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Geometry

Introduction

The fundamental purpose of the Geometry course is to introduce students to formal geometric proof and the study of plane figures, culminating in the study of right triangle trigonometry and circles. They begin to prove results about the geometry of the plane formally, by using previously defined terms and notions. Similarity is explored in greater detail, with an emphasis on discovering trigonometric relationships and solving problems with right triangles. The correspondence between the plane and the Cartesian coordinate system is explored when students connect algebraic concepts with geometric ones. Students explore probability concepts and use probability in real-world situations. The major mathematical ideas in the Geometry course include geometric transformations, proving geometric theorems, congruence and similarity, analytic geometry, right-triangle trigonometry, and probability.

The standards in the traditional Geometry course come from the following conceptual categories: Modeling, Geometry, and Statistics and Probability. The content of the course will be expounded on below according to these conceptual categories, but teachers and administrators alike should note that the standards are not listed here in the order in which they should be taught. Moreover, the standards are not simply topics to be checked off a list during isolated units of instruction, but rather content that should be present throughout the school year through rich instructional experiences.

What Students Learn in Geometry

Overview

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). In the higher mathematics courses, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. In the standards for grades seven

31 and eight, students began to see two-dimensional shapes as part of a generic plane
32 (the Euclidean Plane) and began to explore transformations of this plane as a way to
33 determine whether two shapes are congruent or similar. In the Geometry course, these
34 notions are formalized and students use transformations to prove geometric theorems.
35 The definition of congruence in terms of rigid motions provides a broad understanding of
36 this notion, and students explore the consequences of this definition in terms of
37 congruence criteria and proofs of geometric theorems.

38 Students investigate triangles and decide when they are similar; with this
39 newfound knowledge and their prior understanding of proportional relationships, they
40 define trigonometric ratios and solve problems using right triangles. They investigate
41 circles and prove theorems about them. Connecting to their prior experience with the
42 coordinate plane, they prove geometric theorems using coordinates and describe
43 shapes with equations. Students extend their knowledge of area and volume formulas
44 to those for circles, cylinders and other rounded shapes. Finally, continuing the
45 development of statistics and probability, students investigate probability concepts in
46 precise terms, including the independence of events and conditional probability.

47 **Examples of Key Advances from Previous Grades or Courses**

- 48 • Because concepts such as rotation, reflection and translation were treated in the
49 grade 8 standards mostly in the context of hands-on activities, and with an
50 emphasis on geometric intuition, high school Geometry will put equal weight on
51 precise definitions.
- 52 • In grades K–8, students worked with a variety of geometric measures (length,
53 area, volume, angle, surface area and circumference). In high school Geometry,
54 students apply these component skills in tandem with others in the course of
55 modeling tasks and other substantial applications (MP.4).
- 56 • The skills that students develop in Algebra I around simplifying and transforming
57 square roots will be useful when solving problems that involve distance or area
58 and that make use the Pythagorean Theorem.
- 59 • In grade 8, students learned the Pythagorean Theorem and used it to determine
60 distances in a coordinate system (8.G.6–8). In high school Geometry, students

61 will build on their understanding of distance in coordinate systems and draw on
 62 their growing command of algebra to connect equations and graphs of circles (G-
 63 GPE.1).

- 64 • The algebraic techniques developed in Algebra I can be applied to study analytic
 65 geometry. Geometric objects can be analyzed by the algebraic equations that
 66 give rise to them. Some basic geometric theorems in the Cartesian plane can be
 67 proven using algebra.

68

69 **Connecting Standards for Mathematical Practice and Content**

70 The Standards for Mathematical Practice apply throughout each course and,
 71 together with the content standards, prescribe that students experience mathematics as
 72 a coherent, useful, and logical subject that makes use of their ability to make sense of
 73 problem situations. The Standards for Mathematical Practice (MP) represent a picture
 74 of what it looks like for students to *do mathematics* and, to the extent possible, content
 75 instruction should include attention to appropriate practice standards. There are ample
 76 opportunities for students to engage in each mathematical practice in Geometry; the
 77 table below offers some examples.

78

Standards for Mathematical Practice <i>Students...</i>	Examples of each practice in Geometry
<i>MP1. Make sense of problems and persevere in solving them.</i>	Students construct accurate diagrams of geometry problems to help make sense of them. They organize their work so that others can follow their reasoning, e.g. in proofs.
<i>MP2. Reason abstractly and quantitatively.</i>	Students understand that the coordinate plane can be used to represent geometric shapes and transformations and therefore connect their understanding of number and algebra to geometry.
<i>MP3. Construct viable arguments and critique the reasoning of others.</i> Students build proofs by induction and proofs by contradiction. CA 3.1 (for higher mathematics only).	Students construct proofs of geometric theorems. They write coherent logical arguments and understand that each step in a proof must follow from the last, justified with a previously accepted or proven result.
<i>MP4. Model with mathematics.</i>	Students apply their new mathematical understanding to real-world problems. They learn how transformational geometry and trigonometry can

	be used to model the physical world.
<i>MP5. Use appropriate tools strategically.</i>	Students make use of visual tools for representing geometry, such as simple patty paper or transparencies, or dynamic geometry software.
<i>MP6. Attend to precision.</i>	Students develop and use precise definitions of geometric terms. They verify that a specific shape has certain properties justifying its categorization (e.g. a rhombus as opposed to a quadrilateral).
<i>MP7. Look for and make use of structure.</i>	Students construct triangles in quadrilaterals or other shapes and use congruence criteria of triangles to justify results about those shapes.
<i>MP8. Look for and express regularity in repeated reasoning.</i>	Students explore rotations, reflections and translations, noticing that certain attributes of different shapes remain the same (e.g. parallelism, congruency, orientation) and develop properties of transformations by generalizing these observations.

79

80 MP 4 holds a special place throughout the higher mathematics curriculum, as
 81 Modeling is considered its own conceptual category. Though the Modeling category
 82 has no specific standards listed within it, the idea of using mathematics to model the
 83 world pervades all higher mathematics courses and should hold a high place in
 84 instruction. Readers will see some standards marked with a star symbol (★) to indicate
 85 that they are *modeling standards*, that is, they present an opportunity for applications to
 86 real-world modeling situations more so than other standards.

87 Examples of places where specific MP standards can be implemented in the
 88 Geometry standards will be noted in parentheses, with the standard(s) indicated.

89

90 **Geometry Content Standards by Conceptual Category**

91 The Geometry course is organized by conceptual category, domains, clusters
 92 and standards. Below, the overall purpose and progression of the standards included in
 93 Geometry are described according to these conceptual categories. Standards that are
 94 considered to be new to secondary grades teachers will be discussed in more depth
 95 than others.

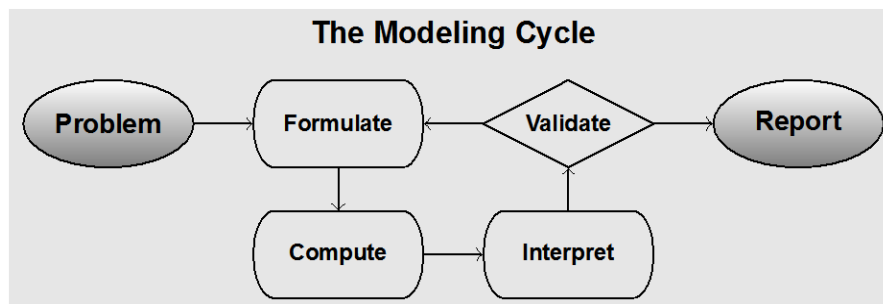
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97 **Conceptual Category: Modeling**

98 Throughout the higher mathematics CA CCSSM, certain standards are marked
 99 with a (★) symbol to indicate that they are considered modeling standards. Modeling at
 100 the higher mathematics level goes beyond the simple application of previously

101 constructed mathematics to real-world problems. True modeling begins with students
102 asking a question about the world around them, and mathematics is then constructed in
103 the process of attempting to answer the question. When students are presented with a
104 real-world situation and challenged to ask a question, all sorts of new issues arise:
105 which of the quantities present in this situation are known and unknown? What can I
106 generalize? Is there some way to introduce a known shape into this diagram that gives
107 more information? Students need to decide on a solution path, which may need to be
108 revised. They make use of tools such as calculators, dynamic geometry software, or
109 spreadsheets. They validate their work by moving between calculations by hand and
110 software assisted computation.

111 Modeling problems have an element of being genuine problems, in the sense
112 that students care about answering the question under consideration. In modeling,
113 mathematics is used as a tool to answer questions that students really want answered.
114 This will be a new approach for many teachers and will be challenging to implement, but
115 the effort will produce students who can appreciate that mathematics is relevant to their
116 lives. From a pedagogical perspective, modeling gives a concrete basis from which to
117 abstract the mathematics and often serves to motivate students to become independent
118 learners.



119
120 Figure 1: The modeling cycle. Students examine a problem and formulate a
121 *mathematical model* (an equation, table, graph, etc.), compute an answer or rewrite
122 their expression to reveal new information, interpret their results, validate them, and
123 report out.

124 Throughout the Geometry chapter, the included examples will be framed as
125 modeling situations whenever appropriate, to serve as illustrations of the concept of

126 mathematical modeling. The big ideas of proving geometric theorems, congruence and
127 similarity, analytic geometry, right-triangle trigonometry, and probability can be explored
128 in this way. The reader is encouraged to consult the Appendix, “Mathematical
129 Modeling,” for a further discussion of the modeling cycle and how it is integrated into the
130 higher mathematics curriculum.

131

132 **Conceptual Category: Geometry**

133 The standards of the Geometry conceptual category comprise the bulk of
134 instruction in the Traditional Geometry course. Here, students develop the ideas of
135 congruence and similarity through transformations. They prove theorems, both with and
136 without the use of coordinates. They explore right triangle trigonometry, and circles and
137 parabolas. Throughout the course, Mathematical Practice 3, “Construct viable
138 arguments and critique the reasoning of others,” plays a predominant role.

139

140 **Congruence**

G-CO

141 **Experiment with transformations in the plane.**

- 142 1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based
143 on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- 144 2. Represent transformations in the plane using, e.g., transparencies and geometry software;
145 describe transformations as functions that take points in the plane as inputs and give other points
146 as outputs. Compare transformations that preserve distance and angle to those that do not (e.g.,
147 translation versus horizontal stretch).
- 148 3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and
149 reflections that carry it onto itself.
- 150 4. Develop definitions of rotations, reflections, and translations in terms of angles, circles,
151 perpendicular lines, parallel lines, and line segments.
- 152 5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure
153 using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of
154 transformations that will carry a given figure onto another.

155

156 **Understand congruence in terms of rigid motions.** [Build on rigid motions as a familiar starting point for 157 development of concept of geometric proof.]

- 158 6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a
159 given rigid motion on a given figure; given two figures, use the definition of congruence in terms
160 of rigid motions to decide if they are congruent.
- 161 7. Use the definition of congruence in terms of rigid motions to show that two triangles are
162 congruent if and only if corresponding pairs of sides and corresponding pairs of angles are
163 congruent.
- 164 8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of
165 congruence in terms of rigid motions.

166
 167 In Geometry, the commonly held—but imprecise—definition that shapes are
 168 congruent when they “have the same size and shape” is replaced by a more
 169 mathematically precise one (MP.6): *two shapes are congruent if there is a sequence of*
 170 *rigid motions in the plane that takes one shape exactly onto the other.* This definition
 171 has been explored intuitively in the grade eight standards, but is now investigated more
 172 closely. Earlier, students experimented with transformations in the plane, but now,
 173 students build more precise definitions for the *rigid motions* (rotation, reflection, and
 174 translation) based on previously defined and understood terms, such as point, line,
 175 between, angle, circle, perpendicular, etc. (G-CO.1, 3, 4). Students base their
 176 understanding of these definitions on their experience with transforming figures using
 177 patty paper, transparencies, or geometry software, (G-CO.2, 3, 5, MP.5), something
 178 they started doing in Grade 8. These transformations should be investigated both in a
 179 general plane as well as on a coordinate system—especially when explicitly describing
 180 transformations using precise names of points, translation vectors, and lines of
 181 symmetry or reflection.

Example: Defining Rotations. Mrs. B wants to help her class understand the following definition of a *rotation*:

A *rotation* about a point P through angle α is a transformation $A \mapsto A'$ such that (1) if point A is different from P , then $PA = PA'$ and the measure of $\angle APA' = \alpha$; and (2) if point A is the same as point P , then $A' = A$.

She gives her students a handout with several geometric shapes on it and a point P indicated on the page. In pairs, students are to copy the shapes onto a transparency sheet and rotate them through various angles about P . Students then transfer the rotated shapes back onto the original page, and measure various lengths and angles as indicated in the definition.

While justifying that the properties of the definition hold for the shapes she has given them, the students also make some observations about the effects of a rotation on the entire plane, for instance that:

- Rotations preserve lengths.
- Rotations preserve angle measures.
- Rotations preserve parallelism.

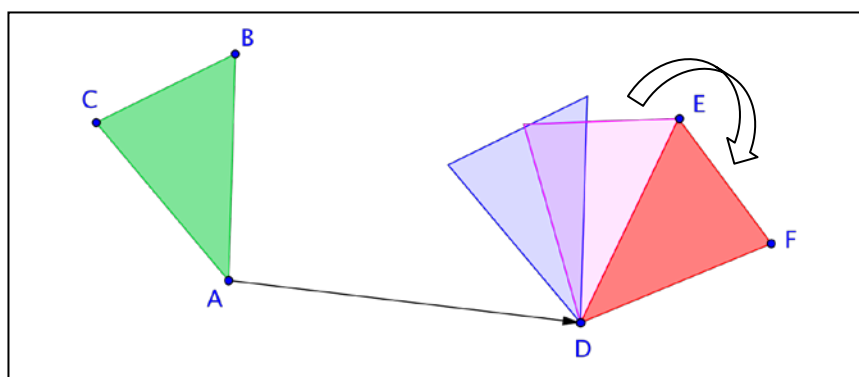
Later, Mrs. B plans to allow students to explore more rotations on dynamic geometry software, asking them to create a geometric shape and rotate it by various angles about various points P , both part of the object and not.

182

183 In standards G-CO.6-8, geometric transformations are given a more prominent
184 role in the higher mathematics geometry curriculum than perhaps ever before. The new
185 definition of congruence in terms of rigid motions applies to *any* shape in the plane,
186 whereas previously congruence seemed to depend on criteria that were specific only to
187 certain shapes. For example, the side-side-side (SSS) congruence criterion for triangles
188 did not extend to quadrilaterals, seemingly suggesting that congruence was a notion
189 specific to the shape one was considering. While it is true that there are specific criteria
190 for determining congruence of certain shapes, the *basic notion of congruence* is the
191 same for all shapes. In the CCSSM, the SSS criteria for triangle congruence is a
192 *consequence* of the definition of congruence, just as is the fact that if two polygons are
193 congruent, then their sides and angles can be put into a correspondence such that each
194 corresponding pair of sides and angles is congruent. This concept comprises the
195 content of standards G-CO.7 and G-CO.8, which derive congruence criteria for triangles
196 from the new definition of congruence.

197 Standard G-CO.7 explicitly states that students show that two triangles are
198 congruent *if and only if* corresponding pairs of sides and corresponding pairs of angles
199 are congruent (MP.3). The depth of reasoning here is fairly substantial at this level; put
200 in other words, students must be able to show using rigid motions that congruent
201 triangles have congruent corresponding parts, and that, conversely, if the corresponding
202 parts of two triangles are congruent, then there is a sequence of rigid motions that takes
203 one triangle to the other. The second statement may be more difficult to justify than the
204 first for most students, so we present a justification of it here. Suppose we have two
205 triangles $\triangle ABC$ and $\triangle DEF$ such that the correspondence $A \leftrightarrow D, B \leftrightarrow E, C \leftrightarrow F$ results in
206 pairs of sides and pairs of angles being congruent. If one triangle was drawn on a fixed
207 piece of paper, and the other drawn on a separate transparency, then a student could
208 illustrate a translation, T , that takes point A to point D . A simple rotation R about the
209 point A , if necessary, takes point B to point E , which we can be certain will occur since
210 $\overline{AB} \cong \overline{DE}$ and rotations preserve lengths. Finally, the last step that may be needed is a
211 reflection S about the side AB , to take point C to point F . It is nontrivial why the image of
212 point C is actually F . Since $\angle A$ is reflected about line \overleftrightarrow{AB} , its measure is preserved.

213 Therefore, the image of side \overline{AC} at least lies on line \overleftrightarrow{DF} , since $\angle A \cong \angle D$. But since
 214 $\overline{AC} \cong \overline{DF}$, it must be the case that the image of point C coincides with F . The previous
 215 discussion amounts to the fact that the sequence of rigid motions, T , followed by R ,
 216 followed by S , maps $\triangle ABC$ exactly onto $\triangle DEF$. Therefore, if we know that the
 217 corresponding parts of two triangles are congruent, then there is a sequence of rigid
 218 motions carrying one onto the other; that is, they are congruent. The informal proof
 219 presented here should be accessible to Geometry students. See the figure for an
 220 illustration of the steps in this reasoning.



221 Figure 2: Illustration of the reasoning that corresponding
 222 parts being congruent implies triangle congruence, in which
 223 point A is translated to D , the resulting image of $\triangle ABC$ is
 224 rotated so as to place B onto E , and finally, the image is
 225 then reflected along line segment DE to match point C to F .

226 Similar reasoning applies for standard G.CO.8, in which students justify the typical
 227 triangle congruence criteria such as ASA, SAS and SSS. Experimentation with
 228 transformations of triangles where only two of the criteria are satisfied will result in
 229 counterexamples, and geometric constructions of triangles of prescribed side lengths,
 230 for example, in the case of SSS, will leave little doubt that any triangle constructed with
 231 these side lengths will be congruent to another, and that therefore SSS holds (MP.7).

232

233 Congruence

G-CO

234 **Prove geometric theorems.** [Focus on validity of underlying reasoning while using variety of ways of writing proofs.]

- 235 9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a*
236 *transversal crosses parallel lines, alternate interior angles are congruent and corresponding*
237 *angles are congruent; points on a perpendicular bisector of a line segment are exactly those*
238 *equidistant from the segment's endpoints.*
- 239 10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum*
240 *to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two*
241 *sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at*
242 *a point.*
- 243 11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite*
244 *angles are congruent, the diagonals of a parallelogram bisect each other, and conversely,*
245 *rectangles are parallelograms with congruent diagonals.*

246

247 **Make geometric constructions.** [Formalize and explain processes.]

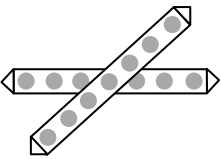
- 248 12. Make formal geometric constructions with a variety of tools and methods (compass and
249 straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying*
250 *a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular*
251 *lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a*
252 *given line through a point not on the line.*
- 253 13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

254

255

256 It is important to note here that once the triangle criteria for congruence are
257 established, students can begin proving geometric theorems. Examples of such
258 theorems can be found in standards G.CO.9-11. The triangle congruence criteria are
259 established results that can be used to prove new results. Instructors are encouraged
260 to use a variety of strategies for engaging students in understanding and writing proofs,
261 including: using ample pictures to demonstrate results and generate strategies; using
262 patty paper, transparencies, or dynamic geometry software to explore the steps in a
263 proof; creating flow charts and other organizational diagrams for outlining a proof; and
264 writing step-by-step or paragraph formats for the completed proof (MP.5). Above all
265 else, the reasoning involved in connecting one step in the logical argument to the next
266 should be emphasized. Students should be encouraged to make conjectures based on
267 experimentation, to justify their conjectures, and to communicate their reasoning to their
268 peers (MP.3). The following example illustrates how students can be encouraged to
269 experiment and construct hypotheses based on their observations.

270

<p>Example. <i>The Kite Factory.</i> The hypothetical situation of a kite factory is presented to students, wherein kite engineers wish to know how the shape of a kite affects how it flies (e.g. the lengths of the rods, where they are attached, the angle at which</p>  <p>the rods are attached, etc.). In this activity, students are given pieces of cardstock of various lengths, hole-punched at regular intervals so they can be attached in different places.</p>	<p>These two “rods” form the frame for a kite at the kite factory. By changing the angle at which the sticks are held, and the places where they are attached, students discover different properties of quadrilaterals.</p> <p>Students are challenged to make conjectures and use precise language to describe their findings about which diagonals result in which quadrilaterals. They can discover properties unique to certain quadrilaterals, such as the fact that diagonals that are perpendicular bisectors of each other imply the quadrilateral is a rhombus. See videos of this lesson being implemented in a high school classroom at insidemathematics.org.</p>
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Similarity, Right Triangles, and Trigonometry

G-SRT

Understand similarity in terms of similarity transformations.

1. Verify experimentally the properties of dilations given by a center and a scale factor:
 - a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
 - b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two triangles to be similar.

286

287

Prove theorems involving similarity.

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

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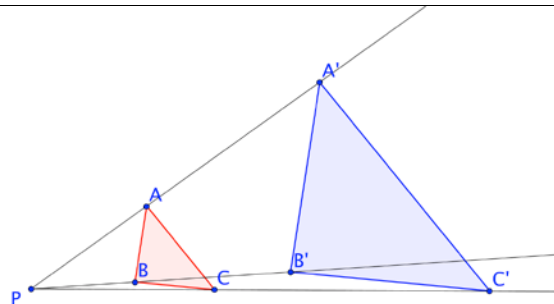
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294 As right triangles and triangle relationships play such an important role in
 295 applications and future mathematics learning, they are given a prominent role in the
 296 geometry conceptual category. A discussion of *similarity* is necessary first, and again, a

297 more precise mathematical definition of similarity is given in the higher mathematics
 298 standards. Students have worked with *dilations* as a transformation in the grade eight
 299 standards; now, they explore the properties of dilations in more detail and develop an
 300 understanding of the notion of *scale factor* (G-SRT.1). Whereas it is common to say that
 301 objects that are similar simply have “the same shape,” the new definition taken for two
 302 objects being similar is that there is a sequence of *similarity transformations*¹ that maps
 303 one exactly onto the other. In standards G-SRT.2 and G-SRT.3, students explore the
 304 consequences of two triangles being similar: that they have congruent angles and that
 305 their side lengths are in the same proportion. This new understanding gives rise to
 306 more results, encapsulated in standards G-SRT.4 and G-SRT.5.²

Example: Experimenting with dilations.

Students are given opportunities to experiment with dilations and determine how they affect planar objects. Students first make sense of the definition of a *dilation of scale factor $k > 0$ with center P* as the transformation that moves a point A along the ray \overrightarrow{PA} to a new point A' , so that $|\overline{PA'}| = k \cdot |\overline{PA}|$. For example, students apply the dilation of scale factor 2.5 with center P to the points A , B , and C illustrated using a ruler. Once they've done so, they consider the two triangles $\triangle ABC$ and $\triangle A'B'C'$. What they discover is that the lengths of the corresponding sides of the triangles have the same ratio as dictated by the scale factor. (G-SRT.2) Students learn that parallel lines are taken to parallel lines by dilations; thus corresponding segments of $\triangle ABC$ and $\triangle A'B'C'$ are parallel. After students have proved results about parallel lines



intersected by a transversal, they can deduce that the angles of the triangles are congruent. Through experimentation, they see that the congruence of corresponding angles is a necessary and sufficient condition for the triangles to be similar, leading to the AA criterion for triangle similarity. (G.SRT.3.)

For a simple investigation, students can observe how the distance at which a projector is placed from a screen affects the size of the image on the screen. (MP.4)

307

¹ Translation, rotation, reflection, or dilation.

² The proof of the Pythagorean Theorem by the use of similar triangles is a nontrivial exercise; a good resource for a proof of this result by H. Wu can be found at <http://vimeo.com/45773544>.

308 **Similarity, Right Triangles, and Trigonometry****G-SRT**309 **Define trigonometric ratios and solve problems involving right triangles.**

- 310 6. Understand that by similarity, side ratios in right triangles are properties of the angles in the
311 triangle, leading to definitions of trigonometric ratios for acute angles.
- 312 7. Explain and use the relationship between the sine and cosine of complementary angles.
- 313 8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied
314 problems. ★
- 315 **8.1 Derive and use the trigonometric ratios for special right triangles (30°, 60°, 90 °and 45°,
316 45°, 90°). CA**

317

318 Once the angle-angle (AA) similarity criterion for triangles is established, it
319 follows that any two *right* triangles $\triangle ABC$ and $\triangle DEF$ are similar when at least one pair of
320 angles are congruent (say $\angle A \cong \angle D$), since the right angles are obviously congruent
321 (say $\angle B \cong \angle E$). By similarity, the corresponding sides of the triangles are in
322 proportion:

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

323 Notice the first and third expressions in the statement of equality above can be
324 rearranged to yield that

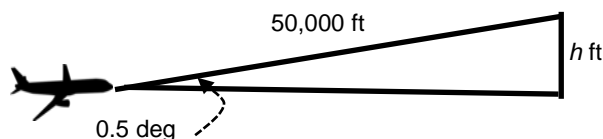
$$\frac{AB}{AC} = \frac{DE}{DF}.$$

325 Since the triangles in question are arbitrary, this implies that for any right triangle with
326 an angle congruent to $\angle A$, the ratio of the side adjacent to $\angle A$ and the hypotenuse of
327 the triangle is a certain constant. This allows us to define unambiguously the *sine* of
328 $\angle A$, denoted by $\sin A$, as the value of this ratio. In this way, students come to
329 understand the trigonometric functions as relationships completely determined by
330 angles (G-SRT.6). They further their understanding of these functions by investigating
331 relationships between sine, cosine, and tangent; by exploring the relationship between
332 the sine and cosine of complementary angles; and by applying their knowledge of right
333 triangles to real-world situations, (MP.4) such as in the example below (G-SRT.6-8).
334 Experience working with many different triangles, finding their measurements, and
335 computing ratios of the measurements found will help students understand the basics of
336 the trigonometric functions.

337

Example: *Using Trigonometric Relationships.*
Planes that fly at high speed and low elevations often have onboard radar systems to detect possible obstacles in the path of the plane. The radar can determine the range of an obstacle and the angle of elevation to the top of the obstacle. Suppose that the radar detects a tower that is 50,000 feet away, with an angle of elevation of 0.5 degree. By how many feet must the plane rise in order to pass above the tower?

Solution: The sketch of the situation below shows that there is a right triangle with hypotenuse 50,000 (ft) and smallest angle 0.5 (degree). To find the side opposite this angle, which represents the minimum height the plane should rise, we use $\sin 0.5^\circ = \frac{h}{50,000}$, so that
 $h = (50,000 \text{ ft}) \sin 0.5^\circ \approx 436.33 \text{ ft}.$



338

339 Similarity, Right Triangles, and Trigonometry

G-SRT

340 Apply trigonometry to general triangles.

- 341 9. (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from
342 a vertex perpendicular to the opposite side.
- 343 10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
- 344 11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown
345 measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
346

347 Students advance their knowledge of right triangle trigonometry by applying
348 trigonometric ratios in non-right triangles. For instance, students see that by dropping
349 an altitude in a given triangle, they divide the triangle into right triangles to which these
350 relationships can be applied. By seeing that the base of the triangle is a and the height
351 is $b \cdot \sin C$, they derive a general formula for the area of any triangle $A = \frac{1}{2} ab \sin C$ (G-
352 SRT.9). In addition, students use reasoning about similarity and trigonometric identities
353 to derive the Laws of Sines and Cosines first in only acute triangles, and use these and
354 other relationships to solve problems (G-SRT.10-11). Instructors will need to address
355 the ideas of the sine and cosine of angles larger than or equal to 90° to fully discuss
356 Laws of Sine and Cosine, though full unit circle trigonometry need not be discussed in
357 this course.

358

359 Circles

G-C

360 Understand and apply theorems about circles.

- 361 1. Prove that all circles are similar.
 362 2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the*
 363 *relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter*
 364 *are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects*
 365 *the circle.*
 366 3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for
 367 a quadrilateral inscribed in a circle.
 368 4. (+) Construct a tangent line from a point outside a given circle to the circle.

369

370

371 Students extend their understanding of the usefulness of similarity

372 transformations through investigating circles (G-C.1). For instance, students can

373 reason that any two circles are similar by describing precisely how to transform one into

374 the other, as the example illustrates with two specific circles. Students continue

375 investigating properties of circles and relationships among angles, radii and chords (G-

376 C.2, 3, 4).

377

Example. Students can show that the two circles C and D given by the equations below are similar.

$$C: (x - 1)^2 + (y - 4)^2 = 9$$

$$D: (x + 2)^2 + (y - 1)^2 = 25$$

Solution. Since the centers of the circles are $(1, 4)$ and $(-2, 1)$, respectively, we first translate the

center of circle C to the center of circle D using the translation $T(x, y) = (x - 3, y - 3)$. Finally, since the radius of circle C is 3 and the radius of circle D is 5, we dilate from the point $(-2, 1)$ by a scale factor of $5/3$.

378

379 Circles

G-C

380 **Find arc lengths and areas of sectors of circles.** [Radian introduced only as unit of measure.]

- 381 5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to
 382 the radius, and define the radian measure of the angle as the constant of proportionality; derive
 383 the formula for the area of a sector. **Convert between degrees and radians. CA**

384

385

386 Another important application of the notion of similarity is the definition of the

387 radian measure of an angle. Students can derive this result in the following way: given

388 a sector of a circle C of radius r and central angle α , and a sector of a circle D of radius

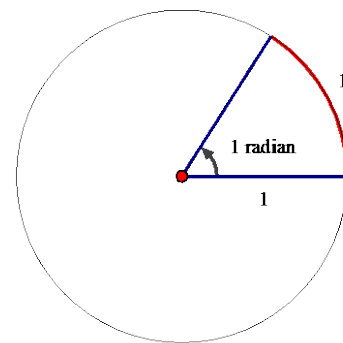
389 s and central angle also α , it stands to reason that since these sectors are similar,

$$\frac{\text{length of arc on circle } C}{r} = \frac{\text{length of arc on circle } D}{s}.$$

390 Therefore, much like when defining the trigonometric functions, there is a constant m
 391 such that for an arc subtended by an angle α on any circle, we have

$$\frac{\text{length of arc subtended by angle } \alpha}{\text{radius of the circle}} = m.$$

392 This constant of proportionality is the *radian measure* of angle α . It follows that an
 393 angle that subtends an arc on a circle that is the same length
 394 as the radius measures 1 radian. By investigating circles of
 395 different sizes, measuring off arcs subtended by the same
 396 angle using string, and finding the ratios described above,
 397 students can apply their proportional reasoning skills to
 398 discover this constant ratio, thereby developing an
 399 understanding of the definition of radian measure.



400

401 **Expressing Geometric Properties with Equations**

G-GPE

402 **Translate between the geometric description and the equation for a conic section.**

- 403 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem;
 404 complete the square to find the center and radius of a circle given by an equation.
 405 2. Derive the equation of a parabola given a focus and directrix.

406

407 **Use coordinates to prove simple geometric theorems algebraically.** [Include distance formula; relate to
 408 Pythagorean Theorem.]

- 409 4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or*
 410 *disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or*
 411 *disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the*
 412 *point $(0, 2)$.*
 413 5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric
 414 problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes
 415 through a given point).
 416 6. Find the point on a directed line segment between two given points that partitions the segment in
 417 a given ratio.
 418 7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g.,
 419 using the distance formula. ★

420

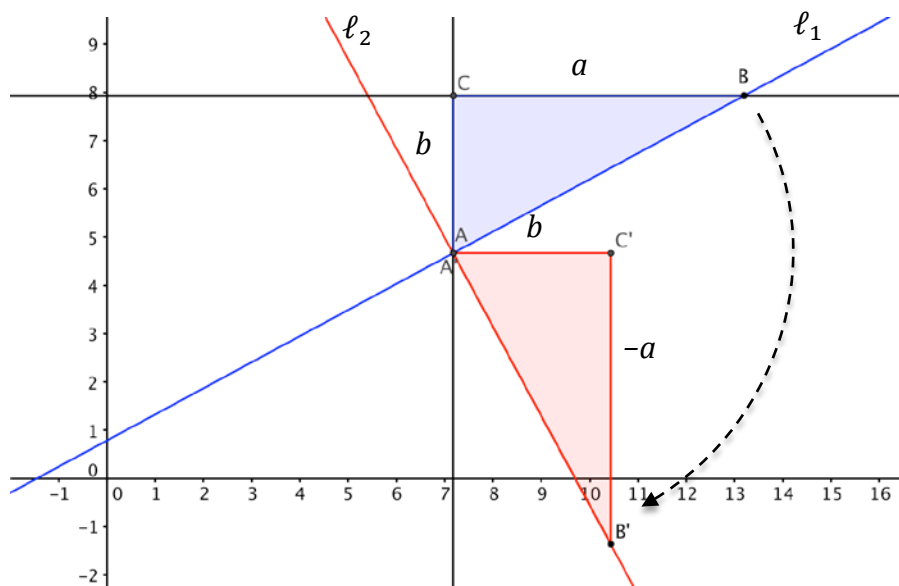
421 The largest intersection of algebra concepts and geometry occurs here, wherein
 422 two-dimensional shapes are represented on a coordinate system and can be described
 423 using algebraic equations and inequalities. Here is a derivation of the equation of a
 424 circle by the Pythagorean Theorem and the definition of a circle (G-GPE.1). Given that
 425 a circle consists of all points (x, y) that are at a distance $r > 0$ from a fixed center (h, k) ,

426 we see that $\sqrt{(x-h)^2 + (y-k)^2} = r$ for any point lying on the circle, so that $(x-h)^2 +$
427 $(y-k)^2 = r^2$ determines this circle. Students can derive this equation, and flexibly
428 change an equation into this form by completing the square as necessary. By
429 understanding the derivation of this equation, the variables h, k and r take on a clear
430 meaning. Students do the same for the definition of a parabola in terms of a focus and
431 directrix in G-GPE.2.

432 In standards G.GPE.4 and G.GPE.6, students continue their work of using
433 coordinates to prove geometric theorems with algebraic techniques. In G.GPE.6, given
434 a directed line segment represented by a vector emanating from the origin to the point
435 $(4,6)$, students may be asked to find the point on this vector that partitions it into a ratio
436 of 2:1. Students may construct right triangles and using triangle similarity for find this
437 point, or, they may represent the vector as $x = 4t, y = 6t$, for $0 \leq t \leq 1$, and reason that
438 the point they seek can be found when $t = \frac{2}{3}$.

439 While many simple geometric theorems can be proved algebraically, perhaps two
440 results of high importance are the slope criteria for parallel and perpendicular lines.
441 Students studied lines and linear equations beginning with the grade seven standards;
442 here, they not only use relationships between slopes of parallel and perpendicular lines
443 to solve problems, but they justify why they are true. An intuitive argument for why
444 parallel lines have the same slope might read: "Since the two lines never meet, each
445 line must *keep up* with the other as we travel along the slopes of the lines. So it seems
446 obvious that their slopes must be equal." This intuitive thought leads us to an
447 equivalent statement: if given a pair of linear equations $\ell_1: y = m_1x + b_1$ and $\ell_2: y =$
448 $m_2x + b_2$ (for $m_1, m_2 \neq 0$) such that $m_1 \neq m_2$, that is, such that *their slopes are different*,
449 then the lines must intersect. Solving for the intersection of the two lines yields the x -
450 coordinate of their intersection to be $x = \frac{b_2 - b_1}{m_1 - m_2}$ which surely exists since $m_1 \neq m_2$. The
451 reasoning here is important: both understanding the steps of the argument and
452 understanding why proving this statement is equivalent to proving the statement, "if
453 $\ell_1 \parallel \ell_2$, then $m_1 = m_2$ " (MP.1, MP.2).

454 In addition, students are expected to justify why the slopes of two non-vertical,
 455 perpendicular lines ℓ_1 and ℓ_2 satisfy the relationship $m_1 = -\frac{1}{m_2}$, or $m_1 \cdot m_2 = -1$. While
 456 there are various ways to do this, one is shown here, and dynamic geometry software
 457 can be used to illustrate it well (MP.4). Let ℓ_1 and ℓ_2 be any two non-vertical
 458 perpendicular lines. Let A be the intersection of the two lines, and let B be any other
 459 point on ℓ_1 above A . We draw a vertical line through A , a horizontal line through B , and
 460 we let C be the intersection of those two lines. $\triangle ABC$ is a right triangle. If a is the
 461 horizontal displacement Δx from C to B and b is the length of \overline{AC} , then the slope of ℓ_1 is
 462 $m_1 = \frac{\Delta y}{\Delta x} = \frac{b}{a}$. By rotating $\triangle ABC$ clockwise around A by 90 degrees, the hypotenuse $\overline{AB'}$
 463 of the rotated triangle $\triangle AB'C'$ lies on ℓ_2 . Using the legs of $\triangle AB'C'$, we see that the slope
 464 of ℓ_2 is $m_2 = \frac{\Delta y}{\Delta x} = \frac{-a}{b}$. Thus $m_1 \cdot m_2 = \frac{b}{a} \cdot \frac{-a}{b} = -1$. See the figure for an illustration of
 465 this proof (MP.1, MP.7).



466
 467

468 The previous proofs make use of several ideas that students have learned about
 469 in this and prior courses, including the relationship between equations and their graphs
 470 in the plane (A.REI.10) and solving equations with variable coefficients (A.REI.3). An
 471 investigative approach that at first uses several examples of lines that are perpendicular

472 and their equations to find points, construct triangles, and decide if the triangles formed
473 are right triangles will help ramp up to the second proof (MP.8). Once more, the
474 reasoning required to make sense of such a proof and to communicate the essence of it
475 to a peer is an important goal of geometry instruction (MP.3).

476

477 **Geometric Measurement and Dimension**

G-GMD

478 **Explain volume formulas and use them to solve problems.**

- 479 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle,
480 volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and*
481 *informal limit arguments.*
- 482 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

483

484 **Visualize relationships between two-dimensional and three-dimensional objects.**

- 485 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify
486 three-dimensional objects generated by rotations of two-dimensional objects.
- 487 5. **Know that the effect of a scale factor k greater than zero on length, area, and volume is to**
488 **multiply each by k , k^2 , and k^3 , respectively; determine length, area and volume measures**
489 **using scale factors. CA**
- 490 6. **Verify experimentally that in a triangle, angles opposite longer sides are larger, sides**
491 **opposite larger angles are longer, and the sum of any two side lengths is greater than the**
492 **remaining side length; apply these relationships to solve real-world and mathematical**
493 **problems. CA**

494

495 The ability to visualize two- and three-dimensional shapes is a useful skill. This
496 group of standards addresses that skill and includes understanding and using volume
497 and area formulas for curved objects. Students also have the opportunity to make use
498 of the notion of a *limiting process*, an idea that plays a large role in calculus and
499 advanced mathematics courses, when they investigate the formula for the area of a
500 circle. By experimenting with grids of finer and finer mesh, they can repeatedly
501 approximate the area of a unit circle, and thereby get a better and better approximation
502 for the irrational number π . They also dissect shapes and make arguments based on
503 the dissections. For instance, a cube can be dissected into three congruent pyramids,
504 as shown in figure 2, which can lend weight to the formula that the volume of a pyramid
505 of base area B and height h is $\frac{1}{3}Bh$. (MP.2).

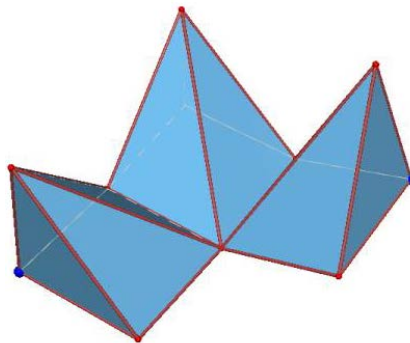


Figure 3: Three congruent pyramids that make a cube.

(Park City Mathematics Institute 2013)

506

507

508

509

510 **Modeling with Geometry**

G-MG

511 **Apply geometric concepts in modeling situations.**

- 512 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a
513 tree trunk or a human torso as a cylinder). ★
- 514 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per
515 square mile, BTUs per cubic foot). ★
- 516 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to
517 satisfy physical constraints or minimize cost; working with typographic grid systems based on
518 ratios). ★

519

520 This set of standards is rich with opportunities for students to apply modeling
521 (MP.4) with geometric concepts. The implementation of these standards should not be
522 limited to the end of a Geometry course simply because they are later in the sequence
523 of standards; they should be employed throughout the geometry curriculum. In
524 standard G-MG.1, students use geometric shapes, their measures, and their properties
525 to describe objects. This standard can involve two- and three-dimensional shapes, and
526 is not relegated to simple applications of formulas. In standard G-MG.3, students solve
527 design problems by modeling with geometry, such as the one illustrated below.

528

Example (Illustrated Mathematics 2013). *Ice Cream Cone.* You have been hired by the owner of a local ice cream parlor to assist in his company's new venture. The company will soon sell its ice

- b. Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of

<p>cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use a real ice cream cone or the dimensions of a real ice cream cone to complete the following tasks.</p> <p>a. Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly.</p>	<p>length, r, and a slant height, s.</p> <p>c. Using measurements of the radius of the base and slant height of your cone, and your equation from the previous step, find the surface area of your cone.</p> <p>d. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.</p> <p>Solutions can be found at illustrativemath.org</p>
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529

530

Conceptual Category: Statistics and Probability

531

532 Students learned some basics of probability, including chance processes,
 533 probability models, and sample spaces in the grade seven and eight standards. In high
 534 school, the relative frequency approach to probability is extended to conditional
 535 probability and independence, rules of probability and their use in finding probabilities of
 536 compound events, and the use of probability distributions to solve problems involving
 537 expected value (The University of Arizona Progressions Documents for the Common
 538 Core Math Standards [Progressions], High School Statistics and Probability 2012).
 539 Building on probability concepts that began in the middle grades, students use the
 540 language of set theory to expand their ability to compute and interpret theoretical and
 541 experimental probabilities for compound events, attending to mutually exclusive events,
 542 independent events, and conditional probability. Students should make use of geometric
 543 probability models wherever possible. They use probability to make informed decisions
 544 (CCSSI 2010).

545

546

Conditional Probability and the Rules of Probability

S-CP

547

548

Understand independence and conditional probability and use them to interpret data. [Link to data from simulations or experiments.]

- 549 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or
550 categories) of the outcomes, or as unions, intersections, or complements of other events (“or,”
551 “and,” “not”). ★
- 552 2. Understand that two events A and B are independent if the probability of A and B occurring
553 together is the product of their probabilities, and use this characterization to determine if they are
554 independent. ★
- 555 3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret
556 independence of A and B as saying that the conditional probability of A given B is the same as
557 the probability of A , and the conditional probability of B given A is the same as the probability
558 of B . ★
- 559 4. Construct and interpret two-way frequency tables of data when two categories are associated
560 with each object being classified. Use the two-way table as a sample space to decide if events
561 are independent and to approximate conditional probabilities. *For example, collect data from a*
562 *random sample of students in your school on their favorite subject among math, science, and*
563 *English. Estimate the probability that a randomly selected student from your school will favor*
564 *science given that the student is in tenth grade. Do the same for other subjects and compare the*
565 *results.* ★
- 566 5. Recognize and explain the concepts of conditional probability and independence in everyday
567 language and everyday situations. ★
- 568
- 569

570 To develop student understanding of conditional probability, students should
571 experience two types of problems: ones in which the uniform probabilities attached to
572 outcomes lead to independence of the outcomes and ones in which they do not (S-
573 CP.1-3). Below are two examples wherein these two possibilities occur.

574

<p>Example (Adapted from Progressions, High School Statistics and Probability 2012). <i>Guessing On a True-False Quiz.</i> If there are four T-F questions on a quiz, then the possible outcomes based on guessing on each question can be arranged as in the table below:</p> <table border="1"> <thead> <tr> <th colspan="6">Possible outcomes: Guessing on four true–false questions</th> </tr> <tr> <th>Number correct</th> <th>Outcomes</th> <th>Number correct</th> <th>Outcomes</th> <th>Number correct</th> <th>Outcomes</th> </tr> </thead> <tbody> <tr> <td>4</td> <td>CCCC</td> <td>2</td> <td>CCII</td> <td>1</td> <td>CIII</td> </tr> <tr> <td>3</td> <td>ICCC</td> <td>2</td> <td>CICI</td> <td>1</td> <td>ICII</td> </tr> <tr> <td>3</td> <td>CICC</td> <td>2</td> <td>CIIC</td> <td>1</td> <td>IICI</td> </tr> <tr> <td>3</td> <td>CCIC</td> <td>2</td> <td>ICCI</td> <td>1</td> <td>IIIC</td> </tr> <tr> <td>3</td> <td>CCCI</td> <td>2</td> <td>ICIC</td> <td>0</td> <td>IIII</td> </tr> <tr> <td></td> <td></td> <td>2</td> <td>IICC</td> <td></td> <td></td> </tr> </tbody> </table> <p><i>C indicates a correct answer; I indicates an incorrect answer.</i></p>	Possible outcomes: Guessing on four true–false questions						Number correct	Outcomes	Number correct	Outcomes	Number correct	Outcomes	4	CCCC	2	CCII	1	CIII	3	ICCC	2	CICI	1	ICII	3	CICC	2	CIIC	1	IICI	3	CCIC	2	ICCI	1	IIIC	3	CCCI	2	ICIC	0	IIII			2	IICC			<p>By simple counting outcomes one can find various probabilities. For example,</p> $P(\text{C on first question}) = \frac{1}{2}$ <p>and</p> $P(\text{C on second question}) = \frac{1}{2}$ <p>as well. Noticing that</p> $P[(\text{C on first}) \text{ AND } (\text{C on second})] = \frac{4}{16} = \frac{1}{2} \cdot \frac{1}{2}$ <p>shows that the two events, getting the first question correct and the second correct, are independent.</p>
Possible outcomes: Guessing on four true–false questions																																																	
Number correct	Outcomes	Number correct	Outcomes	Number correct	Outcomes																																												
4	CCCC	2	CCII	1	CIII																																												
3	ICCC	2	CICI	1	ICII																																												
3	CICC	2	CIIC	1	IICI																																												
3	CCIC	2	ICCI	1	IIIC																																												
3	CCCI	2	ICIC	0	IIII																																												
		2	IICC																																														

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Example (Adapted from Progressions, High School Statistics and Probability 2012).

Working-Group Leaders. Suppose a 5-person working group consisting of three girls (April, Briana, and Cyndi) and two boys (Daniel and Ernesto) wants to randomly choose two people to lead the group. The first person is the discussion leader and the second is the recorder, so order is important in selecting the leadership team. There are 20 outcomes for this situation, shown below:

Selecting two students from three girls and two boys

Number of girls	Outcomes	
2	AB	BA
2	AC	CA
2	BC	CB
1	AD	DA
1	AE	EA
1	BD	DB
1	BE	EB
1	CD	DC
1	CE	EC
0	DE	ED

Notice that the probability of selecting two girls as

the leaders is:

$$P(\text{two girls chosen}) = \frac{6}{20} = \frac{3}{10}$$

whereas

$$P(\text{girl selected on first draw}) = \frac{12}{20} = \frac{3}{5}$$

while

$$P(\text{girl selected on second draw}) = \frac{3}{5}$$

as well. But since $\frac{3}{5} \cdot \frac{3}{5} \neq \frac{3}{10}$, the two events are not independent.

One can also use the conditional probability perspective to show these events are not independent. Since

$$P(\text{girl on second} \mid \text{girl on first}) = \frac{6}{12} = \frac{1}{2}$$

and $P(\text{girl on second}) = \frac{3}{5}$, these events are seen to be dependent.

577

578

579 Conditional Probability and the Rules of Probability**S-CP**

580 **Use the rules of probability to compute probabilities of compound events in a uniform probability**
581 **model.**

- 582 6. Find the conditional probability of A given B as the fraction of B 's outcomes that also belong to A ,
583 and interpret the answer in terms of the model. ★
- 584 7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms
585 of the model. ★
- 586 8. (+) Apply the general Multiplication Rule in a uniform probability model,
587 $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. ★
- 588 9. (+) Use permutations and combinations to compute probabilities of compound events and solve
589 problems. ★

590

591

592 Students also explore finding probabilities of compound events (S-CD.6-9) by

593 using the Addition Rule ($P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$), and the general

594 Multiplication Rule ($P(A \text{ AND } B) = P(A) \cdot P(B|A) = P(B) \cdot P(A|B)$). A simple experiment

595 involving rolling two number cubes and tabulating the possible outcomes can shed light
 596 on these formulas before they are extended to application problems.
 597

<p>Example (Adapted from Illustrative Mathematics 2013). On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Some believe that the rescue procedures favored the wealthier first class passengers. Data on survival of passengers are summarized in the table below. We will use this data to investigate the validity of such claims. Students can use the fact that two events A and B are independent if $P(A B) = P(A) \cdot P(B)$. Let A represent the event a passenger survived, and B represent the event that the passenger was in first class. We compare the conditional probability $P(A B)$ with the probability $P(A)$. The probability of surviving, given that the passenger was in first class, is the fraction of first</p>	<p>class passengers who survived. That is, we restrict the sample space to only first class passengers to obtain $P(A B) = \frac{202}{325} \approx 0.622$. The probability that the passenger survived is the number of all passengers who survived divided by the total number of passengers, that is $P(A) = \frac{498}{1316} \approx 0.378$. Since $0.622 \neq 0.378$, the two given events are not independent. Moreover, we can say that being a passenger in first class increased the chances of surviving.</p> <p style="text-align: center;">Students can be challenged to further investigate where similar reasoning would apply today. For example, what are similar statistics for Hurricane Katrina, and what would a similar analysis conclude about the distribution of damages? (MP.4)</p>

598

599

600 Using Probability to Make Decisions

S-MD

601 **Use probability to evaluate outcomes of decisions.** [Introductory; apply counting rules.]

- 602 6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number
 603 generator).★
- 604 7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical
 605 testing, pulling a hockey goalie at the end of a game).★

606

607

608 Standards S-MD.6 and S-MD.7 involve students' use of probability models and
609 probability experiments to make decisions. These standards set the stage for more
610 advanced work in Algebra II, i.e. where the ideas of statistical inference are introduced.
611 See the "High School Progression on Statistics and Probability" for more explanation
612 and examples: <http://ime.math.arizona.edu/progressions/>.
613

614

615 **Geometry Overview**

616

617 **Geometry**

618

619 **Congruence**

- 620 • Experiment with transformations in the plane.
- 621 • Understand congruence in terms of rigid motions.
- 622 • Prove geometric theorems.
- 623 • Make geometric constructions.

624

625 **Similarity, Right Triangles, and Trigonometry**

- 626 • Understand similarity in terms of similarity transformations.
- 627 • Prove theorems involving similarity.
- 628 • Define trigonometric ratios and solve problems involving right triangles.
- 629
- 630 • Apply trigonometry to general triangles.

631

632 **Circles**

- 633 • Understand and apply theorems about circles.
- 634 • Find arc lengths and areas of sectors of circles.

635

636 **Expressing Geometric Properties with Equations**

- 637 • Translate between the geometric description and the equation for a conic section.
- 638 • Use coordinates to prove simple geometric theorems algebraically.

639

640 **Geometric Measurement and Dimension**

- 641 • Explain volume formulas and use them to solve problems.
- 642 • Visualize relationships between two-dimensional and three-dimensional objects.

643

644 **Modeling with Geometry**

- 645 • Apply geometric concepts in modeling situations.

646

647 **Statistics and Probability**

648

649 **Conditional Probability and the Rules of Probability****Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- 650 • Understand independence and conditional probability and use them to interpret data.
- 651 • Use the rules of probability to compute probabilities of compound events in a uniform
652 probability model.

653 **Using Probability to Make Decisions**

- 654 • Use probability to evaluate outcomes of decisions.

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690 ★ Indicates a modeling standard linking mathematics to everyday life, work, and decision-
691 making

692 (+) Indicates additional mathematics to prepare students for advanced courses.

693

694 Geometry

695

696 **Geometry**697 **Congruence****G-CO**698 **Experiment with transformations in the plane.**

- 699 1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based
700 on the undefined notions of point, line, distance along a line, and distance around a circular arc.
- 701 2. Represent transformations in the plane using, e.g., transparencies and geometry software;
702 describe transformations as functions that take points in the plane as inputs and give other points
703 as outputs. Compare transformations that preserve distance and angle to those that do not (e.g.,
704 translation versus horizontal stretch).
- 705 3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and
706 reflections that carry it onto itself.
- 707 4. Develop definitions of rotations, reflections, and translations in terms of angles, circles,
708 perpendicular lines, parallel lines, and line segments.
- 709 5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure
710 using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of
711 transformations that will carry a given figure onto another.
- 712

713 **Understand congruence in terms of rigid motions.** [Build on rigid motions as a familiar starting point for
714 development of concept of geometric proof.]

- 715 6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a
716 given rigid motion on a given figure; given two figures, use the definition of congruence in terms
717 of rigid motions to decide if they are congruent.
- 718 7. Use the definition of congruence in terms of rigid motions to show that two triangles are
719 congruent if and only if corresponding pairs of sides and corresponding pairs of angles are
720 congruent.
- 721 8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of
722 congruence in terms of rigid motions.
- 723

724 **Prove geometric theorems.** [Focus on validity of underlying reasoning while using variety of ways of writing proofs.]

- 725 9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a*
726 *transversal crosses parallel lines, alternate interior angles are congruent and corresponding*
727 *angles are congruent; points on a perpendicular bisector of a line segment are exactly those*
728 *equidistant from the segment's endpoints.*
- 729 10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum*
730 *to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two*
731 *sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at*
732 *a point.*
- 733 11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite*
734 *angles are congruent, the diagonals of a parallelogram bisect each other, and conversely,*
735 *rectangles are parallelograms with congruent diagonals.*
- 736

737 **Make geometric constructions.** [Formalize and explain processes.]

- 738 12. Make formal geometric constructions with a variety of tools and methods (compass and
739 straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying*
740 *a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular*

741 *lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a*
 742 *given line through a point not on the line.*

743 13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

744
 745

746 **Similarity, Right Triangles, and Trigonometry**

G-SRT

747 **Understand similarity in terms of similarity transformations.**

- 748 1. Verify experimentally the properties of dilations given by a center and a scale factor:
 749 a. A dilation takes a line not passing through the center of the dilation to a parallel line, and
 750 leaves a line passing through the center unchanged.
 751 b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
 752 2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if
 753 they are similar; explain using similarity transformations the meaning of similarity for triangles as
 754 the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs
 755 of sides.
 756 3. Use the properties of similarity transformations to establish the Angle-Angle (AA) criterion for two
 757 triangles to be similar.

758

759 **Prove theorems involving similarity.**

- 760 4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides*
 761 *the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle*
 762 *similarity.*
 763 5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in
 764 geometric figures.

765

766 **Define trigonometric ratios and solve problems involving right triangles.**

- 767 6. Understand that by similarity, side ratios in right triangles are properties of the angles in the
 768 triangle, leading to definitions of trigonometric ratios for acute angles.
 769 7. Explain and use the relationship between the sine and cosine of complementary angles.
 770 8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied
 771 problems. ★
 772 8.1 **Derive and use the trigonometric ratios for special right triangles (30°, 60°, 90° and 45°, 45°, 90°). CA**
 773

774

775 **Apply trigonometry to general triangles.**

- 776 9. (+) Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from
 777 a vertex perpendicular to the opposite side.
 778 10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
 779 11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown
 780 measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

781

782

783 **Circles**

G-C

784 **Understand and apply theorems about circles.**

- 785 1. Prove that all circles are similar.
 786 2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the*
 787 *relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter*

- 788 *are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects*
 789 *the circle.*
 790 3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for
 791 a quadrilateral inscribed in a circle.
 792 4. (+) Construct a tangent line from a point outside a given circle to the circle.
 793

794 **Find arc lengths and areas of sectors of circles.** [Radian introduced only as unit of measure.]

- 795 5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to
 796 the radius, and define the radian measure of the angle as the constant of proportionality; derive
 797 the formula for the area of a sector. **Convert between degrees and radians. CA**
 798
 799

800 **Expressing Geometric Properties with Equations**

G-GPE

801 **Translate between the geometric description and the equation for a conic section.**

- 802 1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem;
 803 complete the square to find the center and radius of a circle given by an equation.
 804 2. Derive the equation of a parabola given a focus and directrix.
 805

806 **Use coordinates to prove simple geometric theorems algebraically.** [Include distance formula; relate to
 807 Pythagorean Theorem.]

- 808 4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or*
 809 *disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or*
 810 *disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the*
 811 *point $(0, 2)$.*
 812 5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric
 813 problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes
 814 through a given point).
 815 6. Find the point on a directed line segment between two given points that partitions the segment in
 816 a given ratio.
 817 7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g.,
 818 using the distance formula. ★
 819

820 **Geometric Measurement and Dimension**

G-GMD

821 **Explain volume formulas and use them to solve problems.**

- 822 1. Give an informal argument for the formulas for the circumference of a circle, area of a circle,
 823 volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and*
 824 *informal limit arguments.*
 825 3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★
 826

827 **Visualize relationships between two-dimensional and three-dimensional objects.**

- 828 4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify
 829 three-dimensional objects generated by rotations of two-dimensional objects.
 830 5. **Know that the effect of a scale factor k greater than zero on length, area, and volume is to**
 831 **multiply each by k , k^2 , and k^3 , respectively; determine length, area and volume measures**
 832 **using scale factors. CA**
 833 6. **Verify experimentally that in a triangle, angles opposite longer sides are larger, sides**
 834 **opposite larger angles are longer, and the sum of any two side lengths is greater than the**

835 remaining side length; apply these relationships to solve real-world and mathematical
836 problems. CA

837

838 **Modeling with Geometry****G-MG**839 **Apply geometric concepts in modeling situations.**

- 840 1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a
841 tree trunk or a human torso as a cylinder). ★
- 842 2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per
843 square mile, BTUs per cubic foot). ★
- 844 3. Apply geometric methods to solve design problems (e.g., designing an object or structure to
845 satisfy physical constraints or minimize cost; working with typographic grid systems based on
846 ratios). ★

847

848 **Statistics and Probability**849 **Conditional Probability and the Rules of Probability****S-CP**850 **Understand independence and conditional probability and use them to interpret data.** [Link to data
851 from simulations or experiments.]

- 852 1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or
853 categories) of the outcomes, or as unions, intersections, or complements of other events (“or,”
854 “and,” “not”). ★
- 855 2. Understand that two events A and B are independent if the probability of A and B occurring
856 together is the product of their probabilities, and use this characterization to determine if they are
857 independent. ★
- 858 3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret
859 independence of A and B as saying that the conditional probability of A given B is the same as
860 the probability of A , and the conditional probability of B given A is the same as the probability
861 of B . ★
- 862 4. Construct and interpret two-way frequency tables of data when two categories are associated
863 with each object being classified. Use the two-way table as a sample space to decide if events
864 are independent and to approximate conditional probabilities. *For example, collect data from a
865 random sample of students in your school on their favorite subject among math, science, and
866 English. Estimate the probability that a randomly selected student from your school will favor
867 science given that the student is in tenth grade. Do the same for other subjects and compare the
868 results.* ★
- 869 5. Recognize and explain the concepts of conditional probability and independence in everyday
870 language and everyday situations. ★

871

872 **Use the rules of probability to compute probabilities of compound events in a uniform probability
873 model.**

- 874 6. Find the conditional probability of A given B as the fraction of B 's outcomes that also belong to A ,
875 and interpret the answer in terms of the model. ★
- 876 7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms
877 of the model. ★
- 878 8. (+) Apply the general Multiplication Rule in a uniform probability model,
879 $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. ★
- 880 9. (+) Use permutations and combinations to compute probabilities of compound events and solve
881 problems. ★

882

883

884 Using Probability to Make Decisions**S-MD****885 Use probability to evaluate outcomes of decisions.** [Introductory; apply counting rules.]

- 886 6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number
887 generator).★
- 888 7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical
889 testing, pulling a hockey goalie at the end of a game).★
- 890 •