the teacher shows the students two objects in sequence; the students must raise the appropriate card to compare the *second object to the first*. Several rounds are played with several different objects.  **Figure 1 Sample Cards** **Classroom Connections:** In alignment with Standard K.MD.2, the purpose of this task is to give students several opportunities to compare measureable attributes of objects. Teachers can use a variety of objects and a variety of attributes, comparing between two different objects and even within the same object (e.g. comparing the width of a cereal box to its length). These informal comparisons of attributes lead to the development of estimation strategies for measurement and the use of standard units (e.g. how many smaller unit squares fit into a given rectangle?). **Connecting to the Standards for Mathematical Practice:** (MP.2) Students reason abstractly when they imagine the attributes of given objects and attempt to compare them, even in the absence of physical objects at hand. They mentally attribute quantities to features of objects when they compare them. (MP.3) Students may be asked to justify why they think a comparison is the correct comparison, and if students disagree they can try to explain their reasoning.

[Note: The vocabulary on the cards may be too difficult for students. The teacher can introduce the lesson by holding up real objects, such as a book or a pencil, and state, "This book is heavier than the pencil" and then pass the two objects to the students to hold. After students are familiar with the concepts (e.g., heavier or lighter) through hands on experiences, the teacher can reinforce the concept by using the cards].

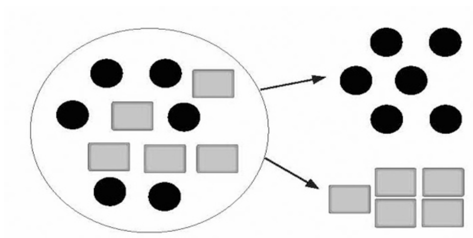
Measurement and Data K.MD

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[4]

Kindergarten students connect counting and ordering skills and understandings to help them classify objects or people into given categories, count the numbers of objects in each category, and sort the categories by count (**K.MD.3**).

Students identify similarities and differences between objects (e.g., size, color, shape.) and use these attributes to sort a collection of objects.
(**MP.2, MP.6, MP.7**)
For example:



Once the objects are sorted, students count the amount in each set and then sort each of the sets by the amount in each set.

For example, when given a collection of buttons, students separate buttons into different piles based on color. Next, they count the number of buttons in each pile [e.g., blue (5), green (4), orange (3), and purple (4)]. Finally, they organize the groups by the quantity in each group (e.g., from the smallest group (orange) to the largest group (blue) and groups with the same number (green and purple) are grouped together).
Students should be able to explain their thinking, for example:

- Explain how they sorted the objects
- Explain how they labeled each set with a category

[4] Limit category counts to be less than or equal to 10.

- Answer a variety of counting questions such as, “How many …”
- Compare the sorted groups using words such as, “most”, “least”, “same” and “different”.

(Adapted from KATM K FlipBook 2012)

Focus, Coherence, and Rigor:

As kindergarteners classify objects they build a foundation for collecting data and creating and analyzing graphical representations in later grades. Also as students count the number of objects in each category and sort the categories by count they reinforce important comparing skills and understanding, which are part of the major work at this grade in the “Counting and Cardinality” domain (**K.CC.4-7▲**). Students can also reinforce mathematical practices as they make sense of problems by counting and recounting (**MP.1**) and explaining their process and reasoning (**MP.3**).

Domain: Geometry

A critical area of instruction in kindergarten is for students to describe shapes and space. Students develop geometric concepts and spatial reasoning from experience with the shapes of objects and the relative positions of objects.

Measurement and Data	K.MD

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

Students use positional words to describe objects in the environment, such as “in” and “out”, “inside” and “outside”, “down” and “up”, “above” and “below”, “over” and “under”, “before” and “after”, “top” and “bottom”, “front” and “”back, “”right and “left”, “on” and “off”, “begin” and “end”, and “near” and “far”.

Students develop spatial sense by connecting geometric shapes to their everyday lives. Students need opportunities to identify and name two- and three-dimensional shapes in and outside of the classroom and describe relative positions by answering questions, such as:

- Which way?
- How far?
- Where?
- What type of objects?

They begin to name and describe three-dimensional shapes with mathematical vocabulary, such as “sphere,” “cube,” “cylinder,” and “cone,” and answer related questions. (**MP.6, MP.7**). For example:

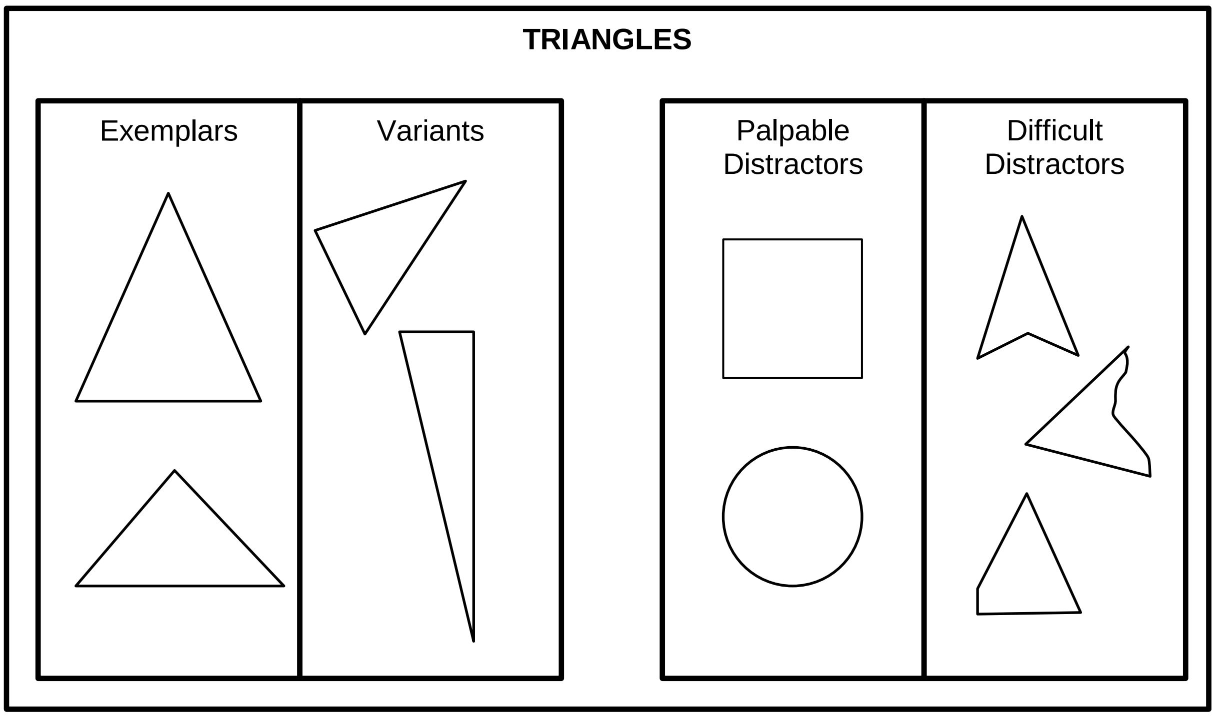
- Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see [(Possible answer: The rectangle (a poster) is over the sphere (globe)].
- The teacher holds up objects such as an ice cream cone, a number cube or ball, and asks students to identify each shape.
- The teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions--Where is the object?

(Adapted from Arizona 2010, KATM K Flip Book 2010, and Progressions K-6 Geometry 2012)

Kindergarten students work with a variety of shapes and in different sizes (**K.G.2**). They learn to match two-dimensional shapes even when the shapes have different orientations. Students name shapes such as circles, triangles, and squares, whose names occur in everyday language, and distinguish them from non-examples of these categories.

Students develop an intuitive image of each shape category. The following table includes samples of:

- Exemplars--typical visual prototypes of the shape category
- Non-examples
  - o Variants--other examples of the shape category
  - o Palpable Distractors--non-examples with little or no overall resemblance to the exemplars
  - o Difficult distractors--visually similar to examples but lack at least one defining attribute



Common Misconceptions:

- Most kindergarten students are not able to recognize an "upside down" triangle as a triangle, because of its orientation. Students should be exposed to many types of triangles, in many different orientations, to eliminate the misconception that a triangle is always vertex-up and equilateral.
- Many times a square with a vertex pointing downward is labeled as a "diamond." This needless introduction of a new shape name should be avoided, as it only serves to confuse the fact that such a shape is still a square, though its orientation is atypical.

Some strategies to support learning shapes include: (**MP.6, MP.7**)

- Kindergartners form visual templates or refer to models for shape categories (e.g., children recognize a shape as rectangle because it looks like a door).
- Students see examples of rectangles that are long and skinny and contrast rectangles with non-rectangles that appear similar but do not have an important defining attribute.
- Students see examples of triangles that have sides with three different lengths and then contrast triangles with non-triangles.
- The teacher hands out pairs of paper shapes in different sizes. Each student is given one shape. Then students need to find the partner who has the same shape.
- The teacher brings in a variety of spheres (tennis ball, basketball, globe, ping pong ball, etc.) to demonstrate that size does not change the name of a shape.

(Adapted from Arizona 2010, KATM K Flip Book 2010, and Progressions K-6 Geometry 2012).

Students identify shapes as two dimensional (lying in a plane, “flat”) or three dimensional (“solid”) (**K.G.3**). Students differentiate between two-dimensional and three-dimensional shapes. (**MP.6, MP.7**) For example:

- Students name a picture of a shape as two dimensional because it is flat and can be measured in only **two** ways (length and width).
- Students name an object as three dimensional because it is not flat (it is a solid object/shape) and can be measured by length, width, height/depth.

(Adapted from Arizona 2010)

Measurement and Data	K.MD
Analyze, compare, create, and compose shapes. 4. Analyze and compare two- and three-dimensional shapes, in different sizes and	

orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length). 5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. 6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

Kindergarten students connect their work with identifying and classifying simple shapes (refer to standards K.G.1–3) to help them compare shapes and manipulate two or more shapes to create a new shape. This understanding also builds foundations for students to "reason with shapes and their attributes" in grade one (refer to standards 1.G.1–3).

Students describe similarities and differences between and among shapes using informal language **(K.MD.4)**. These experiences help young students begin to understand how three-dimensional shapes are composed of two-dimensional shapes (e.g., the base and the top of a cylinder is a circle; the "face" of a cube is a square; a circle is formed in the shadow of a sphere). In early explorations of geometric properties students discover how categories of shapes are subsumed within other categories.

Example: Sorting Shapes.
Students sort a pile of squares and rectangles into two groups. They discuss how the rectangles and squares are alike and how they are different. After students demonstrate an understanding of the differences between squares and rectangles, the teacher hands out either a square or a rectangle cut-out to each student. The teacher directs students with the square cut outs to one side of the room and the students with the rectangle cut outs to the opposite side of the room. The rectangle and square cut outs differ in size and color so that students focus on the shape attributes. Students learn informal language such as "a square is a special rectangle that has 4 sides of equal length" to avoid the misconception that a square is not a rectangle.

Students work with various triangles, rectangles, and hexagons with sides that are not all congruent. Initially, students describe shapes using everyday language and then they expand their vocabulary to include geometric terms such as sides and vertices/corners. Opportunities to work with pictorial representations and concrete objects, as well as technology, will help students develop their understanding and descriptive vocabulary for both two- and three-dimensional shapes. (**MP.4, MP.6, MP.7**)

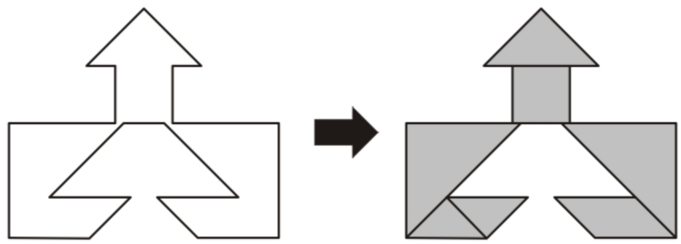
In kindergarten students model shapes in the world by building shapes from physical components (e.g., clay, sticks, toothpicks, marshmallows, gumdrops, straws) and drawing shapes (**K.G.5**) Two-dimensional shapes are flat and three-dimensional shapes are not-flat (and can be "solid"), so students should draw or create two-dimensional shapes and build three-dimensional shapes. (**MP.1, MP.4, MP.7**)

Students compose simple shapes to form larger shapes and answer questions such as, "Can you join these two triangles with full sides touching to make a rectangle?" (**K.G.6**)
Composing shapes is an important concept in kindergarten. Students move from identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. Students rotate, flip, and arrange puzzle pieces, and they move shapes to make a design or picture. Finally, students manipulate simple shapes to make a new shape (**MP.1, MP.3, MP.4, MP.7**) (Adapted from KATM K FlipBook).

Puzzles provide opportunities for students to apply spatial relationships and develop problem solving skills in an entertaining and meaningful way. Pattern blocks and tangrams are often utilized when students work with two-dimensional shapes.

Example. Exploring Shapes with Tangrams.

Students could make a school house using tangrams. The teacher models how to place the pieces and discusses how it is necessary to turn over, rotate, or slide pieces to complete a puzzle.



Each student or pair of students is then provided with a set of tangrams and a simple puzzle such as the outlined version of the school house above. Students use their pieces to complete the puzzle (Adapted from National Council of Teachers of Mathematics Illuminations, Puzzling Relationships 2013).

Composing and decomposing shapes with right angles (squares, rectangles, and right triangles that also make isosceles triangles) provides important foundations for central geometric concepts in later grades.

Examples of an interactive tangram puzzle can be found at the National Council of Teachers of Mathematics at http://www.nctm.org/standards/content.aspx?id=25012.

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In kindergarten through grade two students focus on addition and subtraction, and measurement using whole numbers. To be prepared for grade one mathematics students should be able to demonstrate they have acquired certain mathematical concepts and procedural skills by the end of kindergarten and have met the fluency expectations. For kindergarteners, the expected fluencies are to add and subtract within 5 (**K.OA.5▲**). Addition and subtraction are introduced in kindergarten and these fluencies and the conceptual understandings that support them are foundational for work in later grades

Of particular importance for students to attain in kindergarten are the concepts, skills and understandings necessary to know the number names and the count sequence **(K.CC.1-3▲**); count to tell the number of objects **(K.CC.4-5▲**); compare numbers **(K.CC.6-7▲**); understand addition as putting together and adding to, and understand subtraction as taking apart and taking from **(K.OA.1-5▲**). Also, working with numbers to gain foundations for place value **(K.NBT.1▲**) is essential to understanding the base-ten system of numbers.

Counting and Cardinality

In kindergarten, students learn to count. Students should connect counting to cardinality—knowing that the number word tells the quantity you have and that the number you end on when counting represents the entire amount counted. Until this concept is developed, counting is merely a routine procedure done when a number is needed and students will not understand how to apply numbers to solve problems.

By the end of kindergarten, important number concepts and skills for students include: count by ones and tens to 100 (rote counting); continue a counting sequence when beginning from a number greater than 1 (counting on); count objects to 20; sequence numbers to 20; understand one-to-one correspondence; identify a quantity using both numerals and words; represent numbers with numerals (and pictures and words); understand numbers and the relationships between quantities; and understand the concept of “more” and “less.” Counting to 100 and representing numbers with numerals (0 to 20) will prepare students to read and write numbers to 120 in grade one.

Addition and Subtraction

By the end of kindergarten, students are expected to add and subtract within 10 and solve addition and subtraction word problems. Students are also expected to be fluent with addition and subtraction within 5. Fluency with addition and subtraction will prepare students to add within 100 in grade one. Addition and subtraction is a major instructional focus for kindergarten through grade two.

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade K

Counting and Cardinality K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

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[1] Include groups with up to ten objects.
[2] Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

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[3]Limit category counts to be less than or equal to 10.