Transitional Kindergarten

The Kindergarten Readiness Act of 2010 (Senate Bill 1381, Chapter 705, Statutes of 2010) changed the entry-age requirements for kindergarten in California's public schools.[1] It also required local educational agencies to offer transitional kindergarten classes in addition to kindergarten classes starting in the 2012-2013 school year. Transitional kindergarten is defined in California *Education Code* section 48000(d) as, "the first year of a two-year kindergarten program that uses a modified kindergarten curriculum that is age and developmentally appropriate." Traditional kindergarten, in contrast, is a one-year program with grade-specific curriculum.

The Kindergarten Readiness Act requires that districts provide students in transitional kindergarten instruction in a modified curriculum that is age and developmentally appropriate but it does not specify what that curriculum should be. It leaves it to districts to determine what age and developmentally appropriate means in terms of curriculum. The law also defines transitional kindergarten as the first year of a two-year program kindergarten program, so districts should not replicate a preschool program or traditional kindergarten program.

In determining what modified curriculum will be implemented, districts need to consider how transitional kindergarten fits into the district's early education system and how it

[1] California *Education Code* section 48000(a) specifies that a child shall be admitted to kindergarten at the beginning of the school year if the child will have his or her fifth birthday on or before the following dates: November 1 for the 2012–2013 school year, October 1 of the 2013–2014 school year, and September 1 of the 2014–2015 school year.
The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. *The Mathematics Framework* has not been edited for publication.

differs from instruction in preschool and kindergarten. It will take instructional leadership at both the district and the school level to support the ongoing collaboration and articulation necessary to ensure that transitional kindergarten meets the instructional and developmental needs of young learners. Ideally, teachers and curriculum developers who know mathematics content and are well versed in child development theories will be engaged in developing the curriculum. Articulation with preschool programs in the district and in the community and with traditional kindergarten classes is essential if transitional kindergarten is going to serve as a bridge between preschool and traditional kindergarten.

WHAT STUDENTS LEARN IN TRANSITIONAL KINDERGARTEN

Overview

Unlike preschool or kindergarten, transitional kindergarten does not have grade-specific content standards. Therefore, the guidelines in this document reflect the range of abilities that students may possess in the period between preschool and kindergarten, but are not specific to a grade-level standard. Each domain section will include the Preschool Foundation (60 months) and the corresponding kindergarten standard from the California Common Core State Standards for Mathematics (CA CCSSM). Examples of activities that illustrate this spectrum will be provided.

To build the foundation for success in traditional kindergarten and beyond, mathematics instructional time in transitional kindergarten should focus on two critical areas of

mathematics. One area is representing, relating, and operating on whole numbers. The other area is geometry with a focus on identifying and describing shapes and objects and analyzing, comparing, and composing shapes. To help students gain a deeper understanding of mathematics, the Standards for Mathematical Practice should be connected to content instruction. [Adapted from CCSESA 2011, 26. See text below.]

Instructional time should focus on two critical mathematical areas. One area is representing, relating, and operating on whole numbers. Young students work initially with sets of objects focusing on the topics of subitizing (instantly seeing how many objects are in a small group without counting) (Clements, 1999); counting and cardinality (knowing the number names and the count sequence, understanding that the last number name said when counting a set of objects tells the number of objects counted, and comparing numbers); and operations and algebraic thinking (understanding addition as putting together and adding to, and subtraction as taking apart and taking from). The second important area is geometry with a focus on identifying and describing shapes and space; and analyzing, comparing, creating, and comparing shapes. These two areas are intricate and complex and build the foundation for future learning in mathematics. While both prepare the young learner for more formal mathematics instruction, learning time should be devoted to number sense more than any other topic in mathematics.

CCSESA 2011, 26

It is important to keep in mind that all students should receive instruction that meets them at their developmental level and provides opportunities for growth. In the classroom, this means that if a student is struggling with some of the Preschool Learning Foundations, s/he should be provided opportunities to develop her/his abilities in these areas. This also means that if a student meets some or all of the kindergarten standards, that students should be provided with opportunities for learning beyond these standards.

Instruction in transitional kindergarten should be developmentally appropriate and reflect the needs of the students. This means hands-on activities and instruction primarily in small to middle-sized groups. Particularly important are opportunities to support mathematical vocabulary acquisition in teacher-student and student-student interactions. Questions of all kinds support mathematical thinking and problem-solving, especially open-ended and more challenging questions. Although the CA CCSSM do not include much of what is traditionally taught with calendar-time activities, these activities can be valuable for students' social and academic development if they support important goals. Academic calendar-time activities should develop students' language, understanding of sequence, and number concepts.

The section that follows includes important factors for high quality support of math:

- mathematically-enriched environment,
- frequent opportunities for mathematical discourse,
- engaging and meaningful mathematics activities,
- explicit instruction,
- modeling mathematical thinking, and
- nurturing students' mathematical explorations.

A *mathematically-rich environment* includes a mathematics center that is refreshed on a regular basis, posters (e.g., shapes, numbers with sets) or wall sections devoted to interesting mathematics problems ("Are there more ducks or geese?" "Fewer brown or

gray birds?"), a variety of manipulatives (teddy bears, snap-together cubes, dinosaurs, vehicles, etc.), unit blocks, shopping paraphernalia (money, cash register, labels for prices, grocery store items), two- and three-dimensional shapes, attribute blocks, and etc.

Frequent opportunities for mathematical discourse and "math talk" build mathematical understanding and vocabulary*. Mathematical discourse* requires thinking on one's feet and knowing the vocabulary, as well as definitions for mathematics terms and concepts that make sense to students. The examples below are full of mathematical vocabulary (triangle, sides, corners, count, more, bigger, longer, and number words 1-7).

Example 1:

Student: "Is this a triangle?" [holds up a square]

Teacher: "What do you think, children?" [asking other children in the small group to contribute]

Students in unison: "No!"

Teacher: "Why not?"

Student #2: "Because a triangle doesn't have four sides."

Teacher: "That's right. How many sides does a triangle have?"

Student #3: "Three!"

Teacher: "How many corners does a triangle have?"

Student #3: "Three, just like the sides!"

Example 2:

Student Nora: “Sami isn’t being fair; he has more trains than I do.”

Teacher: “How do you know?”

Student Nora: “His pile looks bigger!”

Student Sami: “I don’t have more!”

Teacher: “How can we figure out if one of you has more?”

Student Nora: “We could count them”

Teacher: “Okay, why don’t both of you count your trains.”

Student Sami: “One, two, three, four, five, six, seven.”

Student Nora: “One, two, three, four, five, six, seven.” [fails to tag and count one of her eight trains]

Student Sami: “She skipped one! That’s not fair!”

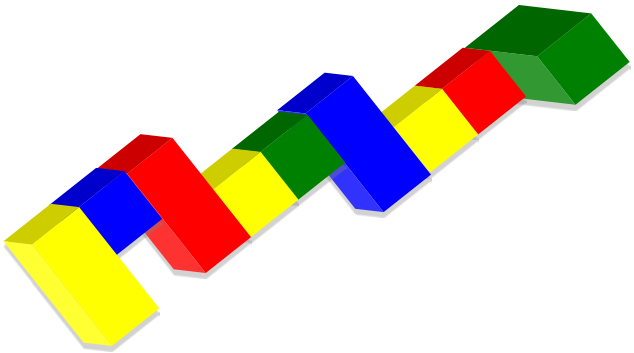
Teacher: “You are right, she did skip one. We could count again and be very careful to make sure not to skip, but can you think of another way that we can figure out if one of you has more?”

Student Sami: “We could line them up against each other and see who has a longer train.”

Teacher: “Okay, show me how you do that. Sami, you line up your trains, and Nora, you line up your trains.”

Engaging and meaningful mathematics activities are those that encourage students to think mathematically about the world around them. These frequently take careful planning. For a student who is interested in dinosaurs, helping him/her make a t-chart of herbivores and carnivores (using pictures or toy versions of the dinosaurs or writing

the names of the dinosaurs from a book) and then counting how many in each category can be a highly engaging activity. Some students love the challenge of recreating structures with building blocks that connect or snap together or magnetic builders. Create a structure with one of these sets and ask a student to recreate it, including exact shapes, colors and positions. Then ask a student to create a set that you or other students duplicate – tell the student to make it as "hard" as possible. Then have them analyze whether you and the other students created it correctly.



Explicit instruction is vital in transitional kindergarten. It allows teachers to support students' acquisition of concepts that may not come up in play or other classroom activities. Explicit instruction does not mean didactic. It means purposefully providing activities that support the understanding of a mathematical concept. The example of the dinosaur sorting activity above is explicit (but, still highly engaging to a dinosaur fan!). Explicit instruction includes dividing a set of toy trucks into three equal (and fair) shares and measuring how many children laid end-to-end it would take to equal the length of a whale shark and then, how many mice. Does it take more mice or more children? All of these activities are purposeful, explicit, and contain important mathematical concepts.

Modeling mathematical thinking provides students with strategies, techniques, and a path to deeper and more flexible understanding of mathematical concepts. There are a variety of ways to sort students into groups - color of clothing, girls and boys, laced shoes versus non-laced shoes, counting off, or the first or last letter of students' names. Visually and verbally modeling these sorting techniques helps students understand that there is more than one way to solve a problem. Teachers encounter mathematics problems throughout the day. Pencils are needed at each table (How many total? How many at each table?). More milk cartons are needed from the cafeteria (How many more?). How many minutes before lunch time? How many cotton balls are needed for an activity? Solving these and other problems out loud and with students provides them with opportunities to see the use of mathematics in solving real-world problems. Visual supports such as a list of numbers with dots in 5-patterns above them can support analysis and learning of number words and quantities.

Finally, *nurturing student's mathematical explorations* can create a classroom atmosphere where students believe that they can solve problems and learn fun new concepts. Discovering repeating numbers in a hundreds chart is eye-opening for a young student. Realizing that one plus any whole number equals the next number in the counting sequence is magical. Activities like these nurture students' interest and encourage future mathematical investigations.

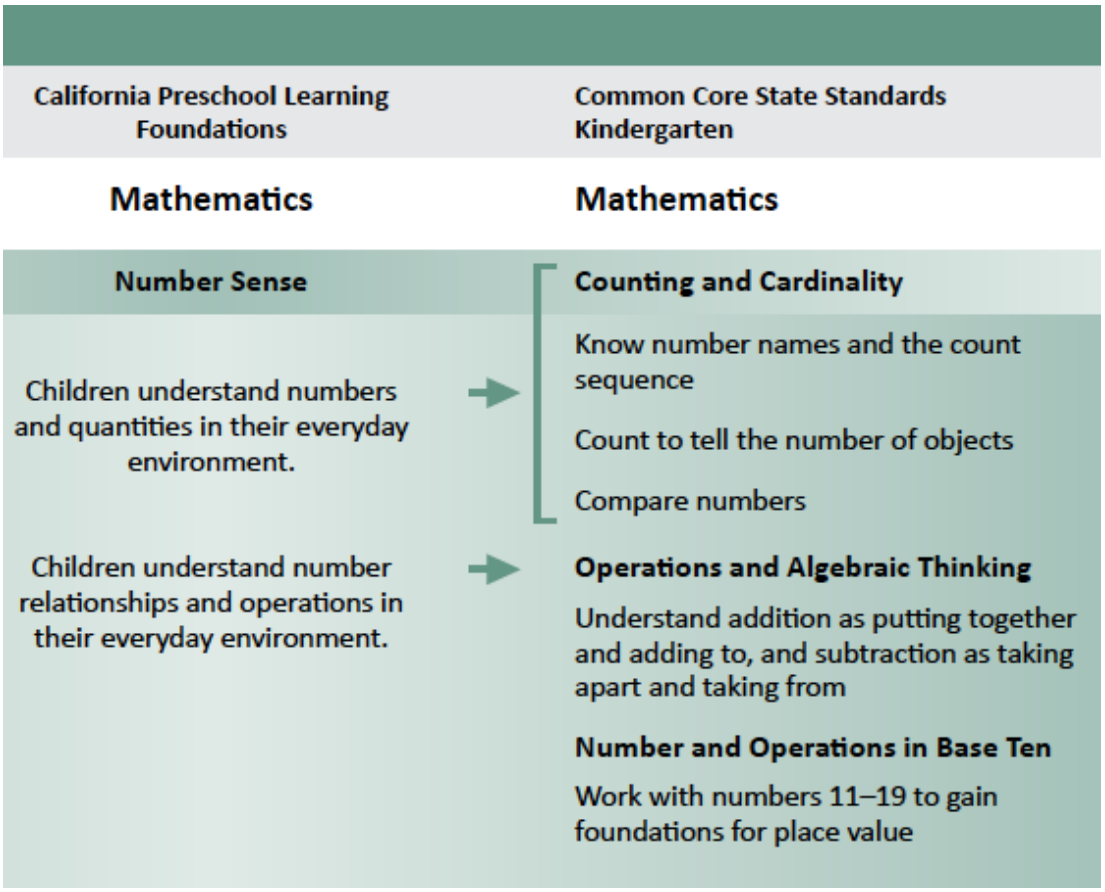
Creating a learning environment that supports foundational mathematics is critical for the acquisition of later, more complex mathematical knowledge and skills. Research

shows that early mathematics skills at entry to kindergarten are predictive of later academic success in both reading and mathematics (Duncan, 2007). Transitional kindergarten provides an excellent opportunity to continue building on students' mathematical understandings. Because children arrive at school with varied mathematical experiences, differentiated instruction is an essential part of classroom teaching. (See the Universal Access chapter for more information.) Knowing where each student is in development and fine-tuning instruction to meet each student's needs are critical in providing quality education for all students. This is also true for English learners and students with disabilities. While whole group activities can be useful for introducing a concept or playing a game, smaller groups or one-on-one interactions are necessary for student acquisition of in-depth knowledge of a concept, and teacher access to students' mathematical thinking, knowledge and skills.

Woven throughout the activity examples are references to the Standards for Mathematical Practice (MP). The MP standards are process standards that describe how mathematically proficient students engage in mathematics. These standards describe behaviors to nurture in students. The MP standards are appropriate for transitional kindergarten students and should be integrated throughout instruction. Examples of these practices that are appropriate to transitional kindergarten are provided (see the chart, "Standards for Mathematical Practice and Explanation and Examples," on page 24).

One approach to developing a modified curriculum that is age and developmentally appropriate is to consider the "intersections between the *California Preschool Learning Foundations* with the *California's Common Core State Standards* for kindergarten." (CCSESA 2011, 25) Transitional kindergarten can be thought of as an opportunity to expose students to the kindergarten standards rather than striving for mastery of the more difficult aspects of those standards. A modified curriculum could provide, in the beginning especially, more hands-on activities, more learning through play and exploration, and more time to develop students' mathematical skills and conceptual understandings in core lessons about smaller numbers. It should focus on developing skills and habits of mind that lead to success in traditional kindergarten, including problem solving, persistence, and reasoning.

The California Department of Education has published a document, *The Alignment of the California Preschool Learning Foundations with Key Early Education Resources* (http://www.cde.ca.gov/sp/cd/re/documents/psalignment.pdf), that connects the California Infant/Toddler Learning and Development Foundations, Head Start Child Development and Early Learning Framework, California Preschool Learning Foundations, and CA CCSSM The chart below provides an alignment between the California Preschool Learning Foundations and CA CCSSM:


California Preschool Learning Foundations
Common Core State Standards Kindergarten
Mathematics
Mathematics
Number Sense
Counting and Cardinality
Children understand numbers and quantities in their everyday environment.
Know number names and the count sequence
Count to tell the number of objects
Compare numbers
Children understand number relationships and operations in their everyday environment.
Operations and Algebraic Thinking
Understand addition as putting together and adding to, and subtraction as taking apart and taking from
Number and Operations in Base Ten
Work with numbers 11–19 to gain foundations for place value
Algebra and Functions (Classification and Patterning)
Measurement and Data
Children sort and classify objects in their everyday environment.
Classify objects and count the number of objects in categories
Children recognize/expand understanding of simple repeating patterns.

Measurement	Measurement and Data
Children compare, order, and measure objects. →	Describe and compare measurable attributes

Geometry	Geometry
Children identify and use shapes. →	Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). Analyze, compare, create, and compose shapes.
Children understand positions in space. →	Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

213

Children use mathematical thinking to solve problems in their everyday environment. →	Make sense of problems and persevere in solving them. Reason abstractly and quantitatively. Construct viable arguments and critique the reasoning of others. Model with mathematics. Use appropriate tools strategically. Attend to precision. Look for and make use of structure. Look for and express regularity in repeated reasoning.

214

215

216 **INTEGRATION OF DOMAINS**

217 The following tables are an integration of the Preschool Learning Foundations and

218 corresponding domains in the kindergarten CA CCSSM. They are provided to facilitate district

219 discussions on the development of a modified curriculum for mathematics instruction in

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transitional kindergarten that is age and developmentally appropriate. Each of the following tables includes:

- Preschool Learning Foundations and corresponding CA CCSSM kindergarten standards
- Vocabulary—a list of vocabulary words that students should acquire as they expand their understanding of the concepts
- What it looks like—examples of what understanding of the concepts might look or sound like in the classroom (in order of complexity)
- Big ideas—some of the main ideas involved in grasping the concepts involved in the standard
- Instructional issues—misconceptions or common conceptual difficulties that students might have
- Activities—activities that support the acquisition of the abilities embodied in the Preschool Learning Foundations and CA CCSSM (in order of complexity)

Preschool Learning Foundations Number Sense	Kindergarten CA CCSSM Counting and Cardinality
Children expand their understanding of numbers and quantities in their everyday environment. PK. N.S. 1.1 Recite numbers in order to twenty with increasing accuracy.	**Know number names and the count sequence.** K. CC. 1 Count to 100 by ones and by tens K. CC. 2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
Vocabulary: *count***,** number words (i.e., *one*, *two*, *three*, etc. from 1-100), *count by, count from, number, next number, how did you figure that out?*	
What it looks like: • While playing hide-and-seek, Ezra counts to twenty before looking for the other children. • When asked to count as high as she can, Melia counts to 50. • When asked how old he is, Kenji answers, "I'm five, and then I'll be six, seven, eight, nine, ten! MP7, MP8	
Big ideas: Students learn to recite numbers before they can apply one-to-one concepts to counting objects or understand cardinality (i.e., the last number counted represents the numerosity of the set). Encourage students to slow down as they count. After students have had experiences counting from one, have them start counting in the middle of the counting sequence to encourage conceptual understanding of the order of numbers.	
Instructional Issues: An important goal in early mathematics instruction is that students achieve fluency with the counting sequence. Students may learn a short sequence of numbers ("*fourfivesix"*) and not understand that they are separate numbers (similar to the "*lmnop"* issue when learning the alphabet). Numbers eleven through fifteen may be difficult for students to learn as they do not follow the pattern (number followed by –teen) of sixteen through nineteen. Use discussions about how these numbers are kind of funny—calling attention to the irregularity of these number names may make it easier for students to remember that they do not follow the regular number naming pattern. *Saying the counting numbers is sometimes referred to as verbal or rote counting and does not indicate an understanding of object counting with one-to-one correspondence.*	
Activities: Transition times are useful for providing opportunities to learn the counting numbers. Students can count how long it takes to clean up the blocks, get in a circle, etc. These are not precise measures of time, but rather a chance to exercise their newfound rote counting abilities. MP4 Using a puppet (George), tell the students a story about how George has a hard time remembering how to count. Tell them that you would like them to help George figure out when his counting is not right. Ask the students to raise their hands when they hear George make a mistake and to remember George's counting mistake. In George's voice, count to ten, skipping or repeating one number in the sequence. Call on the students who raise their hands to describe George's mistake. Ask questions to make sure that students thoroughly describe the mistake and how George can fix his mistake. MP2, MP3, MP4, MP6 During whole group time, ask the students to sit in a circle and tell them that they are going to play a counting game. Tell them that this is a fancy game of counting called, "Everybody Gets a Number." Choose a child to start the counting sequence. That child says "one" aloud; the child sitting to his/her left (clockwise around the circle) says the next number and the counting continues around the circle. When a child says an incorrect number or doesn't know the next number, ask for the child to his/her right to help out. If that child doesn't know the number, ask the child to his/her right (keep asking to the right until you find a child that can help). As students advance in knowledge, increase the difficulty of this game by asking the students to count faster, make the number goal higher, start with a number other than one, or count by tens. MP1, MP4, MP7	

234

235

<table>
<tr><th>Preschool Learning Foundations
Number Sense</th><th>Kindergarten CA CCSSM
Counting and Cardinality</th></tr>
<tr><td>Children expand their understanding of numbers and quantities in their everyday environment.

PK. N.S. 1.2 Recognize and know the name of some written numerals.</td><td>Know number names and the count sequence.

K. CC. 3 Write numbers from 0–20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).</td></tr>
<tr><td colspan="2">Vocabulary: zero</td></tr>
<tr><td colspan="2">What it looks like:
Thomas sees the numeral 4 on the wall and says, “I’m that number!”
Zeke paints the numeral 5 multiple times at the easel.
Using a number and object matching puzzle, Susan correctly matches the numerals 6–10 with pictures of sets of animals that number 6-10. MP2, MP4, MP6
After drawing a pumpkin with four teeth, Maria draws a pumpkin with no teeth, laughs and says, “Look, zero teeth!” MP2, MP4</td></tr>
<tr><td colspan="2">Big ideas: Numerals (written or printed numbers) can describe the numerosity of a set of objects. Zero represents an empty set (in other words, no objects to count).</td></tr>
<tr><td colspan="2">Instructional Issues: Students learn to count with smaller sets before they learn to count larger sets. Students may draw numbers backwards or confuse similar looking numbers (e.g., 6 and 9). Zero is a hard concept for students to understand, and this may take a lot of examples and experiences.</td></tr>
<tr><td colspan="2">Activities:
Ask students to go on a Number Hunt around the classroom. This can be played a variety of ways. Students can look for any numeral and then name it when called upon. Alternatively, students can be assigned particular numerals to look for. Number cards (one for each student in the classroom; some numbers may appear on more than one card) can be hidden around the room and then students can be asked to each find a number card and then name the number on their card when called upon.

Students create their own number cards (with the numerals 0–10), decorating them as they wish (can use construction paper, index cards, card stock, plain white paper). Ask them to put the cards in order. Have the students trade sets and put them in order. MP2, MP4, MP6

Give students a number card (or let them choose) and ask them to find the same number of objects in their environment. Students can bring their card and objects back to the rug/table and share their findings with each other. With multiple students and different number cards, they can order their number cards and objects. To further understanding of zero, hold up a number card with the numeral “0” and encourage the students to discuss how many objects they could match with the card. If appropriate, discuss with students how they know their numeral card matches the set they have displayed on the rug/table. MP2, MP3, MP4, MP6</td></tr>
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236

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<tr><th>Preschool Learning Foundations
Number Sense</th><th>Kindergarten CA CCSSM
Counting and Cardinality</th></tr>
<tr><td>Children expand their understanding of numbers and quantities in their everyday environment.

PK. N.S. 1.3 Identify without counting, the number of objects in a collection of up to four objects (i.e., subitize).*

PK. N.S. 1.4 Count up to ten objects, using one-to-one correspondence (one object for each number word) with increasing accuracy.

PK. N.S. 1.5 Understand, when counting, that the number name of the last object counted represents the total number of objects in the group (i.e., cardinality).

* The Alignment of the California Preschool Learning Foundations with Key Early Education Resources places PK. N.S. 1.3 in a separate category that is unaligned with the CCSS. It is retained here to show the connection between naming the numerosity of a set attained through subitizing and learning to count a set.</td><td>Count to tell the number of objects.

K. CC. 4. Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

K. CC. 5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.</td></tr>
<tr><td colspan="2">Vocabulary: how many, one more, all together, in all, total</td></tr>
<tr><td colspan="2">What it looks like:
• Nathan glances at the die on the table and says, “Look! I got three!”
• DeSean lines up his eight toy cars, and touching each one, counts accurately one through eight. MP6
• The teacher asks Talia to count how many students in the group. Talia counts six students and then announces, “There’re six.” MP8
• When one more student joins the group, the teacher asks, “Now how many are in the group?” Talia answers, “That’s easy, one more, that’s seven!” MP2, MP8
• Diamond is passing out pencils to the tables; she accurately places six pencils on each of four tables. MP2, MP4, MP6, MP8</td></tr>
<tr><td colspan="2">Big ideas: Students at this age can subitize (immediately, and without counting, perceive a quantity) up to about four objects. This may increase to six when the objects are in a stereotypical arrangement (e.g., six pips on a domino). Cardinality refers to the ability to determine the numerosity of a set. Students initially count each item in a set, but when asked “How many?” they will count the set again. When students gain an understanding of cardinality, they will answer with the last number that they counted instead of counting the set again and know that the last number tells how many there are.</td></tr>
<tr><td colspan="2">Instructional Issues: It may take a while for students to construct strategies to keep track of what has been counted in a set. Two of these include touching each object in a row until they reach the end or moving aside the objects already counted. Counting an existing set is easier for students than creating a smaller set from a larger set (e.g., taking exactly six teddy bears from a large container of many bears) because they have to remember the number to which they are counting while counting. To encourage conceptual understanding of cardinality, when beginning activities that require a particular number of objects (e.g., five cards) have the students count out their cards from the larger set of cards instead of doing it for them.</td></tr>
<tr><td colspan="2">Activities:
Create a Number Wall with a “Number of the Week” where students display sets of pictures (magazine, drawings, stickers, etc.) that are equal in number (e.g. six magazine pictures of trees). Arrange pictures</td></tr>
</table>

so that they are in groups that can be subitized. Each week change the "Number of the Week." MP2, MP4, MP6

An activity that encourages both numeral recognition and object counting can be created using a numeral strip (piece of paper containing a row of boxes with numerals printed in them, beginning with the numeral 1), dice/spinner, and counters. Students take turns rolling/spinning the die/spinner and then count out that many counters (teddy bears, cars, etc.), show they are right, and then cross that numeral out on their numeral strip. Game is over when all students have all numerals crossed out on their numeral strips (numeral strips should have all possible dice/spinner numerals in order). MP1, MP2, MP4, MP6, MP7.

Play board games that require counting spaces, which usually come with dice or spinners. Note that use of a number line is not formally introduced in the CA CCSSM until Grade 2. All number lists or number paths should have the numbers within a shape (usually squares) that may or may not be connected to adjacent shapes. MP2, MP4, MP6, MP7.

238

Preschool Learning Foundations Number Sense	Kindergarten CA CCSSM Counting and Cardinality
Children expand their understanding of number relationships and operations in their everyday environment. PK. N.S. 2.1 Compare, by counting or matching, two groups of up to five objects and communicate, "more," "same as," or "fewer" (or "less").	**Compare numbers.** K. CC. 6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. K. CC. 7 Compare two numbers between 1 and 10 represented as written numerals.
Vocabulary: *more, fewer, less, same as, greater than, less than, more than*	
What it looks like: • Jasmin and Lucas are playing in the block area, trying to divide up the long blocks equally between themselves. In lining them up in one-to-one correspondence, Lucas says, "You have five and I only have four. You have more than I do!" MP2, MP3, MP5, MP6 • John and Jamal are playing with trains. After counting the trains several times, Jamal says, "I have eight trains and you only have six! That's not fair. If I give you one of my trains then we'll both have seven!" MP2, MP3, MP6 • Angel and Lisa are looking at the graphs with the numbers of students who like various fruits. Angel remarks, "Look, more like apples than oranges, see apples has an 8 and oranges only has a 2." MP2, MP3, MP4, MP6, MP8	
Big ideas: Students may initially compare sets perceptually ("That one has more!") or by lining them up in one-to-one correspondence, or they may count both sets to compare quantitatively ("One, two, three. One, two, three—we both have three!"). Moving from comparing two sets of objects to comparing numerical symbols can be difficult for students. Encourage conceptual understanding by offering many opportunities to use numerals with matching sets of objects.	
Instructional Issues: Fair-sharing in the classroom can play a big part in providing opportunities for students to compare quantities (everyone wants a fair share!). Some students may be confused by the length or size of a set of objects when comparing it to another set. Lining up objects in one-to-one correspondence may help students ascertain whether one set is larger than another. Students may struggle with understanding that the numerosity of a set does not change if nothing is added or taken away. Using the language "Is this group really more or does it just look like more?" can be helpful. Use the words "fewer" and "less" more than the word "more" because children usually have fewer opportunities to learn those words.	

Activities:
Card game of Compare (comparing numerals or sets of icons on cards). MP2

Play a game in which you create a set of counters (1-9). Count the counters with a small group of students. Then either add one more or take away one counter from the set. Then, ask the students to figure out how many there are in the set. Involve the students in a discussion of how they can figure out the answer (there are several ways) and how they know the answer (involve the entire group in the discussion, but make sure that all students participate). Understanding is supported if there is a number list/path up in the room at the students' eye level with the numerals in order from 1 (these are the counting numbers) with dots in 5-groups above them. MP1, MP2, MP3, MP4, MP8

In a small group, play a game with counters (teddy bears, cars, etc.) with the students. Create a set and ask the students to create their own sets of the same number. Work with 1-10 counters. An additional way to increase the complexity is to ask the students to create a set that is one more/less than your set or to create a set that has more/less objects than yours. Have a discussion with the students, asking each child to describe her/his set and whether it is larger or smaller than yours and how they know. If students do not use the phrase "the extra" during the discussion, pointing to the objects that are "the extra" can be helpful here. The group with more has extra objects. MP1, MP2, MP3, MP4, MP7, MP8

239

Preschool Learning Foundations Number Sense	Kindergarten CA CCSSM Operations and Algebraic Thinking
Children expand their understanding of number relationships and operations in their everyday environment. PK. N.S. 2.2 Understand that adding one or taking away one changes the number in a small group of objects by exactly one. PK. N.S. 2.3 Understand that putting two groups of objects together will make a bigger group and that a group of objects can be taken apart into smaller groups. PK N.S. 2.4 Solve simple addition and subtraction problems with a small number of objects (sums up to 10), usually by counting.	**Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.** K. OA. 1 Represent addition and subtraction problems with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. K. OA. 2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. K. OA. 3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). K. OA. 4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. K. OA. 5 Fluently add and subtract within 5.
Vocabulary: *bigger, smaller, add, subtract, take away, addition, subtraction, adding, subtracting, make ten, all together, equals, the same as, in all, total, amount left*	

What it looks like:

- Tony announces, "Look, if we put your blocks with my blocks we have a bigger pile! We have more." (This is an example of an Add To/Result Unknown situation. See Table 1 in the Glossary.) MP8
- Miriam says, "I have three cows and two pigs, that makes one, two, three, four, five. Five animals!" MP2, MP4, MP6. (This is an example of a Put Together/Total Unknown addition situation.)
- While playing in the block area, José says to Antonio, "If we put your cylinders with my cylinders, we'll have, one, two three, four, five, six cylinders—enough for the factory smokestacks!" MP2, MP4 (This is an example of a Put Together/Total Unknown addition situation.)
- Oscar says, "There are five cars, but two are broken, so we can only use three of them." MP2, MP4 (This is an example of a Take From/Results Unknown subtraction situation.)

Big ideas: Most young students use a counting-all strategy to solve addition problems with objects. That is, they count all of the objects in both sets. However, some students will go on to learn the more advanced Grade 1 strategy of counting-on from the larger set (when adding four and two objects, they begin with "Four" and continue, "Five, six"). Provide opportunities for students to take apart groups of objects, examining how many they started with, how many were taken away, and how many are left. Table 1 in the Glossary, *Addition and Subtraction Situations,* illustrates the variety of addition and subtraction situations and difficulty level. Students in transitional kindergarten may work with *Add To/Take From with Result Unknown* and *Put Together/Take Apart* with *Total Unknown* and *Both Addends Unknown*.

Instructional Issues: Students can directly model addition and subtraction situations told by the teacher or taken from their own lives. Provide frequent opportunities to engage in addition and subtraction activities involving story situations; students should learn to tell such stories and not just solve them. Initially working within addends less than five encourages in-depth understanding of addition and subtraction concepts. Encourage problem solving through the use of fingers, drawings, and manipulatives. When introducing the equal sign, emphasize and illustrate that the symbol "=" means equal (not "the answer is"). Stress that the *quantities* represented on the left and right sides of this symbol must be the same (can be objects/numerals/expressions). Use the equation form 5 = 3 + 2 when taking apart a number to show both addends. Using the word "partners" for addends helps students conceptualize these numbers hiding inside a number.

Activities: While reading books, ask questions about numbers. For instance, in a book about dogs, on the page with a picture of two dogs, ask how many dogs there are, and then ask how many legs one dog has? How many legs do two dogs have? If one dog left the page, how many legs would be left? MP1, MP2, MP4, MP6, MP7

In small group or whole group, have students represent the addends in a story problem with their fingers. Call on individual students to explain how they decided how many fingers to choose for each hand ("One day two baby dinosaurs hatched out of their eggs. The Mama Triceratops was so excited that she called to Auntie to come and see! Then four more baby dinosaurs hatched! How many dinosaurs hatched all together? Mirasol, can you show me how many fingers you used?"). Note that children from different cultures learn to show numbers on their fingers in different ways. Children may start with the thumb, the little finger, or the pointing finger. Support all of these ways of showing numbers with fingers. MP2, MP4, MP5

Present story problems to the students and encourage them to solve the problems with manipulatives or drawings. Initially, talking about how one can represent the problem on paper or with manipulatives might be useful ("Four cars are waiting to be repaired at the repair shop. These blocks will be the cars [putting four blocks in front of the students] – now Paul said that his garage only had two car lifts. If we put these two cars up on the lifts [moving the blocks away from the group of four], how many cars are waiting for their turn on the lifts?"). MP2, MP4, MP5

<table>
<tr><th>Preschool Learning Foundations
Measurement</th><th>Kindergarten CA CCSSM
Measurement and Data</th></tr>
<tr><td>Children expand their understanding of comparing, ordering, and measuring objects.

PK. M. 1.1 Compare two objects by length, weight, or capacity directly (e.g., putting objects side by side) or indirectly (e.g., using a third object).

PK. M. 1.2 Order four or more objects by size.

PK. M. 1.3 Measure length using multiple duplicates of the same-size concrete units laid end-to-end.</td><td>Describe and compare measurable attributes.

K. MD. 1 Describe measureable attributes of objects, such as length or weight. Describe several measureable attributes of a single object.

K. MD. 2 Directly compare two objects with a measureable attribute in common, to see which object has “more of”/”less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe on child as taller/shorter.</td></tr>
<tr><td colspan="2">Vocabulary: Longest, shortest, largest, smallest, heaviest, lightest, highest, lowest, most, least, more than, less than, same as, shorter than, longer than, larger than, smaller than, heavier than, lighter than</td></tr>
<tr><td colspan="2">What it looks like:
Jake lines up the four twigs he’s found on the playground in order of height. MP4, MP7
Tyrone pulls his train up beside Malik’s with the engines lined up and says, “My train is longer than your train.” MP4, MP6, MP7
Dylan and Kiara are comparing the pumpkins they’ve drawn and cut out. Kiara puts Dylan’s pumpkin on top of hers says, “My pumpkin is bigger than yours [then measures her pumpkin’s mouth with her index finger and compares to Dylan’s pumpkin’s mouth], but your pumpkin has a longer mouth than mine!”. MP4, MP5</td></tr>
<tr><td colspan="2">Big ideas: Measuring for older grades usually means assigning a numerical quantity to an object (e.g., 4 pounds or 6 inches). Generally, this is referred to as formal measurement. However, for younger students, directly comparing (informal measurement) these attributes forms an important foundation for this later understanding. Using duplicates of the same-size concrete objects prepares students for thinking about repeating units such as inches on a ruler or measuring tape. Emphasizing that these units are all the same size is an important concept that provides scaffolding for later formal measurement.</td></tr>
<tr><td colspan="2">Instructional Issues: Students may not understand that in order to compare the length or height of objects, they must all have the same starting point (e.g., in measuring the height of four objects, they are all placed upright on a table). Help students develop this ability. For vertical measurements, use the table or floor as the starting point. For horizontal measurement, mark the starting point with tape, by drawing a line, or with a straight stick. When students are measuring with non-standard units of measure, encourage them to use the same unit to measure the item(s).</td></tr>
<tr><td colspan="2">Activities:
Make balance scales available in a center in your classroom. Encourage students to use them to compare the weight of objects. Ask students to estimate which is heavier before weighing and then ask them to check their guess. MP5, MP6

Divide your students into 4-6 groups. Choose items from your classroom with measurable attributes (e.g. pencil, doll, truck, etc. for length; pumpkin, ball, beanbag, etc. for weight). Provide each group of students with one of these items. After distributing, ask each group to find objects that are shorter/longer or heavier/lighter than the item that you have given them (one item per student). Give the groups about five to ten minutes to complete this task. Then ask each group to make two piles of objects they have collected—one that consists of longer/heavier objects than the initial object you have provided and one that is shorter/lighter. The goal is for all students in the group to agree on which pile each item goes in. Then have all students in the classroom listen as each group reports to the whole group on their decisions. In order to keep all students engaged, tell them to listen carefully to the decisions made by other groups and be prepared to state whether they agree or not and how they might check the accuracy of the decisions. MP3, MP4, MP8

Young students are fascinated by large creatures. Find a children’s book about dinosaurs or elephants</td></tr>
</table>

that talks about the size of these animals. Create an activity in which the dinosaur or elephant is drawn on the playground to scale. Ask the students how they can measure it with their bodies ("How many children tall is a dinosaur?" "How many hands tall?"). Have a discussion about whether all children are the same height or not and would it make a difference if you measured a dinosaur with different-sized children (would it take fewer third- grade students laid end-to-end than it would if transitional kindergarten students were laid end-to-end?). Be sure to include discussion of the ability or inability to measure exactly the number of children (for instance if it takes four children and a part of a child). This is not to teach fractions, but to highlight the unit of measurement and underscore the importance of using a standardized unit (e.g. "four Sams high" is different than "four Susies high"). MP2, MP3, MP4, MP8

242

Preschool Learning Foundations Algebra and Functions	Kindergarten CA CCSSM Measurement and Data
Children expand their understanding of sorting and classifying objects in their everyday environment. PK. AF. 1.1 Sort and classify objects by one or more attributes, into two or more groups, with increasing accuracy (e.g., may sort first by one attribute and then by another attribute).	**Classify objects and count the number of objects in each category.** K. MD. 3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.
Vocabulary: *sort, group, same, different*	
What it looks like: • Jane is playing with the teddy bear counters. The counters come in three sizes and four colors. First she divides them by color into four groups. Then she says, "Now I'm gonna put all the Daddies together and the Mamas together and the babies together." She sorts all of the bears into these three sizes. MP1, MP3, MP4, MP6, MP8 • Garrett is sorting the buttons. First he sorts by color, then by size. MP4, MP6, MP8 • Demetrius is sorting the trains into engines, coal carriers, and flat train cars. He announces, "I have more train cars than coal carriers, and I only have two engines." MP2, MP4, MP6	
Big ideas: Objects can be sorted by more than one attribute.	
Instructional Issues: Being able to sort a group of objects by more than one attribute is an important ability. Help students develop this ability by encouraging this activity in a variety of settings (not just with manipulatives). If you go on a walk, ask the students to think about how many ways trees could be grouped (shape of leaves, color of trunk, type of fruit, etc.). While eating lunch ask how many ways vegetables can be grouped (color, soft/hard, etc.).	
Activities: Attribute blocks are a good way to encourage students to think of the different ways in which objects can be categorized. They can be grouped by color, shape, size, and thickness. MP1, MP4, MP8 Have students sit in 4-6 small groups. Have each group gather 10-12 objects from the classroom. Have them figure out ways to sort the items. Encourage discussion within the group as part of the decision making process. Then have each group present to the class the groupings that they have decided upon and explain why. MP1, MP3, MP4, MP8 Place 10 objects in 2 groups (items in each group should be related to one another in some aspect). Ask students, "How are these objects grouped together? Why were these objects placed in the same group? Why are there two groups of objects?" Encourage students to discuss the attributes they notice for each group and to explain their reasoning. You can make this activity simple or complex depending on the readiness of the students. MP1, MP3, MP6, MP7	

243

244

Preschool Learning Foundations Geometry	Kindergarten CA CCSSM Geometry
Children identify and use a variety of shapes in their everyday environment. PK. G. 1.1 Identify, describe and construct a variety of different shapes, including variations of a circle triangle rectangle, square, and other shapes. PK. G. 1.2 Combine different shapes to create a picture or design PK. G. 1.3 Identify positions of objects and people in space, including in/on/under, up/down, inside/outside, beside/between, and in front/behind	**Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres).** K. G. 1 Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as *above, below, beside, in front of, behind,* and *next to*. K. G. 2 Correctly name shapes regardless of their orientations or overall size. K. G. 3 Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid"). **Analyze compare, create, and compose shapes** K. G. 4. Analyze and compare two-and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/corners" and other attributes (e.g., having sides of equal length). K.G. 5 Model shapes in the world by building shapes from components (e.g. sticks and clay balls) and drawing shapes. K. G. 6 Compose simple shapes to form larger shapes. For example *"Can you join these two triangles with full sides touching to make a triangle?*
Vocabulary: *in, on, under, up, down, inside, outside, beside, between, in front, behind, below, next to, flat, solid, square, circle, triangle, rectangle, hexagon, cube, cone, cylinder, sphere, side, corner, vertex, vertices*	
What it looks like: • Xavier says, "Look, the window is a rectangle and it has rectangles in it!" MP7 • In a class discussion about shapes, Veronica says, "A sphere is just like a ball, round all around!" MP2 • Cho says in a discussion about the prepositions *above* and *below*, "That's funny, things can be both! Everything is above the floor and below the ceiling!" MP2, MP3	
Big ideas: Shapes have fixed attributes like the number of sides and corners. Knowledge of three-dimensional shapes is important – do not limit exposure to two-dimensional shapes. Two or more shapes can be put together to make new shapes.	
Instructional Issues: Shapes should be provided in all orientations and all permutations (long rectangles, triangles with a vertex pointing down, isosceles triangles, scalene triangles), and these should be discussed to help students focus on the central attributes. Help students to understand the difference between shape-like objects and actual representations of shapes (e.g., an apple has round characteristics, but it is not a sphere). Help students begin to understand that some shapes are special cases of a larger shape category: A square is a special rectangle that has all sides of equal lengths. Students should compose and decompose shapes with right angles and not just pattern blocks made from equilateral triangles.	

Activities:
In a whole group or small group, talk about words that describe where something is. Examples are *in, on, under, up, down, inside, outside, beside, between, in front, behind, below,* and *next to.* Ask your students to each find an example of these positions/prepositions in the classroom. An example would be "over." Ask students to find things that are over something else. Give students about five minutes to find examples. Go around the room and ask each student what the object is and what it is "over" (e.g., "Exit" sign is over the door.). MP1, MP4

Provide opportunities for sorting by shapes. For students who are just learning about shapes, shape-sorters can be useful (a box with holes on the outside in the form of shapes that comes with shapes that can be put through these holes). Pattern blocks and attribute blocks are also useful for sorting. MP4

Provide an activity center where students can create and work with shapes. This center can include shape magnets, clay balls and toothpicks, chopsticks, and paper/pencils/scissors. Encourage students to talk about what they are creating. Provide tangram sets with pictures to compose and parquet blocks with which to create designs. MP4, MP5, MP7

Gather a collection of two- and three-dimensional shapes. In a whole or small group, ask students to describe the shapes one-by-one. For instance, hold up a triangle and ask the students to describe it. They may need help with initially learning the vocabulary (see Vocabulary list above). Some of the questions you can use to prompt students' descriptions are how many sides/corners/vertices/faces. After students are comfortable with providing these sorts of descriptions, switch the activity to describing a hidden shape and asking the students to guess which shape it is. MP3, MP4, MP6, MP7, MP8

Table of Standards for Mathematical Practices, Explanations and Examples

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	Transitional kindergarten provides an opportunity for teachers to instill a joy of problem solving in mathematics. Mathematical activities should be both meaningful and challenging. Some of these activities are games (e.g., board games, card number games, dominoes, etc.) and are useful because mathematics is being used to solve problems. Consider using games in which no one "wins" until every student has finished and games that require collaboration. Encourage students to persevere in solving problems – they will find that those problems that take a bit of time to solve can be the most rewarding. Possible prompts: How do you know? What do you know about …? What would happen if…?
MP.2 Reason abstractly and quantitatively.	Counting things for a reason—or just to get better at it—is important. Young students love to count things and to practice the counting sequence. Competence is the motivation. Many experiences in the manipulative-centered activities of transitional kindergarteners are natural environments that require quantitative reasoning. Fair-sharing, in particular, promotes this sort of thinking in the classroom. As students become more familiar with quantitative reasoning with objects, they become more able to reason abstractly (e.g., "You have five trucks and I have four trucks and since five is more than four, you have more trucks than I do and that's not fair!"). Possible prompts: What do you know about the number …? Let's make a story about these numbers…
MP.3 Construct viable arguments and critique the reasoning of others.	Young students are very capable of stating a point of view and defending it. Help students transfer these abilities to the domain of mathematics. Ask students how they arrived at the answer and discuss with others not only the correct answer, but also the strategies used for finding the answer. Frequently there is more than one "right" answer (e.g. "What number is greater than five?") and more than one strategy. Model how to explain answers and discuss other solutions with classmates. Possible prompts: How did you figure that out? What do you think about…?
MP.4 Model with mathematics.	Modeling with mathematics means that teachers provide models (solving a problem aloud and with manipulatives) and that students use objects to demonstrate their thinking. Possible prompt: What could we use to…? Solve mathematical problems aloud, like dividing up a box of pencils so that each table receives one for each student seated ("Let's see, there are four of you here, so we will need four pencils. One, two, three, four".) Encourage students to use manipulatives to show their thinking ("Mica, can you show me how you know you shared these eight trucks fairly with Charlie?").
MP.5 Use appropriate tools strategically.	The transitional kindergarten classroom is filled with tools. These not only include instruments like balance scales and measuring tapes, but all of the manipulatives and objects that students and teachers use to model mathematics. Students should have frequent opportunities to ponder which of these is appropriate to the task at hand. Possible prompts: What could you use to help you with…? How could you use a … to help you with…?
MP.6 Attend to precision.	Precision is more than the "right" answer. It involves being able to describe strategies, arguments and decisions with increasing skill. Descriptions become more and more precise. Triangle descriptions change from "Because it looks like a triangle" to "It has three sides and three corners." Students learn that if they do not provide accurate representations during problem solving (e.g., in drawing 3 + 5 they only draw two and five objects) then they will have problems determining

	accurate answers. There is a beauty in precision – many students are entranced by this beauty (2 + 3 is *always* 5 – a quite amazing concept!). Possible prompts: What do you know about…? What else do you notice?
MP.7 Look for and make use of structure.	Students in transitional kindergarten will begin to see patterns as they gain experience in mathematics. For instance, one plus any number will always equal the next [whole] number in the sequence. Possible prompts: What do you notice about…? How is this the same as…? What are two different ways we can look at these objects? Tell me about your pattern.
MP.8 Look for and express regularity in repeated reasoning...	Young students delight in finding patterns – to solve addition problems, one can always count all the objects in both sets. One can also count-on from the larger set. In number decomposition, students may find (especially if they record the addends) that if the first addend is decreased by one, then the second is increased by one (3 + 7 = 10; 2 + 8 = 10; 5 = 3 + 2; 5 = 2 + 3). Asking questions of students that help them examine the strategies with which they solve problems will help them see the regularity in the way they solve these problems. Possible prompt: What do you notice?

248

249

Resources for Implementing Transitional Kindergarten

Developing and implementing a modified curriculum is just one of the challenges districts face as they continue to phase-in transitional kindergarten classes. A number of resources are available to support the implementation of TK, some of which are listed below.

The Alignment of the California Preschool Learning Foundations with Key Early Education Resources, California Department of Education. http://www.cde.ca.gov/sp/cd/re/documents/psalignment.pdf

Transitional Kindergarten (TK) Planning Guide: A Resource for Administrators of California Public School Districts, California County Superintendents Educational Services Association. http://www.ccsesa.org/index/attachments/TKGuide_11311_Web.pdf

Transitional Kindergarten FAQs, California Department of Education. http://www.cde.ca.gov/ci/gs/em/kinderfaq.asp

National Council of Teachers of Mathematics. http://www.nctm.org/

National Council of Teachers of Mathematics (NCTM) (2010). *Focus in Prekindergarten: Teaching with Curriculum Focal Points*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (NCTM) (2010). *Focus in Kindergarten: Teaching with Curriculum Focal Points*. Reston, VA: NCTM.

TK California. http://www.tkcalifornia.org/

TK California Mathematics Teaching Tools. http://www.tkcalifornia.org/teaching-tools/mathematics/