Statistics and Probability

Introduction

The Statistics and Probability course offers an alternative fourth course to Precalculus. In Statistics and Probability students continue to develop a more formal and precise understanding of statistical inference, which requires a deeper understanding of probability. Students learn that formal inference procedures are designed for studies in which the sampling or assignment of treatments was random, and these procedures may be less applicable to nonrandomized observational studies. Probability is still viewed as long-run relative frequency but the emphasis now shifts to conditional probability and independence, and basic rules for calculating probabilities of compound events. In the plus (+) standards are the Multiplication Rule, probability distributions, and their expected values. Probability is presented as an essential tool for decision-making in a world of uncertainty.

The course described below can be taught as either a one-semester (half-year) course or a full-year course. Teachers may want to supplement a one-semester course with more modeling experiences to extend it into a full year course.

What Students Learn in Statistics and Probability

Overview

Students extend their work in statistics and probability by applying statistics ideas to real-world situations. They link classroom mathematics and statistics to everyday life, work, and decision-making, by applying these standards in modeling situations. They choose and use appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions.

Students in Statistics and Probability take their understanding of probability further by studying expected values, interpreting them as long-term relative means of a random variable. They use this understanding to make decisions about both probability games and real-life examples using empirical probabilities.

The fact that numerous standards are repeated from previous courses does not imply that those standards should not be covered in those courses. In keeping with the CA CCSSM theme that mathematics instruction should strive for depth rather than breadth, teachers should view this course as an opportunity to delve deeper into those repeated Statistics and Probability standards while addressing new ones.

Connecting Standards for Mathematical Practice and Content

The Standards for Mathematical Practice apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The Standards for Mathematical Practice (MP) represent a picture of what it looks like for students to *do mathematics* and, to the extent possible, content instruction should include attention to appropriate practice standards. The table below gives examples of how students can engage in the MP standards in Statistics and Probability.

Standards for Mathematical Practice Students…	**Examples of each practice in Statistics and Probability**
MP1. Make sense of problems and persevere in solving them.	Students correctly apply statistical concepts to real-world problems. They understand what information is useful and relevant and how to interpret the results they find.
MP2. Reason abstractly and quantitatively.	Students understand that the outcomes in probability situations can be viewed as *random variables*, that is, functions of the outcomes of a random process, with associated probabilities attached to their possible values.
MP3. Construct viable arguments and critique the reasoning of others. ***Students build proofs by induction and proofs by contradiction. CA 3.1*** *(for higher mathematics only).*	Students defend their choice of a function to model data. Students pay attention to the precise definitions of concepts such as *causality* and *correlation* and learn how to discern between the two, becoming aware of potential abuses of statistics.

MP4. Model with mathematics.	Students apply their new mathematical understanding to real-world problems. Students also discover mathematics through experimentation and examining patterns in data from real-world contexts.
MP5. Use appropriate tools strategically.	Students continue to use spreadsheets and graphing technology as aids in performing computations and representing data.
MP6. Attend to precision.	Students pay attention to approximating values when necessary. They understand margins of error know how to apply them in statistical problems.
MP7. Look for and make use of structure.	Students make use of the normal distribution when investigating the distribution of means. They connect their understanding of theoretical probabilities and finding expected values to situations involving empirical probabilities, and correctly apply expected values.
MP8. Look for and make use of regularity in repeated reasoning.	Students observe that repeatedly finding random sample means results in a distribution that is roughly normal, and begin to understand this as a process for approximating true population means.

MP standard 4 holds a special place throughout the higher mathematics curriculum, as Modeling is considered its own conceptual category. Though the Modeling category has no specific standards listed within it, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a high place in instruction. Readers will see some standards marked with a star symbol (★) to indicate that they are *modeling standards*, that is, they present an opportunity for applications to real-world modeling situations more so than other standards.

Statistics and Probability Mathematics Content Standards by Conceptual Category

The Statistics and Probability course is organized by conceptual category, domains, clusters, and then standards. Below, the general purpose and progression of the standards included in this course are described according to these conceptual categories. Note that the standards are not listed in an order in which they should be taught.

Conceptual Category: Modeling

Throughout the higher mathematics CA CCSSM, certain standards are marked with a (⋆) symbol to indicate that they are considered modeling standards. Modeling at this level goes beyond the simple application of previously constructed mathematics to real-world problems. True modeling begins with students asking a question about the world around them, and mathematics is then constructed in the process of attempting to answer the question. When students are presented with a real-world situation and challenged to ask a question, all sorts of new issues arise: which of the quantities present in this situation are known and unknown? Students need to decide on a solution path, which may need to be revised. They make use of tools such as calculators, dynamic geometry software, or spreadsheets. They will try to use previously derived models (e.g. linear functions) but may find that a new equation or function will apply. They may see that solving an equation arises as a necessity when trying to answer their question, and that oftentimes the equation arises as the specific instance of the knowing the output value of a function at an unknown input value.

Modeling problems have an element of being genuine problems, in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. This will be a new approach for many teachers and will be challenging to implement, but the effort will produce students who can appreciate that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners.

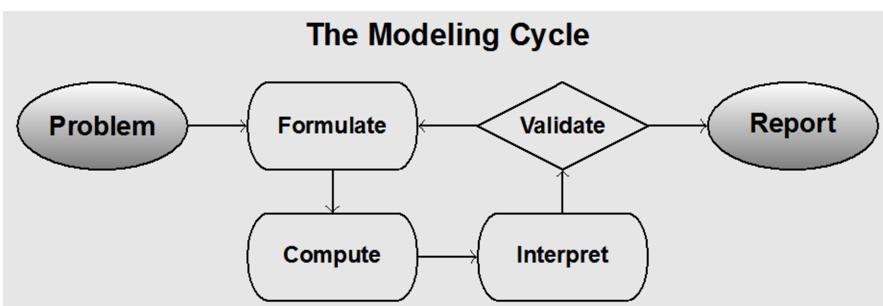


Figure 1: The modeling cycle. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, etc.), compute an answer or rewrite their expression to reveal new information, interpret their results, validate them, and report out.

The reader is encouraged to consult the Appendix, “Mathematical Modeling,” for a further discussion of the modeling cycle and how it is integrated into the higher mathematics curriculum.

Conceptual Category: Statistics and Probability

The standards of the Statistics and Probability conceptual category are all considered modeling standards, providing a rich ground for studying the content of this course through real-world applications. The first set of standards listed below deals with interpreting data, and while students have already encountered standards S-ID.1-6, they can be provided opportunities to refine their ability to represent data and apply their understanding to the world around them. For instance, they may examine current news articles containing data and decide whether the representation used is appropriate or misleading, or they may collect data from students at their school and choose a sound representation for the data.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* ★
    b. Informally assess the fit of a function by plotting and analyzing residuals. ★
    c. Fit a linear function for a scatter plot that suggests a linear association. ★

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation. ★

Students understand that the process of fitting and interpreting models for discovering possible relationships between variables requires insight, good judgment and a careful look at a variety of options consistent with the questions being asked in the investigation. Students work more with the *correlation coefficient*, which measures the "tightness" data points about a line fitted to the data. Students understand that when the correlation coefficient is close to 1 or −1, the two variables are said to be *highly correlated,* and that high correlation does not imply causation (S-ID.9). For instance, in a simple grocery store experiment of measuring the cost of certain frozen pizzas and the calorie content of each, students may find that a scatter plot of this data reveals a relationship that is nearly linear, with a high correlation coefficient. However, students learn to reason that the cost increasing does not necessarily *cause* the calories to increase, any more than an increase in calories would cause an increase in price. It is more likely that the addition of other expensive ingredients causes both to increase together.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★

6. Evaluate reports based on data. ★

Again, students have encountered standards S-IC.1-3 in previous courses, however in the Statistics and Probability course students can build off of these standards, now using the data from sample surveys to estimate such attributes as the population mean or proportion. With their understanding of the importance of random sampling (S-IC.3), students learn that running a simulation and obtaining multiple sample means will yield a roughly normal distribution when plotted as a histogram. They use this to estimate the true mean of the population and can develop a margin of error (S-IC.4).

Furthermore, students' understanding of random sampling can now be extended to the random assignment of treatments to available units in an experiment. A clinical trial in medical research, for example, may have only 50 patients available for comparing two treatments for a disease. These 50 are the population, so to speak, and randomly assigning the treatments to the patients is the "fair" way to judge possible treatment differences, just as random sampling is a fair way to select a sample for estimating a population proportion.

Effects of Caffeine: There is little doubt that caffeine stimulates bodily activity, but how much does it take to produce a significant effect? This is a question that involves measuring the effect of two or more treatments and deciding if the different interventions have differing effects. To obtain a partial answer to the question on caffeine, it was decided to compare a treatment consisting of 200 mg of caffeine with a control of no caffeine in an experiment involving a finger tapping exercise.

Twenty male students were randomly assigned to one of two treatment groups of 10 students each, one group receiving 200 milligrams of caffeine and the other group no caffeine. Two hours later the students were given a finger tapping exercise. The response is the number of taps per minute, as shown in the table.

The mean for the 200 mg data is 3.5 taps larger than that for the 0 mg data. In light of the variation in the data, is that enough to be confident that the 200 mg treatment truly results in more tapping activity than the 0 mg treatment? In other words, could this difference of 3.5 taps be explained simply by the randomization (the luck of the draw, so to speak) rather than any real difference in the treatments? An empirical answer to this question can be found by "re-randomizing" the two groups many times and studying the distribution of differences in sample means. If the observed difference of 3.5 occurs quite frequently, then we can safely say the difference could simply be due to the randomization process. If it does not occur frequently, then we have evidence to support the conclusion that the 200 mg treatment has increased mean finger tapping count.

The re-randomizing can be accomplished by combining the data in the two columns, randomly splitting them into two different groups of ten, each representing 0 and 200 mg, and then calculating the difference between the sample means. This can be expedited with the use of technology.

The plot below shows the differences produced in 400 re-randomizations of the data for 200 and 0 mg. The observed difference of 3.5 taps is equaled or

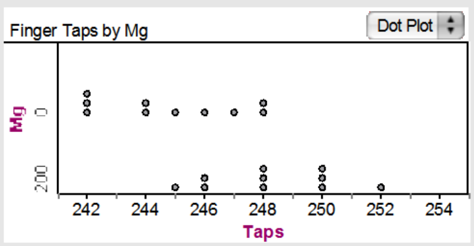
Finger taps per minute in a caffeine experiment

	0 mg caffeine	200 mg caffeine
	242	246
	245	248
	244	250
	248	252
	247	248
	248	250
	242	246
	244	248
	246	245
	242	250
Mean	244.8	248.3

Source: Draper and Smith, *Applied Regression Analysis*, John Wiley and Sons, 1981

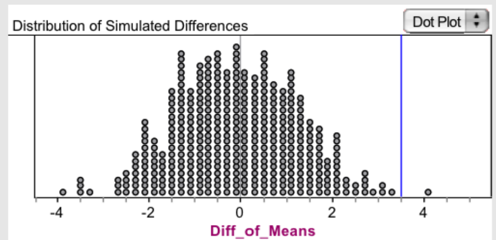
The plot of the finger tapping data shows that the two data sets tend to be somewhat symmetric and have no extreme data points (outliers) that would have undue influence on the analysis. The sample mean for each data set, then, is a suitable measure of center, and will be used as the statistic for comparing treatments.

Plot of finger tapping data



exceeded only once out of 400 times. Because the observed difference is reproduced only 1 time in 400 trials, the data provide strong evidence that the control and the 200 mg treatment do, indeed, differ with respect to their mean finger tapping counts. In fact, we can conclude with little doubt that the caffeine is the cause of the increase in tapping because other possible factors should have been balanced out by the randomization (S-IC.5). Students should be able to explain the reasoning in this decision and the nature of the error that may have been made.

Differences in re-randomized means for finger tapping data



(From Progression on High School Statistics and Probability, 10-11.)

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). ★
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★
3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. ★
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* ★

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model. ★
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. ★
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. ★
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. ★

Students can deepen their understanding of the rules of probability, especially when finding probabilities of compound events in standards S-CP.7-9. Students can generalize from simpler events exhibiting independence (such as rolling number cubes) to understand that independence is often used as a simplifying assumption in constructing theoretical probability models that approximate real situations. For example, suppose a school laboratory has two smoke alarms as a built-in redundancy for safety. One has probability of 0.4 of going off when steam (not smoke) is produced by running hot water and the other has probability 0.3 for the same event. The probability that they both go off the next time someone runs hot water in the sink can be reasonably approximated as the product $0.4 \times 0.3 = 0.12$, even though there may be some dependence between the two systems in the same room.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. ★
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* ★

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ★

Use probability to evaluate outcomes of decisions.

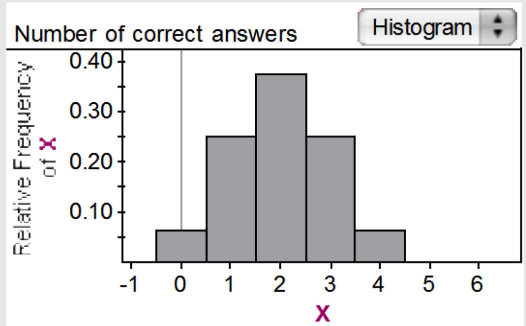
5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★
    a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* ★
    b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* ★
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★
7. (+) Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game). ★

The standards of the S-MD domain allow students the opportunity to apply concepts of probability to real-world situations. For example, a political pollster will want to know how many people are likely to vote for a particular candidate while a student may want to know the effectiveness of guessing on a true-false quiz. They begin to see the outcomes in such situations as *random variables*, functions of the outcomes of a random process, with associated probabilities attached to their possible values.

For example, after students have calculated the probabilities of obtaining 0, 1, 2, 3, or 4 correct answers by guessing on a four-question true-false quiz, they can construct the following probability distribution using statistical software (MP.5):

True–false test

Number correct, X	Probab-ility
0	$\frac{1}{16}$
1	$\frac{4}{16}$
2	$\frac{6}{16}$
3	$\frac{4}{16}$
4	$\frac{1}{16}$



Considering the probabilities as long-run frequencies, they can average them to come up with a mean score of:

$$0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16} = 2.$$

Students interpret this as saying that someone who guessed on four-question true-false tests can expect over the long run to get two correct answers per test.

Students can generalize this example to develop the general rule that for any discrete random variable *X*, the *expected value of X* is given by:

$$E(X) = \sum (\text{value of } X)(\text{probability of that value}).$$

Students interpret the expected value of a random variable in such situations as games of chance or insurance payouts based on the probability of having an automobile accident.

While the probability distribution shown above comes from theoretical probabilities, students can also use probabilities based on empirical data to make similar calculations in applied problems.

For more information about this collection of standards and student learning expectations, the reader should consult the document “Progressions for the Common Core State Standards in Mathematics: High School Statistics and Probability.”

Statistics and Probability Overview★

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments and observational studies.

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems.
- Use probability to evaluate outcomes of decisions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
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4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* ★
   b. Informally assess the fit of a function by plotting and analyzing residuals. ★
   c. Fit a linear function for a scatter plot that suggests a linear association. ★

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation. ★

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

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2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* ★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★

The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. *The Mathematics Framework* has not been edited for publication.

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★
6. Evaluate reports based on data. ★

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5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* ★

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A,* and interpret the answer in terms of the model. ★
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. ★
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2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★
3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* ★
4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* ★

Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★
    a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* ★
    b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* ★
6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★
7. (+) Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game). ★