Overview of the Standards Chapters

"These Standards are not intended to be new names for old ways of doing business."

CCSSI Introduction 2010, 5)

In 2009, the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA) committed to developing a set of standards that would help prepare students for success in career and college. The CCSS Initiative was a voluntary, state-led effort coordinated by the CCSSO and NGA to establish clear and consistent education standards. Development of the standards began with research-based learning progressions detailing what is known about how students' mathematical knowledge, skill, and understanding develop over time.

California adopted the California Common Core State Standards for Mathematics (CA CCSSM) in June 2010, replacing the 1997 statewide mathematics standards. In January 2013, the California State Board of Education adopted changes to the CA CCSSM (in accordance with Senate Bill 1200), including organizing standards into model courses for higher mathematics aligned with Appendix A in the CCSS Initiative.

These standards define what students should understand and be able to do in their study of mathematics. California's implementation of the CA CCSSM demonstrates a continued commitment to providing a world-class education for all students that supports lifelong learning and the skills and knowledge necessary to be ready to fully participate in the 21st century global economy.

Understanding the California Common Core State Standards for Mathematics

The CA CCSSM are designed for students to gain proficiency with and understanding of mathematics across grade levels. The development of the standards began with research-based learning progressions detailing what is known about how students' mathematical knowledge, skill, and understanding develop over time. The standards call for learning mathematical content in the context of real-world situations, using mathematics to solve problems, and developing “habits of mind” that foster mastery of mathematics content as well as mathematical understanding.

The standards for kindergarten through grade eight prepare students for higher mathematics that begins with Mathematics I or Algebra I, and serve as the foundation on which to build more advanced mathematical knowledge. The standards for higher mathematics (high-school-level standards) prepare students for college, career and productive citizenship. In short, the standards are a progression of mathematical learning.

The three major principles on which the standards are based are **focus, coherence,** and **rigor**. These principles are meant to fuel greater achievement in a rigorous curriculum, in which students acquire conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems.

[Note: Sidebar]

Three Major Principles of the Standards

Focus: Place strong emphasis where the standards focus

Coherence: Think across grades, and link to major topics in each grade

Rigor: In major topics, pursue with equal intensity

- conceptual understanding,
- procedural skill and fluency, and
- applications

Focus is necessary so that students have sufficient time to think about, practice, and integrate new ideas into their growing knowledge structure. Focus is also a way to allow time for the kinds of rich classroom discussion and interaction that support the Standards for Mathematical Practice (MP) and develop students’ broader mathematical understanding. Instruction should focus deeply on only those concepts that are emphasized in the standards so that students can gain strong foundational conceptual understanding, a high degree of procedural skill and fluency, and the ability to apply the math they know to solve problems inside and outside the math classroom.

Coherence arises from mathematical connections. Some of the connections in the standards knit topics together at a single grade level. Most connections are vertical, as the standards support a progression of increasing knowledge, skill, and sophistication across the grades.

- Thinking across grades: The standards are designed to help administrators and teachers connect learning within and across grades. For example, the standards develop fractions and multiplication across grade levels, so that students can build new understanding onto foundations built in previous years. Each standard is not a new event, but an extension of previous learning.

- Linking to major topics: Connections between the standards at a single grade level can be used to improve the instructional focus, by linking supporting or additional topics to the major work of the grade. For example, in grade 3, bar graphs are not "just another topic to cover," students use information presented in bar graphs to solve word problems using the four operations of arithmetic.

[Note: Sidebar]

Grades	**Priorities in Support of Rich Instruction:** **Expectations of Fluency and Conceptual Understanding in the CA CCSSM**
K–2	Addition and subtraction – concepts, skills and problem solving; measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions—concepts, skills and problem solving.
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra

Rigor requires that conceptual understanding, procedural skill and fluency, and application be approached with equal intensity.

- Conceptual understanding: The word “understand” is used in the standards to set explicit expectations for conceptual understanding. Teachers teach more than “how to get the answer” and support students’ ability to access concepts from a number of perspectives. Students might demonstrate deep conceptual understanding of core mathematics concepts by solving short conceptual problems, applying mathematics in new situations, and speaking and writing about their understanding. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, help other students understand a given method or find and correct an error, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

[Note: Sidebar]

Examples of Understanding in the CA CCSSM

Grade	Standards
K	**Understand** that each successive number name refers to a quantity that is one larger. **(K.CC.4.c)** (Note: partial standard)
2	**Understand** that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds. (**2.NBT.7**)
4	**Understand** addition and subtraction of fractions as joining and separating parts referring to the same whole. (**4.NF.3a**)
6	**Understand** the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. **(6.RP.1)**
8	**Understand** that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (**8.F.1)**
Higher Mathematics	**Understand** that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. **(F-IF.1)** (Note: partial standard)

Higher Mathematics	**Understand** that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **(G-SRT.6)**

- Procedural skill and fluency: The standards are explicit where fluency is expected. In grades K–6 students should make steady progress toward procedural skill and computational fluency (accurate and reasonably fast), including knowing single-digit products and sums from memory (see, e.g., 2.OA.2 and 3.OA.7). The word “fluently” as used in the standards refers to fluency with a written or mental method, not a method using manipulatives or concrete representations. Progress toward fluency should be woven into instruction in grade appropriate ways, along with developing conceptual understanding of the four operations.[1]

  Manipulatives and concrete representations such as diagrams that enhance conceptual understanding can help students make connections to written and symbolic methods (see, e.g., 1.NBT.1). Methods and algorithms should be general and based on principles of mathematics (e.g., place value and properties of operations).

  Developing fluency with single-digit computations can involve a mixture of just knowing some answers, knowing some answers from understanding patterns, and knowing some answers from understanding and using strategies. In grades 4, 5, and 6, moving to fluency with multi-digit computations and operations with decimals and fractions requires developing a base of understanding in previous years about how to use place value in carrying out and interpreting operations with the single-digit numbers within a multi-digit number, and understanding how to use unit fractions and equivalence for meaningful fraction operations. Students examine various methods relating them to visual models, but from the beginning students develop, discuss, and use efficient, accurate, and generalizable methods that are

[1] For more about how students develop fluency in tandem with understanding, see the *Progressions* for Operations and Algebraic Thinking, http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf and for Number and Operations in Base Ten, http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf.

or will lead to a variation of the standard algorithm. Students drop the visual models when they can, though they may continue to use models if needed. Fluency means working without visual models. It is important to ensure that sufficient practice and extra support are provided at each grade to allow all students to meet the standards that call explicitly for fluency.

[Note: Sidebar]

Grade	Examples of Expectations of Fluency in the K–6 CA CCSSM
K	Add/subtract within 5
1	Add/subtract within 10
2	Add/subtract within 20 Add/subtract within 100
3	Multiply/divide within 100 Add/subtract within 1,000 (using algorithms[2])
4	Add/subtract whole numbers within 1,000,000 (using the standard algorithm[3])
5	Multi-digit multiplication (using the standard algorithm) Add/subtract fractions
6	Multi-digit division (using the standard algorithm) Multi-digit decimal operations (add, subtract, multiply and divide using the standard algorithm for each operation).

- Application: Students are expected to use mathematics to solve “real-world problems”. In the standards, the phrase “real-world problems” and the star symbol (★) are used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a Conceptual Category in higher mathematics). Real-world problems and standards that support

[2] A range of algorithms may be used.
[3] Minor variations of writing the standard algorithm are acceptable.

modeling are also opportunities to provide activities related to careers and the work-world. Teachers in content areas outside of math, particularly science, ensure that students use mathematics – at all grade levels – to make meaning of and access content (adapted from Achieve the Core 2012).

Progression to Higher Mathematics

The progression from kindergarten standards to standards for higher mathematics, beginning with Mathematics I or Algebra I, exemplifies the three principles of focus, coherence, and rigor that are the basis for the CA CCSSM.

In kindergarten through grade five, the focus is on addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills, and problem solving. Arithmetic is viewed as an important set of skills and also as a thinking subject that prepares students for higher mathematics. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In middle school, multiplication and division develop into the powerful forms of ratio and proportional reasoning. The properties of operations take on prominence as arithmetic matures into algebra. The theme of quantitative relationships also becomes explicit in grades six through eight, developing into the formal notion of a function by grade eight. Meanwhile, the foundations of deductive geometry are laid in the middle grades. Finally, the gradual development of data representations in kindergarten through grade five leads to statistics in middle school: the study of shape, center, and spread of data distributions; possible associations between two variables; and the use of sampling in making statistical decisions.

In higher mathematics, algebra, functions, geometry, and statistics develop with an emphasis on modeling. Students continue to take a thinking approach to algebra, learning to see and make use of structure in algebraic expressions of growing complexity (PARCC 2012).

Mathematics is a logically progressing discipline, which has intricate connections among the various domains and clusters in the standards, and requires sustained practice to master grade-level and course-level content. The major work (or emphases) in the grade-level standards are identified in the standards chapters that follow. The following chart summarizes an important subset of the major work in grades K–8, as the progression of learning in the standards leads toward Mathematics I or Algebra I.

Table 1: Progression to Algebra I and Mathematics I in Kindergarten through Grade Eight

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five	Grade Six	Grade Seven	Grade Eight
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent and solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply and divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities*

* Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the two indicated exceptions, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents.

Two Types of Standards

The CA CCSSM include two types of standards: Standards for Mathematical Practice and Standards for Mathematical Content. These standards address both “habits of mind” that students should develop to foster mathematical understanding and expertise and also concepts, skills and knowledge – what students need to understand, know and be able to do. The standards also require that mathematical practices and mathematical content be connected. These connections are essential to support the development of students’ broader mathematical understanding, as students who lack understanding of a topic may rely on procedures too heavily. The Standards for Mathematical Practice must be taught as carefully and practiced as intentionally as the Standards for Mathematical Content. Neither should be isolated from the other; impactful mathematics instruction occurs when these two aspects of the CA CCSSM come together in a powerful whole.

The eight **Standards for Mathematical Practice** (**MP**) describe the attributes of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students. Mathematical practices provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, mathematical practices are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

[Note: Sidebar]

Standards for Mathematical Practice (MP)

These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (CCSSI 2010).

Table 2: Standards for Mathematical Practice (MP)

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically, and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meanings of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples.

They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. **Students build proofs by induction and proofs by contradiction CA.3.1** [for higher mathematics only].
4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
MP.5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can

enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MP.6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
MP.7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square, and use that to realize that its value cannot be more than 5 for any real numbers x and y.
MP.8 Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually

evaluate the reasonableness of their intermediate results.

The following table, Table 3, summarizes the eight MP standards and provides examples of questioning strategies teachers might use to support mathematical thinking and student engagement as called for in the MP standards.

Table 3

Summary of the Standards for Mathematical Practice	**Questions to Develop Mathematical Thinking**
MP.1 Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. • Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their own progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, “Does this make sense?” • Can understand various approaches to solutions.	• How would you describe the problems in your own words? • How would you describe what you are trying to find? • What do you notice about...? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you’ve used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize...represent... show...?
MP.2 Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and flexibly flexible use operations and their properties. • Create a logical representation of the problem. • Attend to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is _______ related to ________? • What is the relationship between ______and ______? • What does_______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use...? • Could we have used another operation or property to solve this task? Why or why not?

MP.3 Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? • How can we be sure that...? or How could you prove that...? • Will it still work if...? • What were you considering when...? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about...? • How could you demonstrate a counter-example? • I think it might be clearer if you said … Is that what you meant • Is your method like Shawna's method or how is it different?
MP.4 Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	• What math drawing or diagram could you make and label to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the diagram, number line, chart, table..? • Where did you see one of the quantities in the task in your equation or expression? • How would it help to create a diagram, graph, table...? • What are some ways to visually represent...? • What formula might apply in this situation?
MP.5 Use appropriate tools strategically. • Use available tools including visual models, recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach are you considering trying first? • What estimate did you make for the solution? • In this situation would it be helpful to use a graph, number line, ruler, diagram, calculator, manipulative…? • Why was it helpful to use...? • What can using a ______ show us that _____may not? • In what situations might it be more informative or helpful to use...?
MP.6 Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy?

mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	• How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties…can you use to explain...? • Can you say it in a different way? • Can you say it in your own words? And now say it in math words. • How could you test your solution to see if it answers the problem?
MP.7 Look for and make use of structure. • Look for the overall structures and patterns in mathematics. Think about how to describe these in words, math symbols, or visual models. • See complicated things as single objects or as being composed of several objects. Compose and decompose conceptually. • Apply general mathematical patterns, rules, or procedures to specific situations.	• What observations can you make about...? • What do you notice when...? • What parts of the problem might you eliminate, simplify...? • What patterns do you find in...? • How do you know if something is a pattern? • What ideas that we have learned before were useful in solving this problem? • What are some other problems that are similar to this one? • How does this relate to...? • In what ways does this problem connect to other mathematical concepts?
MP.8 Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details in the problem solving steps. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	• Explain how this strategy works in other situations. • Is this always true, sometimes true or never true? • How would we prove that...? • What do you notice about...? • What is happening in this situation? • What would happen if...? • Is there a mathematical rule for...? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice? • How is this situation like and different from other situations using this operation?

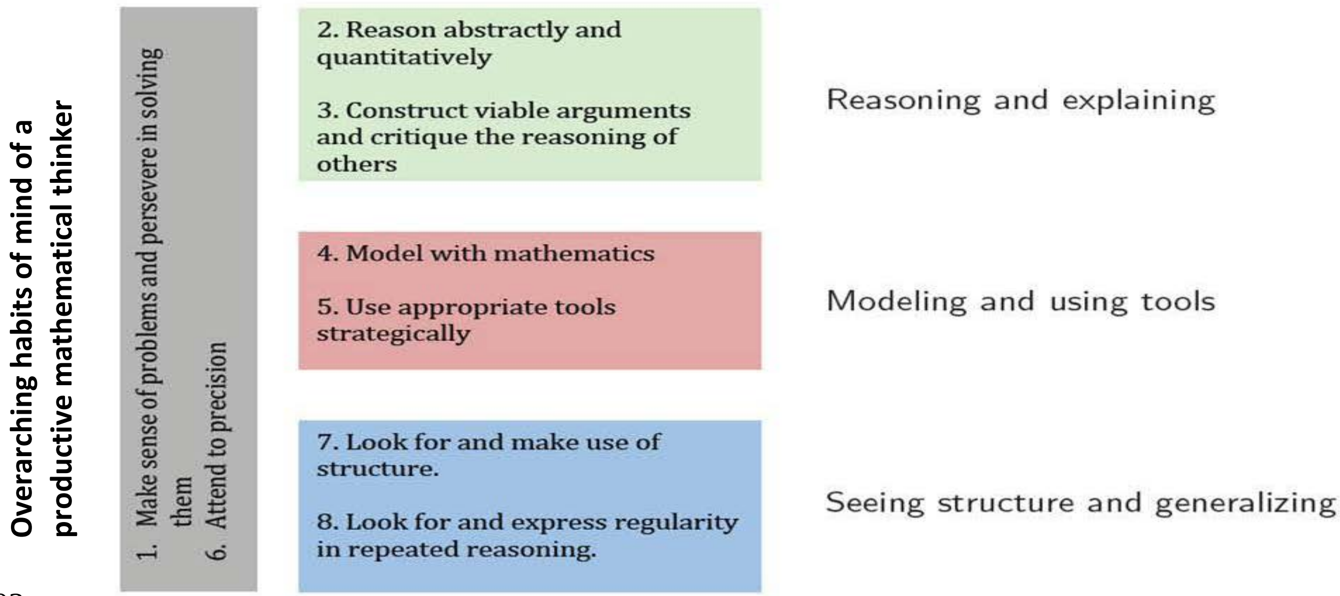
(Adapted from KATM 3rd FlipBook 2012)

Ideally, several MP standards will be evident in each lesson as they interact and overlap with each other. The MP standards are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MP.1), teachers need to allow their students to struggle productively and they must be attentive to the type of feedback they provide to students. In her research, Dr. Carol Dweck found feedback that praises effort and perseverance seems to engender and reinforce the growth mindset.[4] "Grow-minded teachers tell students the truth [about being able to close the learning gap between them and their peers] and then give them the tools to close the gap." (Dweck, 2007, 199

Structuring the MP standards can help educators recognize opportunities for students to engage with mathematics in grade appropriate ways. The eight MP standards can be grouped into the four categories in the following chart. These four pairs of standards can also be given names beginning with the grey (vertical) rectangle and then moving up from the blue (bottom) to the green (top) rectangle. These names can become a sentence teachers might ask at the end of every day: Did I do *Math Sense Making* about *Math Structure* using *Math Drawings* to support *Math Explaining?* This approach can help teachers to continually incorporate the core of the MP standards in classroom practices.

Structuring the Standards for Mathematical Practice (MP)

[4] A person with a "growth" mindset believes that intelligence is something that can nurtured and gained. When a growth mindset person does not meet the expected level of performance on a test or an assignment, or has difficulty understanding a concept, they work hard at it, believing that if they just try hard enough, they'll "get it."



(McCallum 2011)

The **Standards for Mathematical Content** were built on progressions of topics across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics.

Kindergarten through Grade Eight

In kindergarten through grade eight the standards are organized by grade level and then by **domains** (clusters of standards that address “big ideas” and support connections of topics across the grades), **clusters** (groups of related standards inside domains) and finally by the **standards** (what students should understand and be able to do). The standards do not dictate curriculum or pedagogy. For example, just because Topic A appears before Topic B in the standards for a given grade, it does not mean that Topic A must be taught before Topic B. (CCSSI 2010)

How to read the standards for kindergarten through grade eight:

Domain

Number and Operations in Base Ten	3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Standard

Cluster

Throughout the framework specific standards or groups of standards are identified in the narrative. For example, to identify the third grade standards from the previous graphic, the chapter narrative includes notations such as **3.NBT.1-3**, referring to a standard from grade 3, in the domain Number and Operations in Base Ten (**NBT**), in particular the three standards in the cluster numbered 1, 2, and 3.

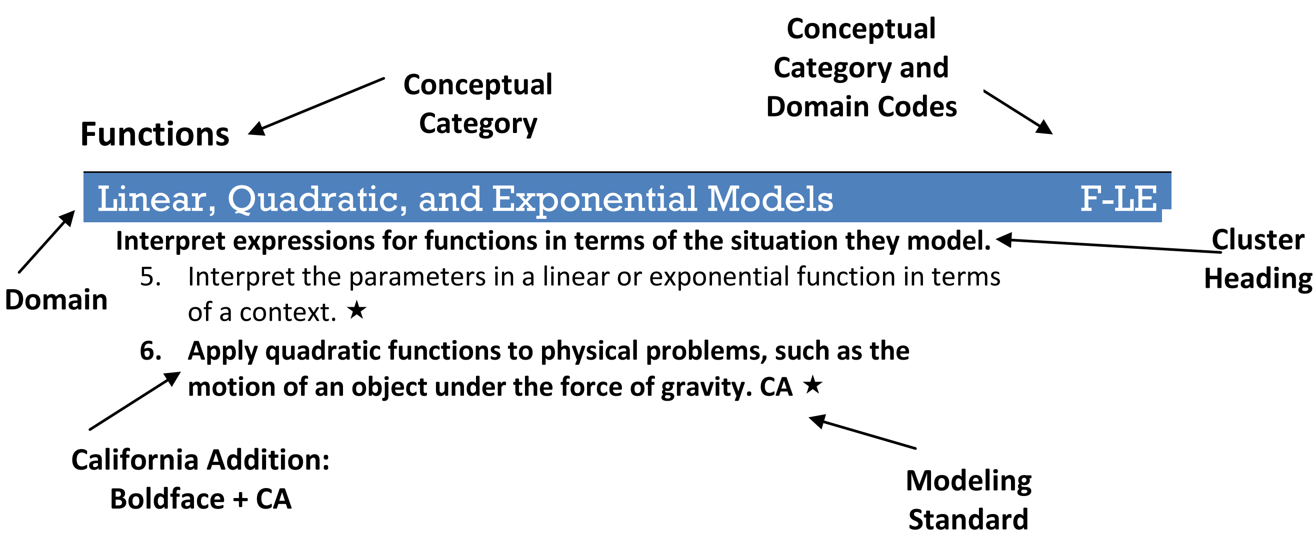
Higher Mathematics

When developed by the CCSSO, the standards for higher mathematics were organized differently than the K-8 standards. The higher mathematics standards were not organized into courses but listed according to conceptual categories:

- Number and Quantity (N)
- Algebra (A)
- Functions (F)
- Modeling (★)
- Geometry (G)
- Statistics and Probability (S)

Conceptual categories present a coherent view of higher mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Similar to the grade level content standards, each conceptual category (except Modeling, see explanation following the illustration) is further subdivided into several domains, and each domain is subdivided into clusters. (See page 22 in this chapter for information on the model courses in higher mathematics in the CA CCSSM.)

How to read the standards for higher mathematics:
The higher mathematics content standards are identified first by conceptual category, rather than by grade as for the kindergarten through grade 8 content standards. The code for each high school standard begins with the identifier for the conceptual category code (N, A, F, G, S), followed by the domain code, and the standard number.



The two standards in the figure would be referred to as F-LE.5 and F-LE.6, respectively. The star symbol (★) following the standards in the previous graphic indicates those are also Modeling standards. Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. The reader is encouraged to see the Appendix, “Mathematical Modeling,” for an extensive explanation of the Modeling conceptual category.

The following table, Table 4, illustrates how the domains and conceptual categories are distributed across the K–12 mathematical content standards. The corresponding abbreviations for each are also identified, for example Geometry (G).

Table 4: Mathematical Content Domains (K–8) and Conceptual Categories (Higher Mathematics)

Grade	K	1	2	3	4	5	6	7	8	Higher Mathematics Conceptual Categories	
Domains K-8	Counting & Cardinality (CC)						Ratios & Proportional Relationships (RP)		Functions (F)	Functions (F)	Modeling (★)
	Operations and Algebraic Thinking (OA)						Expression and Equations (EE)			Algebra (A)	
	Number and Operations in Base Ten (NBT)						The Number System (NS)			Number & Quantity (N)	
				Number and Operations - Fractions (NF)							
	Measurement and Data (MD)						Statistics and Probability (SP)			Statistics & Probability (S)	
	Geometry (G)						Geometry (G)			Geometry (G)	

Overview: Kindergarten through Grade Eight (K–8) Chapters

The kindergarten through grade eight chapters provide guidance on instruction and learning aligned to the CA CCSSM. Chapters present a brief summary of prior learning and an overview of what students learn at the grade. A section on the Standards for Mathematical Content highlights the instructional focus of the standards at the grade and includes a "Grade-Level Emphases" chart that designates clusters of standards as "Major" or "Additional/Supporting" work at the grade. The section "Connecting Mathematical Practices and Content" provides grade-level explanations and examples of how the MP standards may be integrated into grade-level appropriate tasks.

By far the largest section of each chapter is a description of "Standards-based Learning" organized by domains and clusters with exemplars to explain the content standards, highlight connections to the various mathematical practice standards, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Also noted are opportunities to link concepts in the additional/supporting clusters to major work at the grade (based on grade level "Cluster Level Emphases" charts) and examples of focus, coherence and rigor. Finally, each chapter summarizes "Essential Learning for the Next Grade" to provide guidance on important knowledge, skills, and understanding for students' success in future grades. The grade level content standards are included throughout the narrative and also at the end of each chapter. Standards unique to California (California additions) are coded by "**CA**."

Overview: Higher Mathematics Chapters

When first adopted in August 2010, the CA CCSSM for higher mathematics were organized differently than the K-8 standards—in conceptual categories rather than in courses. In January 2013, the California State Board of Education adopted changes to the CA CCSSM, including organizing content standards into model courses for higher mathematics, in accordance with Senate Bill 1200.

The model courses are organized into two pathways: Traditional and Integrated. The framework includes a description of these courses. The content of these courses are the same regardless of the grade level at which they are taught.

Standards for Mathematical Practice

The MP standards are interwoven throughout the higher mathematics curriculum. Instruction should focus equally on developing students' ability to engage in the practice standards and on developing conceptual understanding and procedural fluency with regard to the content standards. The MP standards are the same at each grade level, with the exception of an additional practice standard included in the CA CCSSM for higher mathematics only:

> **MP3.1: Students build proofs by induction and proofs by contradiction.** This standard can be seen as an extension of Mathematical Practice 3, in which students construct viable arguments and critique the reasoning of others.

In the higher mathematics courses, the levels of sophistication of each MP standard increases as students integrate grade appropriate mathematical practices with the content standards. Examples of the MP standards appear in each higher mathematics course narrative.

Standards for Mathematical Content

The entire catalog of higher mathematics standards is presented in the *California Common Core State Standards for Mathematics* (forthcoming), organized by both model courses and conceptual category. In this framework, the standards are organized into model courses that were adopted by the State Board of

Education in January 2013. The higher mathematics content standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by a (+) symbol, as in this example:

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Note that standards with a (+) symbol may also appear in courses intended for all students.

Higher Mathematics Chapters

The higher mathematics chapters are organized into courses according to two pathways:

- **Traditional Pathway** - consists of the higher mathematics standards organized along more traditional lines into Algebra I, Geometry and Algebra II courses. In this sequence, almost the entire Geometry conceptual category is separated into a single course and treated as a separate subject. It is important to note that while these courses have the same names as their traditional counterparts, the nature of the CA CCSSM results in very different courses. In the past, the label “Geometry” referred to a specific course, but now it may also refer to the conceptual category. Care will be taken throughout the higher mathematics chapters to make the distinction clear.

- **Integrated Pathway** - consists of the courses Mathematics I, II and III. The integrated pathway presents higher mathematics as a connected

> subject, in that each course contains standards from all six of the conceptual categories. For example, in Integrated Mathematics I, students will focus on linear functions. Students contrast them with exponential functions, solve linear equations, and model with functions. Students investigate the geometric properties of graphs of linear functions (lines) and model statistical data with lines of best fit. This is the way most other high performing countries present higher mathematics, and it maintains the theme developed in grades K-8 of mathematics being a multifaceted, connected subject.

In California, regardless of the grade level at which it is taught, an Algebra I course or Mathematics I course is aligned with the Algebra I or Mathematics I course presented in the higher mathematics chapters of the framework.

In addition, the framework contains suggested courses in Pre-Calculus and Statistics and Probability comprised of CA CCSSM and an appendix on Mathematical Modeling (See Appendix D.). The Pre-Calculus course consists of mainly the (+) standards that have not yet been taught in either the Integrated or Traditional Pathways and is designed to be an appropriate preparation course for Calculus. The 1997 Calculus and Advanced Placement Probability and Statistics courses are also included.

Local educational agencies are not limited to offering the higher mathematics courses described in this framework. Beyond providing the courses necessary for students to fulfill the state requirements for high school graduation, local districts make decisions about which courses to offer their students. For example, career technical education (CTE) courses that integrate the higher mathematics CA CCSSM with technical and work-related knowledge and skills can make mathematics more relevant to students and can be an alternate rigorous pathway which prepares students for technical education programs after high school. CTE courses provide opportunities for students to engage in hands-on activities, problem-solving, and decision-making while learning in a work-world setting. The *California Career Technical Education Model Curriculum Standards* (http://www.cde.ca.gov/ci/ct/sf/ctemcstandards.asp) are a vital resource for designing CTE courses that incorporate the CA CCSSM. There are also CTE courses developed by groups of educators at the University of California Curriculum Integration (UCCI) Institutes that balance academic rigor with career technical content and meet the mathematics component of the A–G requirements for college admission. (For additional information, go to http://www.ucop.edu/agguide/career-technical-education/index.html). In addition, "Appendix D: Mathematical Modeling," provides guidance to assist local educational agencies in designing a higher mathematics course in modeling.

Each CA CCSSM course is described in its own chapter, starting with an overview of the course followed by a detailed description of the mathematics content standards comprising the course. Throughout, there are examples illustrating the mathematical ideas and connecting the MP standards to the content standards. In particular, standards that are expected to be new to existing secondary teachers are elaborated on more fully than standards that have appeared in the curriculum prior to the adoption of the CA CCSSM.

It is important to note that specific CA CCSSM standards are often broad in scope, and as a result are sometimes included in more than one course. When this occurs, a parenthetical remark will be included within the standard that serves to clarify the intent of the standard in that course. For example, the following standard appears in both Algebra I and Algebra II and has a different parenthetical remark for each course:

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials. [Linear and quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials**.**

In Algebra II, the notation specifies that the standard applies to all equations beyond quadratic.

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials. [Beyond quadratic]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

In addition, some standards have an italicized example appended to them, which may or may not be appropriate for a given course. When it is not clear from the context of a course whether an example fits in that course, the narrative course chapter will attempt to clarify any confusion that may arise.