# Technology in the Teaching of Mathematics

## The Role of Technology in Education

The field of mathematics education has changed greatly due to technology. Educational technology can facilitate simple computation and the visualization of mathematics situations and relationships, allowing students to better comprehend mathematical concepts in practice. The use of technology can be a tool for students to model mathematical relationships in real-world situations. Technology is also an integral part of the Common Core State Standards Initiative, and its effort to prepare students for college and 21st century careers.

Technology pervades our modern society. In this environment, the question is not whether educational technology will be used in the classroom, but rather how best to use it (Cheung and Slavin 2011). Current-generation students are digital natives, and the generation of teachers that will enter into the profession over the next few decades will likewise be the product of a culture where technology is a constant presence and its use in education a fundamental assumption. Training and supporting teachers in the use of technology is essential to its effective use in the classroom.

Education technology is a broad category, inclusive of both a wide range of electronic devices and the applications that deliver content and support learning. Technology is an essential tool for learning mathematics in the 21st century, but it is a tool; it cannot replace conceptual understanding, computational fluency, or problem-solving skills.
Technology tools include both content-specific technologies like computer programs

and computational devices and content-neutral technologies like communication and collaboration tools (NCTM, Technology in Teaching and Learning Mathematics 2011). According to guidelines adopted by Massachusetts to guide the construction and evaluation of curriculum, “Technology changes the mathematics to be learned, as well as when and how it is learned… Some mathematics becomes more important because technology requires it, some becomes less important because technology replaces it, and some becomes possible because technology allows it.” (Massachusetts Department of Elementary and Secondary Education 2011)

Research completed over the last decade has confirmed the potential benefits of education technology applied to the teaching and learning of mathematics. Education technology, when used effectively, can enhance student understanding of mathematical concepts, bolster student engagement, and strengthen problem-solving skills. Most of the recent meta-analyses of research studies in this area, however, note that these benefits are contingent upon how education technology is implemented, that it is integrated with instruction, and that teachers are trained and interested in its use (Guerrero, Walker, and Dugdale 2004; Kahveci and Imamoglu 2007; Goos and Bennison 2007; Li and Ma 2010; Cheung and Slavin 2011). This chapter will provide some suggestions and cautions on how to manage that implementation to accomplish those goals.

## Education Technology and the Common Core

The use of technology is directly integrated into the California Common Core State Standards for Mathematics (CA CCSSM). The mathematics content standards encourage the use of multiple representations and modeling to help students understand the mathematical concepts behind a problem. This is an area where the use of technology can be helpful. The standards make specific reference to using technology tools in a number of cases, especially in the middle grades and high school. For example, Geometry standard 7.G 2. states,

> Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Similarly, the higher mathematics standards for algebra, functions, geometry, and statistics and probability include references to using technology to develop mathematical models, test assumptions, and conduct appropriate computations.

Technology is also an integral part of the standards for mathematical practice that are emphasized throughout the CA CCSSM, starting in kindergarten and continuing through grade twelve. It is expected that students will be able to integrate technology tools into their mathematical work. For example, the descriptive text for MP.5, “Use Appropriate Tools Strategically,” states the following:

> Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and

> paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Students who gain proficiency in the CA CCSSM are expected to know not only how to use technology tools, but also when to use them.

## Technology and the Common Core: Illustrative Examples

| **Grade Level/ Course** | **Content Standards** | **Practice Standards** | **Instructional Strategy Using Technology** |
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| **Elementary Grades** | | | |

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| K | K.CC.4. | 2, 6, 7 | Using a free application like “Concentration” or “Okta’s Rescue” from the National Council of Teachers of Mathematics (NCTM), students work in pairs to match number names with the corresponding numeral. |
| 1 | 1.OA.6. | 2, 7, 8 | Using a free application like “Deep Sea Duel,” students work in pairs to find various number combinations that sum to a given number. |
| 2 | 2.OA.2. | 1, 6, 7 | Using a free application like “Pick a Path” from the NCTM, assign a group of students to solve problems on tablet computers while others work directly with the teacher. |
| 3 | 3.OA.3. | 2, 3 | Students record their solutions on a tablet and present to other students who in turn describe their recorded solutions on another tablet. |
| **Middle Grades** | | | |
| 6 | 6.SP.3.<br>6.SP.4. | 3, 5 | Using a computer, students find a data set online and using a spreadsheet formula students calculate measures of center and variability, create a graphical representation, and write a description of the data based on the numerical and graphical evidence. |
| 8 | 8SP.1.<br>8.SP.2.<br>8.SP.3. | 4, 5 | Students work in pairs using two graphing calculators and one ultrasonic ranging device collect data. The first student walks *toward* his/her partner who uses the ranging device partner to record the distance between them, attempting to produce a graph that is a straight line, repeating the measurements until both partners are happy with the result. The pair now reverses roles, but with the second student walking *away* from his/her recording partner. Once the data are collected the pair answer the following questions by manipulating the Time List and Distance List data stored in their two calculators:<br>• How far away did your partner start?<br>• How far away was your partner at the end of the experiment?<br>• How long did the experiment last?<br>• By computing $\frac{Dist(last)-Dist(first)}{Time(last)-Time(first)}$ calculate your velocity and your partner's velocity. How |

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| | | | are these alike? How are these different? Explain you observations.<br>• Compute your partner‘s velocity over the first half, second half, first quarter, second quarter, third quarter, and fourth quarter of the experiment to determine if your velocity was constant. How did they do?<br>• Manually (or otherwise, *e.g.* Median-Median of Least Squares) fit a line to your partner‘s data and obtain an equation for the line. What are the slope and intercept of your line? How do these compare to your earlier calculations? |
| **Higher Mathematics** | | | |
| Geometry/ Mathematics I/Mathematics II | G-CO 9.<br>G-CO 10.<br>G-CO 11.<br>G-CO 12.<br>G-CO 13. | 5, 7 | Using dynamic geometry software and an interactive whiteboard, students investigate and create conjectures of geometric theorems and constructions. |
| Mathematics I/Algebra I | S-ID.6.<br>S-ID.7.<br>S-ID.8.<br>S-ID.9. | 4, 5 | Students use a computer to locate a bivariate data set and use statistical software to create a scatterplot and calculate the least squares regression line. Students explore the properties of this line and use it to predict and interpret relevant results. |
| Mathematics I/Algebra I | F-LE.3.<br>S ID.6.a. | 2, 4, 5 | As a whole class activity you need a graphing calculator, one ultrasonic ranging device, a six to nine foot wooden plank, and a large (family or industrial) sized can of a non-liquid, such as refried beans or ravioli. The plank is raised to a small incline, by propping one end on one or two textbooks. The experiment consists of collecting data on the distance between the ranging device placed at the top of the ramp and the can placed at the bottom of the ramp. The can’s position on the ramp and its velocity are recorded by the ultrasonic ranging device as the can is rolled up and allowed to roll back down the ramp. Preparing for the experiment, an assistant practices rolling the can up and down. From these practice rolls, the class decides on the length of the experiment, and is asked to describe what they see. [The can's speed slows on the way up, there is an apparent pause at the |

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| | | | top, and the can speeds up as it descends the ramp.] Having decided on the length of the experiment (the number of trials) and possibly the rate of sampling, data are collected. The rolling process is repeated until a clean run is obtained. (It is not uncommon for the can to roll off the ramp.)<br>The resulting graph is discussed. How close did the can get to the ranging device? The descending part of the graph corresponds to the assent of the can. When did the can turn around? Students perform a quadratic regression and plot the resulting equation. Students compute and examine the residuals and examine them, the number negative, the number positive and the Mean Absolute Deviation to discuss the goodness of fit. |

Technology will also be an integrated part of the new assessment systems under development. The multistate Smarter Balanced Assessment Consortium (Smarter Balanced), of which California is a governing member, is developing computer-adaptive assessments that can respond to a student’s initial performance to more rapidly and accurately identify which skills the student has mastered. These assessments will also allow for a faster turnaround of test results, so that they can be used to inform instruction. The Smarter Balanced test protocols will allow the use of calculators on certain test items on the middle and high school assessments, including integrating calculators directly into the assessment software. (For additional information, see the “Assessment” Chapter.)

## Education Technology in the Classroom

### Handheld Devices

For decades one of the biggest questions and largest controversies with regard to educational technology was the use of calculators in classroom instruction and assessment. Previous studies (Hembree and Dessart 1986, Loveless 2004) raised cautions about the use of calculators in the elementary grades, especially in terms of undermining students' skills at basic computation. Ellington (2003) found that calculators had the greatest benefit when used for both instruction and assessment. She noted, however, that "Students received the most benefit when calculators had a pedagogical role in the classroom and were not just available for drill and practice or checking work." Instruction needs to be structured to use technology in nonroutine ways (i.e., not in drill and computational practice), in other words where students are using it to make decisions and solve problems (Guerrero, Walker, and Dugdale 2004). More powerful graphing calculators can be used in conjunction with the technology emphasis in the CA CCSSM to allow students to actively participate in the process of developing mathematical ideas and solve problems. These devices can help students better understand spatial concepts, connect functions with graphs, and visualize written problems to develop solutions (Ellington 2003).

The National Council of Teachers of Mathematics, in its 2011 research brief on "Using Calculators for Teaching and Learning Mathematics," stated the following conclusion after a synthesis of nearly 200 studies conducted from 1976 through 2009:

> In general, we found that the body of research consistently shows that the use of calculators in the teaching and learning of mathematics does not contribute to any negative outcomes for skill development or procedural

> proficiency, but instead enhances the understanding of mathematics concepts and student orientation toward mathematics.

While these findings do not eliminate the need to ensure that calculators are used to enhance rather than supplant students' computational and procedural skills, they do provide reassurance that calculators can be integrated into instruction and assessment without harming student progress toward mathematical proficiency. It is important to remember that instruction and curriculum using calculators should be designed to emphasize the problem-solving and conceptual skills of students.

The next generation of handheld devices with networking capabilities also offers opportunities for using technology effectively in a classroom environment. Clark-Wilson (2010) conducted a study of seven teachers using handheld graphing computers connected via wireless network to the teacher's computer. These devices allowed teachers to monitor student work and provide live feedback and enabled students to lead the classroom discussion via a projector connected to the network. The advantages of using these devices included promoting a "collaborative classroom" where students were able to learn from each other. Clark-Wilson also noted the benefits of added student engagement, a finding that was duplicated in many of the other studies on the use of classroom technology.

Smart phones and tablet computers are other forms of handheld technology that are becoming increasingly common in schools. Educational applications, or "apps," designed to work with these devices are proliferating. They offer the advantages of builtin networking capability and access to the Internet, which enables immediate access to content and feedback from the teacher as noted above. Tablet computers are being used by some school districts to provide delivery of instructional materials, with advantages in terms of weight and convenience.

However, there are challenges associated with the use of handheld devices to provide instruction. Recent studies suggest that schools need to educate both students and parents about the need for policies regarding student access to and use of technology at school, but that comprehensive bans on cell phones may not be the most effective means of addressing these problems. Smart phones and tablet computers are easily lost or stolen and can be expensive to replace. Furthermore, the advantages of networked technology that is “always on” can also be disadvantages. In a 2009 national survey of middle- and high-school students, 35 percent admitted to using a cell phone to cheat, while 52 percent admitted to cheating using the Internet. The survey indicated the cultural gaps involved with new technology; for example, many of the students did not consider texting a warning about a pop quiz to fellow students, or copying text available online to turn in as their own work, to be “cheating” (Common Sense Media 2009). This same survey also found that while 92 percent of parents indicated that they believed that cheating using cell phones happens at their child’s school, just 3 percent of parents said that their child had engaged in such behavior. Multiple measures of assessment with an emphasis on problem solving will provide avenues for students to demonstrate content mastery beyond rote memorization (Challenge Success 2012).

## Computers and Software

Computers have become a more ubiquitous presence in schools over the last fifteen years. The number of computers has increased from approximately one computer for every eleven students in the late nineties to one computer per 5.8 students in 2010-11 (CDE Educational Demographics Office 2011).

Research on the use of computer-mediated learning tools has demonstrated the potential for increased student achievement and proficiency in mathematical concepts. However, those benefits are dependent upon the teaching approaches, types of programs, and the learners themselves (Li and Ma 2010). Kahveci and Imamoglu (2007) note that mathematics instruction works best when there are high levels of interactivity between students and the content and that the most successful instructional systems are those that adapt to students, allow them to work collaboratively, and give immediate feedback. Similarly Li and Ma (2010) found that computer technology was most effective when used with constructivist teaching methods where students gain conceptual understanding through an inquiry-based, collaborative learning model.

As a more concrete example, Ruthven, Deaney, and Hennessy (2009) studied the use of graphing software in secondary schools for investigating algebraic expressions. They found that the use of the software to enable students to graph linear and quadratic equations in the classroom had positive results in terms of efficiency of instruction, student engagement, and understanding. Students could use the software to explore the topic and share their results, for example by immediately seeing the effects on a graph of altering the coefficient in a formula-defining function. However, the authors of this study noted that despite the fact that the software had been “designed to do things easily,” the teacher’s role was still vital in structuring the activity and designing tasks that would help students master the mathematical content that was at the core of the lesson.

## Online Learning

Online delivery of instruction is becoming increasingly popular. More than one million kindergarten through grade twelve students enrolled in at least one online course in 2007-08 (USDE 2010). Online courses offer distinct advantages to districts in terms of cost and convenience, especially for districts where students are distributed across a wide geographic area and there might be challenges in delivering instruction in specific content areas.

While more research is being conducted on the efficacy of online instruction, preliminary findings provide reason for optimism. The United States Department of Education, in a 2009/10 study of online learning, found only five studies dealing with K–12 education in its survey of research from 1996 through 2008. Of those five, only one dealt with mathematics, but in general, the study’s authors found that the outcomes for online learning were not significantly different than those from face-to-face instruction, and that programs that combined online and face-to-face learning (a “blended” or “hybrid” model) could actually produce higher outcomes in terms of student performance. The study noted that newer online applications are able to combine asynchronous tools, such as e-mail, newsgroups, and discussion boards, with synchronous tools such as Web casting, chat, and audio/video presentations. These combinations allow students to approach the subject with more interaction between themselves and the content, their peers, and their teacher, which is more conducive to the “deep learning” that is the goal of mathematics learning. This is consistent with a sociocultural perspective on learning which states that learning takes place in social environments where social activity provides support and assistance for learning (Vygotsky, 1978, Cobb, 1994). However, the relative newness of online learning and the limited number of studies available suggest that districts should approach online instruction with caution, especially when the material is intended to replace face-to-face instruction, rather than to enhance it.

## Professional Development and Teacher Support

The various research studies cited in this chapter share a consensus that educational technology cannot be effective at improving student outcomes without the classroom teacher playing a central role. The teacher must ensure that technological tools are used to support student understanding of mathematical concepts and practices, rather than simply entertaining students or shielding them from following mathematical practices.

Teacher attitudes toward technology can affect its implementation and effectiveness. Guerrero, Walker, and Dugdale (2004) have noted that many teachers are cautious about technology and believe that its use may potentially hinder students’ understanding and learning of mathematics. Some fear, for example, that the inappropriate use of technology may interfere with students' ability to learn number facts or basic computational skills. Others are wary of the changes to instruction that are necessary to make use of new technology in the classroom. In some cases teachers themselves are willing to use technology, but face opposition in the form of reluctant administrators or an organizational culture that is resistant to change (Goos and Bennison 2007). It is also important for administrators to ensure that teachers have the technical support necessary to keep the technology functioning and available.

Simply providing teachers with greater access to technology is not going to lead to its successful use (Goos and Bennison 2007; Walden University 2010). Using technology effectively requires changes in pedagogical approach. Utilizing the technological tools referenced in this chapter can involve changes to the working environment, to the format of lessons and activities, to the time economy of the classroom, and to the curriculum script. Therefore, any innovation in technology must be accompanied by adaptions to the teacher's craft knowledge (Ruthven, Deaney, and Hennessy 2009).

A study by Walden University (2010) has examined a number of myths about the relationship between educators, technology, and 21st century skills. That study found that it is not true that newer teachers necessarily use technology more frequently. The study also suggested that teachers and administrators often have very different ideas about classroom technology, with administrators more likely to assume that technology is used and is more effective than is actually the case. Teachers surveyed indicated that they did not feel particularly well-prepared by their pre-service training programs for implementing technology and 21st century skills. However, the study also reiterated the importance of the teacher’s role in successful implementation of classroom technology.

These findings emphasize the critical importance of training teachers in the effective use of educational technology. Specifically, mathematics teachers must be trained in using technology to enhance mathematics learning, not just how the tools work. This training should be ongoing, and not just a one-time event. Simply using technology to teach the same mathematical topics in fundamentally the same way does not take advantage of the capabilities of technology, and it may even be harmful in that it can show that technology is not worth the cost or effort of implementation (Garofalo, and others 2000).

## The Digital Divide and the Achievement Gap

The term “digital divide” was coined in the 1990s to reference the gap in access to computers and the Internet that separated different demographic and socioeconomic groups in the United States. The concept was popularized by a series of reports conducted by the National Telecommunications and Information Administration called, “Falling through the Net” (NTIA 1995, 1998, 1999, 2000). These reports found that rural Americans, the socioeconomically disadvantaged and ethnic minorities tended to have less access to modern information and communication technology and the benefits provided by those connections.

While the gap in access has closed somewhat over the last two decades, especially with regard to access to broadband connections, it remains significant (Smith 2010). In 2009, the percentage of white households that had Internet access was 79.2 percent; for African-American and Hispanic households it was 60.0 and 57.4 percent respectively (U.S. Census Bureau 2009). Furthermore, there are concerns that minorities are less likely to be involved with social media and “Web 2.0” applications that include rich content and technologies for networking and collaboration online (Payton 2003; Trotter 2007). Given the overlap between the groups involved in the digital divide and the achievement gap in student performance, it is important that districts, schools, and teachers remain alert to the issue of equitable access to technology. While federal grants and other funding have helped balance the technology available to schools with disproportionate populations of students from disadvantaged groups, there may still be significant gaps in the technology that students have access to outside of their school environments. Studies have shown that gaps in access to reading material affect outcomes in reading achievement, and gaps in access to technology will have as much impact upon student success in a 21st century learning environment. Solutions to help address these gaps may include giving students access to computer resources outside of school hours, issuing technology devices to students to take home, and training teachers to be aware of these issues and providing them with strategies to address them as part of their professional development (Davis and others 2007).

## Accessibility

Education technology can help ensure that all children have access to the standards-based academic curriculum. Issues of universal access are discussed in more detail in the “Universal Access” chapter of this framework, but the specific ability of technology to support students with special needs should be addressed. One of the advantages of education technology, the ability to differentiate instruction to meet varied learning needs, makes it a potentially effective tool to support the learning goals of these students.

Assistive technology can be used to help students with disabilities gain access to the core curriculum and perform functions that might otherwise be difficult or impossible. This technology can be a hardware device that help a student overcome a physical disability or adaptive software that modifies content so that a student can access the curriculum. An example could be a digital talking book that reads content that a student cannot access due to a visual handicap or a learning disability that affects reading comprehension. A student with motor difficulties might use an enlarged, simplified computer keyboard, a talking computer with a joystick, head-gear, or eye selection devices. Li and Ma (2010) found that special education students were a subgroup that tended to show higher gains than other students when computer technology was used to support instruction. Software that differentiates instruction can also be used to meet the needs of students who are below grade level in mathematics. The CDE’s Clearinghouse for Specialized Media and Translations (http://www.cde.ca.gov/re/pn/sm/) produces accessible versions of textbooks, workbooks assessments, and ancillary student instructional materials. Accessible formats include braille, large print, audio, and digital files ranging from Rich Text Files (RTF), HyperText Markup Language (HTML), Digital Accessible Information System (DAISY), and Portable Document Format (PDF).

Education technology can also be used to support English learners. Software that uses visual cues to assist in the teaching of mathematics concepts can help someone with limited-English proficiency gain understanding. A 2010 study of one district’s Digital Learning Classroom project found that interactive whiteboard technology used in grades three and five increased English learners achievement and helped to close the achievement gap between English learners and students who are proficient in English (Lopez 2010).

Finally, education technology can help to provide a challenging and interesting educational environment for advanced learners. Computer programs that include self-paced options and allow students to explore advanced concepts can keep these students engaged in the learning process. Technology that facilitates a collaborative learning environment can also help advanced students become involved in their peers’ study of mathematics, a more useful outcome than simply giving these students a longer list of problems to solve or sending them off to study independently. Adaptive learning software provides individualized instruction that focuses on the needs of all students and challenges them to improve mathematics achievement.

## Resources

**CDE CalServe K-12 Service Learning Initiative Page** (www.cde.ca.gov/ci/cr/sl)
Provides information about the CalServe K-12 Service learning initiative; including the California STEM Service Learning Initiative which supports secondary school and higher education students working together to meet community needs through a STEM (Science, Technology, Engineering, and Mathematics) design process.

**California Learning Resources Network** (http://www.clrn.org)
Provides reviews of supplemental electronic math learning resources that include software, video, and Internet resources. CLRN also reviews online courses aligned to the Common Core State Standards.

**Math by Design** (http://mathbydesign.thinkport.org/)
Produced as part of a national public television collaborative that was formed to create online resources focused on STEM subjects for middle school students and teachers, this free site focuses on helping middle grades students refine and build upon their knowledge of geometry and measurement through real-world simulations.

**GeoGebra** (http://www.geogebra.org/cms/)
An interactive geometry, algebra, and calculus application, intended for teachers and students. Constructions can be made with points, vectors, segments, lines, polygons, conic sections, inequalities, implicit polynomials and functions. All of them can be

changed dynamically afterwards. Teachers and students can use the software to make conjectures and prove geometric theorems.