Algebra I

Introduction

The following Traditional and Integrated Pathways lay out two sequences of courses that include the "College and Career Readiness" standards of the higher mathematics CA CCSSM. These are the standards deemed appropriate for all students to learn in preparation for pursuing a college education or a career in a technology-infused 21st Century. Most of the additional "plus" (+) standards are treated in courses beyond these pathways and are intended to be learned by those students who wish to pursue higher-level mathematics courses or college majors in a STEM field.

These pathways and the two suggested "fourth courses" presented in the framework do not represent the only courses available for high school students. The higher mathematics CA CCSSM are designed in part as a menu of standards from which educators can create customized courses. Thus, for example, a school or district may opt to create a mathematics course based on certain CA CCSSM and the Career Technical Education Model Curriculum Standards, or they may opt to create a Mathematical Modeling course using starred (★) standards (see the Appendix, "Mathematical Modeling," for more information). Overall, schools and districts have a choice in their course offerings at the high school level and are welcome to take advantage of the focus on real-world applications of the higher mathematics standards and the overall flexibility of the CA CCSSM.

The main purpose of Algebra I is to develop students' fluency with linear, quadratic and exponential functions. The critical areas of instruction involve deepening and extending students' understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. In addition, students engage in methods for analyzing, solving, and using exponential and quadratic functions. Some of the overarching ideas in the Algebra I course include: the notion of function, solving equations, rates of change and growth patterns, graphs as representations of functions, and modeling.

The standards in the Traditional Pathway Algebra I course come from the following conceptual categories: Modeling, Functions, Number and Quantity, Algebra, and Statistics and Probability. The content of the course will be expounded on below according to these conceptual categories, but teachers and administrators alike should note that the standards are not listed here in the order in which they should be taught. Moreover, the standards are not simply topics to be checked off a list during isolated units of instruction, but rather content that should be present throughout the school year through rich instructional experiences.

What Students learn in Algebra I

Overview

In Algebra I, students use reasoning about structure to define and make sense of rational exponents and explore the algebraic structure of the rational and real number systems. They understand that numbers in real world applications often have units attached to them, that is, they are considered *quantities*. Students' work with numbers and operations throughout elementary and middle school has led them to an understanding of the structure of the number system; now, students explore the structure of algebraic expressions and polynomials. They see that certain properties must persist when working with expressions that are meant to represent numbers, now written in an abstract form involving variables. When two expressions with overlapping domains are set equal to each other, resulting in an equation, there is an implied solution set (be it empty or non-empty), and students not only refine their techniques for solving equations and finding the solution set, but they can clearly explain the algebraic steps they used to do so.

Students began their exploration of linear equations in middle school. They first connected proportional equations ($y = kx, k \neq 0$) to graphs, tables and real-world contexts, and then moved towards an understanding of general linear equations ($y = mx + b, m \neq 0$) and their graphs. In Algebra I, students extend this knowledge to working with absolute value equations, linear inequalities, and systems of linear equations. After learning a more precise definition of function in this course, students examine this new idea in the familiar context of linear equations (for example, by seeing the solution of a linear equation as solving $f(x) = g(x)$ for two linear functions f and g).

Students continue building their understanding of functions beyond linear ones by investigating tables, graphs, and equations that build on previous understandings of numbers and expressions. They make connections between different representations of the same function. They learn to build functions in a modeling context, and solve problems related to the resulting functions. Note that the focus in Algebra I is on linear, simple exponential, and quadratic equations.

Finally, students extend their prior experiences with data, using more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Examples of Key Advances from Grades K–8

- Having already extended arithmetic from whole numbers to fractions (grades 4–6) and from fractions to rational numbers (grade 7), students in grade 8 encountered particular irrational numbers such as $\sqrt{5}$ and π. In Algebra I, students will begin to understand the real number *system*. (For more on the extension of number systems, see page 58 of the CCSSI document, *Common Core State Standards for Mathematics*).
- Students in middle grades worked with measurement units, including units obtained by multiplying and dividing quantities. In Algebra I, students apply these skills in a more sophisticated fashion to solve problems in which reasoning about units adds insight (N-Q).

- Algebraic themes beginning in middle school continue and deepen during high school. As early as grades 6 and 7, students began to use the properties of operations to generate equivalent expressions (6.EE.3, 7.EE.1). By grade 7, they began to recognize that rewriting expressions in different forms could be useful in problem solving (7.EE.2). In Algebra I, these aspects of algebra carry forward as students continue to use properties of operations to rewrite expressions, gaining fluency and engaging in what has been called "mindful manipulation."
- Students in grade 8 extended their prior understanding of proportional relationships to begin working with functions, with an emphasis on linear functions. In Algebra I, students will master linear and quadratic functions. Students encounter other kinds of functions to ensure that general principles are perceived in generality, as well as to enrich the range of quantitative relationships considered in problems.
- Students in grade 8 connected their knowledge about proportional relationships, lines and linear equations (8.EE.5, 6). In Algebra I, students solidify their understanding of the analytic geometry of lines. They understand that in the Cartesian coordinate plane:
    - The graph of any linear equation in two variables is a line.
    - Any line is the graph of a linear equation in two variables.
- As students acquire mathematical tools from their study of algebra and functions, they apply these tools in statistical contexts (e.g., S-ID.6). In a modeling context, they might informally fit a quadratic function to a set of data, graphing the data and the model function on the same coordinate axes. They also draw on skills

they first learned in middle school to apply basic statistics and simple probability in a modeling context. For example, they might estimate a measure of center or variation and use it as an input for a rough calculation.

- Algebra I techniques open a huge variety of word problems that can be solved that were previously inaccessible or very complex in grades K–8. This expands problem solving from grades K–8 dramatically.

Connecting Standards for Mathematical Practice and Content

The Standards for Mathematical Practice apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, relevant, and meaningful subject that makes use of their ability to make sense of problem situations. The Standards for Mathematical Practice (MP) represent a picture of what it looks like for students to *do mathematics* and, to the extent possible, content instruction should include attention to appropriate practice standards. There are ample opportunities for students to engage in each mathematical practice in Algebra I; the table below offers some general examples.

Standards for Mathematical Practice *Students…*	Examples of each practice in Algebra I
MP1. Make sense of problems and persevere in solving them.	Students learn that often patience is required to fully understand what a problem is asking. They discern between what information is useful, and what is not. They expand their repertoire of expressions and functions that can used to solve problems.
MP2. Reason abstractly and quantitatively.	Students extend their understanding of slope as the rate of change of a linear function to understanding that the average rate of change of any function can be computed over an appropriate interval.
MP3. Construct viable arguments and critique the reasoning of others.	Students reason through the solving of equations, recognizing that solving an equation is more than simply a matter of rote rules and steps. They use language such as "if… then..." when explaining their solution methods and

Students build proofs by induction and proofs by contradiction. CA 3.1 *(for higher mathematics only).*	provide justification.
MP4. Model with mathematics.	Students also discover mathematics through experimentation and examining patterns in data from real world contexts. Students apply their new mathematical understanding of exponential, linear and quadratic functions to real-world problems.
MP5. Use appropriate tools strategically.	Students develop a general understanding of the graph of an equation or function as a representation of that object, and they use tools such as graphing calculators or graphing software to create graphs in more complex examples, understanding how to interpret the result. They construct diagrams to solve problems.
MP6. Attend to precision.	Students begin to understand that a *rational number* has a specific definition, and that *irrational numbers* exist. They make use of the definition of *function* when deciding if an equation can describe a function by asking, "Does every input value have exactly one output value?"
MP7. Look for and make use of structure.	Students develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Students see that the expression $5 + (x - 2)^2$ takes the form of "5 plus 'something' squared", and so that expression can be no smaller than 5.
MP8. Look for and express regularity in repeating reasoning.	Students see that the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and that the result of evaluating the expression $\frac{y_2 - y_1}{x_2 - x_1}$ for points on the line is always equal to a certain number m. Therefore, if (x, y) is a generic point on this line, the equation $m = \frac{y - y_1}{x - x_1}$ will give a general equation of that line.

MP 4 holds a special place throughout the higher mathematics curriculum, as Modeling is considered its own conceptual category. Though the Modeling category has no specific standards listed within it, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a high place in instruction. Readers will see some standards marked with a star symbol (★) to indicate that they are *modeling standards*, that is, they present an opportunity for applications to real-world modeling situations more so than other standards. In the description of the Algebra I Content Standards below, Modeling comes first to emphasize its important in the higher mathematics curriculum.

Examples of places where specific Mathematical Practice standards can be implemented in the Algebra I standards will be noted in parentheses, with the standard(s) indicated.

Algebra I Content Standards by Conceptual Category

The Algebra I course is organized by conceptual category, domains, clusters, and then standards. Below, the overall purpose and progression of the standards included in Algebra I are described according to these conceptual categories. Standards that are considered to be new to secondary grades teachers will be discussed in more depth than others.

Conceptual Category: Modeling

Throughout the higher mathematics CA CCSSM standards, certain standards are marked with a (★) symbol to indicate that they are considered modeling standards. Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics to real-world problems. True modeling begins with students asking a question about the world around them, and mathematics is then constructed in the process of attempting to answer the question. When students are presented with a real world situation and challenged to ask a question, all sorts of new issues arise: which of the quantities present in this situation are known and unknown? Can I make a table of data? Is there a functional relationship in this situation? Students need to decide on a solution path, which may need to be revised. They make use of tools such as calculators, dynamic geometry software, or spreadsheets. They try to use previously derived models (e.g. linear functions) but may find that a new equation or function will apply. They may see that solving an equation arises as a necessity when trying to answer their question, and that oftentimes the equation arises as the specific instance of knowing the output value of a function at an unknown input value.

Modeling problems have an element of being genuine problems, in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. This will be a new approach for many teachers and will be challenging to implement, but the effort will produce students who can appreciate that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners.

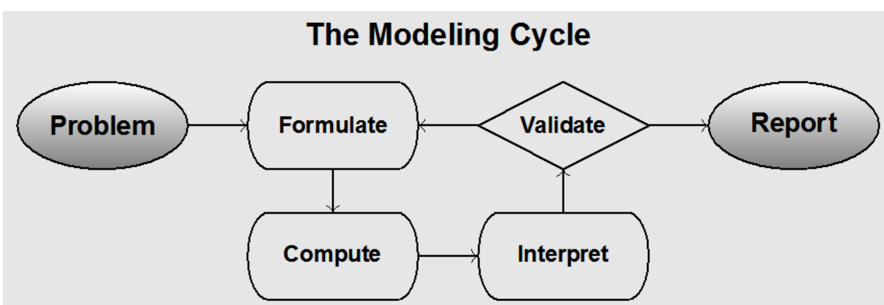


Figure 1: The modeling cycle. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, etc.), compute an answer or rewrite their expression to reveal new information, interpret their results, validate them, and report out.

Throughout the Algebra I chapter, the included examples will be framed as much as possible as modeling situations, to serve as illustrations of the concept of mathematical modeling. The big ideas of linear and exponential functions, graphing, solving equations, and rates of change will be explored through this lens. The reader is encouraged to consult the Appendix on Mathematical Modeling for a further discussion of the modeling cycle and how it is integrated into the higher mathematics curriculum.

Conceptual Category: Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, *v*; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is *T*.

The set of inputs to a function is called its domain. We often assume the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. When describing relationships between quantities, the defining characteristic of a *function* is that the input value determines the output value, or equivalently, that the output value depends upon the input value.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an assignment, such as the fact that each individual is given a unique Social Security

Number; by an algebraic expression like $f(x) = a + bx$; or by a recursive rule, such as $f(n+1) = f(n) + b, f(0) = a$. The graph of a function is often a useful way of visualizing the relationship that the function models, and manipulating a mathematical expression for a function can shed light on the function's properties.

Interpreting Functions F-IF

Understand the concept of a function and use function notation. [Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences]

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context. [Linear, exponential, and quadratic.]

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

While the grade eight standards call for students to work informally with functions, in Algebra I they begin to refine their understanding and use the formal mathematical language of functions. Standards F-IF.1-9 deal with understanding the concept of a function, interpreting characteristics of functions in context, and representing functions in different ways (MP.6). In F.IF.1-3, students learn the language of functions and that a function has a domain that must be specified and a corresponding range. For instance, the function f where $f(n) = 4(n-2)^2$, defined for n an integer, is a different function than the function g where $g(x) = 4(x-2)^2$ and g is defined for all real numbers x. Students make the connection between the graph of the equation $y = f(x)$ and the function itself, namely, that the coordinates of any point on the graph represent an input and output, expressed as $(x, f(x))$, and understand that the graph is a *representation* of a function. They connect the domain and range of a function to its graph (F.IF.5). Note that there is neither an exploration of the notion of *relation vs. function* nor the *vertical line test* in the CA CCSSM. This is by design. The core question when investigating functions is: “Does each element of the domain correspond to exactly one element of the range?”

Standard F.IF.3 represents a topic new to the traditional Algebra I course, that of *sequences*. Sequences are functions with a domain consisting of a subset of the integers. Back in grades four and five, students began to explore number patterns, and this work led into a full progression of ratios and proportional relationships in grades six and seven. Patterns are examples of sequences, and the work here is intended to formalize and extend students’ earlier understandings. For a simple example consider the sequence $4, 7, 10, 13, 16, \ldots$ which might be described as a “plus 3 pattern” because terms are computed by adding 3 to the previous term. If we decided that 4 is the first term of the sequence then we can make a table, a graph, and eventually a recursive rule for this sequence: $f(1) = 4, f(n+1) = f(n) + 3$ for $n \geq 2$. Of course, this sequence can also be described with explicit formula $f(n) = 3n + 1$ for $n \geq 1$. Notice that the domain is included in the description of the rule. In Algebra I, students should have opportunities to work with linear, quadratic and exponential sequences, and to interpret the parameters of the expressions defining the terms of the sequence when they arise in context.

Interpreting Functions **F-IF**

Analyze functions using different representations. [Linear, exponential, quadratic, absolute value, step, piecewise-defined.]

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

In standards F-IF.7-9, students represent functions with graphs and identify key features in the graph. In Algebra I, linear, exponential and quadratic functions are given extensive treatment since they themselves have their own group of standards (the F-LE standards) dedicated to them. Students are expected to develop fluency with only linear, exponential and quadratic functions in Algebra I, which includes the ability to graph them by hand.

In this set of three standards, students represent the same function algebraically in different forms and interpret these differences in terms of the graph or context. For instance, students may easily see that the graph of the equation $f(x) = 3x^2 + 9x + 6$ crosses the y-axis at $(0,6)$ since the terms with x in them are simply 0 when $x = 0$, but they then factor the expression defining f to obtain $f(x) = 3(x+2)(x+1)$, easily revealing that the function crosses the x-axis at $(-2, 0)$ and $(-1, 0)$, since this is where $f(x) = 0$ (MP.7).

Building Functions **F-BF**

Build a function that models a relationship between two quantities. [For F.BF.1, 2, linear, exponential, and quadratic.]

1. Write a function that describes a relationship between two quantities. ★
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ★
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

Build new functions from existing functions. [Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only.]

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Knowledge of functions and expressions is only part of the complete picture. One must be able to understand a given situation and apply function reasoning to model how quantities change together. Often, the function created sheds light on the situation at hand; one can make predictions of future changes, for example. This is the content of standards F-BF.1 and F-BF.2 (starred to indicate they are Modeling standards). A strong connection exists between standard F.BF.1 and standard A.CDE.2, which discusses creating equations. The following example shows that students can create functions based on prototypical ones.

Exponential Growth. When a quantity grows with time by a multiplicative factor greater than 1, it is said the quantity grows exponentially. Hence, if an initial population of bacteria, P_0, doubles each day, then after t days, the new population is given by

$$P(t) = P_0 2^t.$$

This expression can be generalized to include different growth rates, r, as in $P(t) = P_0 r^t$. The following example illustrates the type of problem that students can face after they have worked with basic exponential functions like these.

Example. On June 1, a fast growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area covered by the algae doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

Possible Questions to Ask:

a. When will the lake be covered halfway?

b. Write an equation that represents the percentage of the surface area of the lake that is covered in algae as a function of time (in days) that passes since the algae was introduced into the lake.

Solution and Comment.

a. Since the population doubles each day, and since the entire lake is covered by June 30, this implies that half the lake was covered on June 29.

b. If $P(t)$ represents the *percentage* of the lake covered by the algae, then we know that $P(29) = P_0 2^{29} = 100$ (note that June 30 corresponds to $t = 29$). Therefore, we can solve for the initial percentage of the lake covered, $P_0 = \frac{100}{2^{29}} \approx 1.86 \times 10^{-7}$. The equation for the percentage of the lake covered by algae at time t is therefore $P(t) = (1.86 \times 10^{-7})2^t$.

As mentioned earlier, new to the Algebra I course in California is the study of arithmetic and geometric sequences, written both explicitly and recursively (F-BF.2). Often when presented with a sequence, students can manage to find the recursive pattern of the sequence, i.e. how the sequence changes from term to term. For instance, a simple doubling pattern can lead to an exponential expression of the form $a2^n$, for $n \geq 0$. Ample experience with linear and exponential functions—which show equal differences over equal intervals and equal ratios over equal intervals, respectively—can provide students with tools for finding explicit rules for sequences. Investigating the simple sequence of squares, $f(n) = n^2$, $n \geq 1$ provides a prototype for other basic quadratic sequences. Diagrams, tables, and graphs can help students make sense of the different rates of growth all three sequences exhibit.

Example: *Cellular Growth*. Populations of cyanobacteria can double every 6 hours under ideal conditions, at least until the nutrients in its supporting culture are depleted. This means a population of 500 such bacteria would grow to 1000 in the first 6-hour period, 2000 in the second 6-hour period, 4000 in the third 6-hour period, etc. Evidently, if n represents the number of 6-hour periods from the start, the population at that time $P(n)$ satisfies $P(n) = 2 \cdot P(n-1)$. This is a *recursive* formula for the sequence $P(n)$, which gives the population at a given time period n in terms of the population at time period $n-1$. To find a closed, *explicit*, formula for $P(n)$, students can reason that

$$P(0) = 500, P(1) = 2 \cdot 500, P(2) = 2 \cdot 2 \cdot 500, P(3) = 2 \cdot 2 \cdot 2 \cdot 500, \ldots$$

A pattern emerges, that $P(n) = 2^n \cdot 500$. In general, if an initial population P_0 grows by a factor $r > 1$ over a fixed time period, then the population after n time periods is given by $P(n) = P_0 r^n$.

The content of standard F-BF.3 has typically been left to later courses. In Algebra I, the focus is on linear, exponential and quadratic functions. Even and odd functions are addressed in later courses. In keeping with the theme of the input-output interpretation of a function, students should work towards developing an understanding of the effect on the output of a function under certain transformations, such as in the table below:

Expression	Interpretation
$f(a+2)$	The output when the input is 2 greater than a
$f(a)+3$	3 more than the output when the input is a
$2f(x)+5$	5 more than twice the output of f when the input is x

Such understandings can help in seeing the effect of transformations on the graph of a function, and in particular, can aid in understanding why it appears that the effect on the graph is the opposite to the transformation on the variable (e.g. the graph of $y = f(x+2)$ is the graph of f shifted 2 units to the left, not to the right).

Also new to the Algebra I course is standard F-BF.4, in which students find inverse functions in simple cases. For example, an Algebra I student might solve the equation $C = \frac{9}{5}F + 32$ for F. The student starts with this formula, showing how Celsius temperature is a function of Fahrenheit temperature, and by solving for F finds the formula for the inverse function. This is a contextually appropriate way to find the expression for an inverse function, in contrast with the practice of simply swapping x and y in an equation and solving for y.

Linear, Quadratic, and Exponential Models **F-LE**

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ★
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context. ★ [Linear and exponential of form $f(x)=b^x+k$.]

6. **Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity. CA ★**

Modeling the world often involves investigating rates of change, patterns of growth. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. In standards F-LE.1a-c, students recognize and understand the defining characteristics of linear, quadratic, and exponential functions. Students have already worked extensively with linear equations. They have developed an understanding that an equation in two variables of the form $y = mx + b$ exhibits a special relationship between the variables x and y, namely, that a change of Δx in the variable x, the independent variable, results in a change of $\Delta y = m \cdot \Delta x$ in the dependent variable y. They have seen this informally, in graphs and tables of linear relationships, starting in the grade eight standards (8.EE.5, 8.EE.6, 8.F.3). For example, students recognize that for successive whole number input values, x and $x + 1$, a linear function f, defined by $f(x) = mx + b$, exhibits a constant rate of change:

$$f(x+1) - f(x) = [m(x+1) + b] - (mx + b) = m(x + 1 - x) = m.$$

Standard F-LE.1a requires students to prove that linear functions exhibit such growth patterns.

In contrast, an exponential function exhibits a constant percent change in the sense that such functions exhibit a constant *ratio* between output values for successive input values.[1] For instance, a t-table for the equation $y = 3^x$, illustrates the constant ratio of successive y-values for this equation:

x	$y = 3^x$	Ratio of successive y-values
1	3	
2	9	9/3=3
3	27	27/9=3
4	81	81/27=3

[1] Note: In CA CCSSM Algebra I, only integer values for x are considered in exponential equations such as $y = b^x$.

In general, a function g, defined by $g(x) = ab^x$, can be shown to exhibit this constant ratio growth pattern:

$$\frac{g(x+1)}{g(x)} = \frac{ab^{x+1}}{ab^x} = \frac{b^{x+1}}{b^x} = b^{(x+1)-x} = b.$$

In Algebra I, students are not required to prove that exponential functions exhibit this growth rate, however, they must be able to recognize situations that represent both linear and exponential functions and construct functions to describe the situations (F-LE.2). Finally, students interpret the *parameters* in linear, exponential and quadratic expressions, and model physical problems with such functions. When presented in a modeling situation, often the meaning of parameters becomes much clearer than when presented abstractly.

A graphing utility, spreadsheet, or computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions (MP.5). A real-world example where this can be explored involves investments, mortgages, half-lives of pharmaceuticals and other financial instruments. For example, students can develop formulas for annual compound interest based on a general formula, such as $P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt}$, where r is the interest rate, n is the number of times the interest is compounded per year, and t is the number of years the money is invested. They can explore values after different time periods and compare different rates and plans using computer algebra software or simple spreadsheets (MP.5). This hands-on experimentation with such functions helps develop an understanding of their behavior.

Conceptual Category: Number and Quantity

In the grade eight standards, students encountered some examples of irrational numbers, such as π and $\sqrt{2}$ (or $\sqrt{n}$ for n a nonsquare number). In Algebra I, students extend this understanding beyond the fact that there are numbers that are not rational; they begin to understand that the rational numbers form a closed system themselves. Students have witnessed that with each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—whole numbers, rational numbers, and real numbers— the distributive law continues to hold, and the commutative and associative laws are still valid for both addition and multiplication. However, in Algebra I students go further along this path.

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

With N-RN.1, students make meaning of the representation of radicals with rational exponents. Students are first introduced to exponents in grade six; by now, they should have an understanding of the basic properties of exponents (e.g. that $x^n \cdot x^m = x^{n+m}, (x^n)^m = x^{nm}, \frac{x^n}{x^m} = x^{n-m}, x^0 = 1 \text{ for } x \neq 0,$ etc.). In fact, they may have justified certain properties of exponents by reasoning with other properties (MP.3, MP.7), for example, justifying why any nonzero number to the power 0 is equal to 1:

$$x^0 = x^{n-n} = \frac{x^n}{x^n} = 1, \text{for } x \neq 0.$$

They further their understanding of exponents in Algebra I by using these properties to explain the meaning of rational exponents. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be the same as $5^{[(1/3)\cdot 3]} = 5^1 = 5$, so that $5^{1/3}$ should represent the cube root of 5. In addition, the fact that $(ab)^n = a^n \cdot b^n$ reveals that

$$\sqrt{20} = (4 \cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5},$$

showing that $\sqrt{20} = 2\sqrt{5}$. The intermediate steps of writing the square root as a rational exponent are necessary at first, but eventually students can work more quickly, understanding the reasoning underpinning this process. Students extend such work with radicals and rational exponents to variable expressions as well, e.g. rewriting an expression like $(a^2b^5)^{3/2}$ using radicals (N-RN.2).

In standard N-RN.3, students explain that the sum or product of two rational numbers is rational, by arguing that the sum of two fractions with integer numerator and denominator is also a fraction of the same type, showing that the rational numbers are *closed* under the operations of addition and multiplication (MP.3). The notion that this set of numbers is closed under these operations will be extended to the sets of polynomials and rational functions in later courses. Moreover, students argue that the sum of a rational and irrational is irrational, and the product of a non-zero rational and an irrational is still irrational, showing that the irrational numbers are truly another unique set of numbers that along with the rational numbers forms a larger *system*, the system of real numbers (MP3, MP7).

Quantities N-Q

Reason quantitatively and use units to solve problems. [Foundation for work with expressions, equations and functions.]

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
2. Define appropriate quantities for the purpose of descriptive modeling. ★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

In real-world problems, the answers are usually not pure numbers but *quantities*: numbers with units, which involves measurement. In their work in measurement up through grade eight, students primarily measure commonly used attributes such as length, area, and volume. In higher mathematics, students encounter a wider variety of units in modeling, e.g., when considering acceleration, currency conversions, derived quantities such as person-hours and heating degree-days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.

In Algebra I, students reason through problems with careful selection of units, and they use units to understand problems and make sense of the answers they deduce. The following example illustrates the facility with units that students are expected to attain in this domain.

Example N-Q: As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes, and she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70 mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon. a. Describe an estimate that Felicia might do in her head while driving to decide how many	**Solution:** a. To estimate the amount of gas she needs, Felicia calculates the distance traveled at 70 mph for 1.25 hours. She might calculate $70 \cdot 1.25 = 70 + 0.25 \cdot 70 = 70 + 17.5 = 87.5 \text{ miles}$. Since 1 gallon of gas will take her 30 miles, 3 gallons of gas will take her 90 miles, a little more than she needs. So she might figure that 3 gallons is enough. b. Since Felicia pays $3.50 for one gallon of gas, and one gallon of gas takes her 30 miles, it costs her $3.50 to travel 30 miles, therefore

gallons of gas she needs to make it to the gas station at the other end. b. Assuming she makes it, how much does Felicia spend per mile on the freeway?	$\frac{\$3.50}{30\text{ miles}} \approx \frac{\$0.12}{1\text{ mile}}$, meaning it costs Felicia 12 cents to travel each mile on the freeway.

Conceptual Category: Algebra

In the Algebra conceptual category, students extend the work with expressions they started in the middle grades standards. They create and solve equations in context, utilizing the power of variable expressions to model real-world problems and solve them with attention to units and the meaning of the answers they obtain. They continue to graph equations, understanding the resulting picture as a representation of the points satisfying the equation. This conceptual category comprises a large portion of the Algebra I course and along with the Functions category represents the main body of content.

The Algebra category in higher mathematics is very closely allied with the Functions category:

- An expression in one variable can be viewed as defining a function: the act of evaluating the expression is an act of producing the function's output given the input.
- An equation in two variables can sometimes be viewed as defining a function, if one of the variables is designated as the input variable and the other as the output variable, and if there is just one output for each input. This is the case if the expression is of the form $y = (\text{expression in } x)$ or if it can be put into that form by solving for y.

- The notion of equivalent expressions can be understood in terms of functions: if two expressions are equivalent they define the same function.
- The solutions to an equation in one variable can be understood as the input values that yield the same output in the two functions defined by the expressions on each side of the equation. This insight allows for the method of finding approximate solutions by graphing functions defined by each side and finding the points where the graphs intersect.

Thus, in light of understanding functions, the main content of the Algebra category, solving equations, working with expressions, etc., has a very important purpose.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions. [Linear, exponential, and quadratic.]

1. Interpret expressions that represent a quantity in terms of its context. ★
   a. Interpret parts of an expression, such as terms, factors, and coefficients. ★
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.* ★
2. Use the structure of an expression to identify ways to rewrite it.

An expression can be viewed as a recipe for a calculation, with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor. Students began this work in grades six and seven, and continue this work with more complex expressions.

Write expressions in equivalent forms to solve problems. [Quadratic and exponential.]

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
   a. Factor a quadratic expression to reveal the zeros of the function it defines. ★
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ★
   c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*★

In Algebra I, students work with examples of more complicated expressions, such as those that involve multiple variables and exponents. Students use the distributive property to investigate equivalent forms of quadratic expressions, e.g. by writing $(x+y)(x-y) = x(x-y) + y(x-y) = x^2 - xy + xy - y^2 = x^2 - y^2$, thereby yielding a special case of a factorable quadratic, the difference of squares. Students factor second-degree polynomials by making use of such special forms, and by using factoring techniques based on properties of operations (A-SSE.2, A-SSE.2.1). Note that the standards avoid talking about “simplification,” because it is often not clear what the simplest form of an expression is, and even in cases where it is clear, it is not obvious that the simplest form is desirable for a given purpose. The standards emphasize purposeful transformation of expressions into equivalent forms that are suitable for the purpose at hand, as the example below shows.

Example: *Which is the simpler form?* A particularly rich mathematical investigation involves finding a general expression for the sum of the first n consecutive natural numbers:

$$S = 1 + 2 + 3 + \cdots + (n-2) + (n-1) + n.$$

A famous tale speaks of a young C.F. Gauss being able to add the first 100 natural numbers quickly in his head, wowing his classmates and teachers alike. One way to find this sum is to consider the "reverse" of the sum:

$$S = n + (n-1) + (n-2) + \cdots + 3 + 2 + 1,$$

and to then add the two expressions for S together, obtaining:

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1),$$

where there are n terms of the form $(n+1)$. Thus, $2S = n(n+1)$, so that $S = n(n+1)/2$.

While students may be tempted to transform this expression into $\frac{1}{2}n^2 + \frac{1}{2}n$, they are obscuring the ease with which they can evaluate the first expression. Indeed, since n is a natural number, one of either n or $n+1$ is even, so evaluating $n(n+1)/2$, especially mentally, is often easier. Indeed, in Gauss' case, $\frac{100(101)}{2} = 50(101) = 5050$.

Students also use different forms of the same expression to reveal important characteristics of the expression. For instance, when working with quadratics, they complete the square in the expression $x^2 - 3x + 4$ to obtain the equivalent expression $\left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$. Students can then reason with the new expression that the term being squared is always greater than or equal to 0; hence, the value of the expression will always be greater than or equal to $\frac{7}{4}$ (A-SSE.3, MP.3). A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave, further contributing to students' understanding of work with expressions (MP.5).

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials. [Linear and quadratic.]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

In Algebra I, students begin to explore the set of polynomials in x as a system in its own right, subject to certain operations and properties. To perform operations with polynomials meaningfully, students are encouraged to draw parallels between the set of integers, wherein integers can be added, subtracted, and multiplied according to certain properties, and the set of all polynomials with real coefficients (A-APR.1, MP.7). If the function concept is developed before or concurrently with the study of polynomials, then a polynomial can be identified with the function it defines. In this way, $x^2 - 2x - 3$, $(x+1)(x-3)$, and $(x-1)^2 - 4$ are all the same polynomial because they all define the same function.

In Algebra I, students are only required to add linear or quadratic polynomials and to multiply linear polynomials to obtain quadratic ones, since in later courses they will explore polynomials of higher degree. Students fluently add, subtract, and multiply linear expressions of the form $ax + b$, and add and subtract expressions of the form $ax^2 + bx + c$, with a, b and c real numbers, understanding that the result is yet another expression of one of these forms. The explicit notion of *closure* of the set of polynomials need not be explored in Algebra I.

Understanding addition and subtraction of polynomials and the multiplication of monomials and binomials can be supported using manipulatives, such as “algebra tiles,” which can offer a concrete representation of the terms in a polynomial (MP.5). The tile representation relies on the *area interpretation of multiplication*: the notion that the product ab can be thought of as the area of

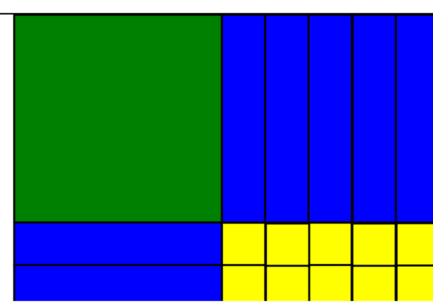


Figure 2: The rectangle above has height $(x+3)$ and base $(x+5)$. The total area represented, the product of these binomials, is seen to be $\mathbf{x^2 + 5x + 3x + 15 = x^2 + 8x + 15}$.

a rectangle of dimensions a units and b units. With this understanding, tiles can be used to represent 1 square unit (a 1×1 tile), x square units (a $1 \times x$ tile), and x^2 square units (an $x \times x$ tile). Finding the product $(x+5)(x+3)$ amounts to finding the area of an abstract rectangle of dimensions $x+5$ and $x+3$ as illustrated in the figure (MP.2).	Care must be taken in the way negative numbers are handled with this representation. The tile representation of polynomials is also very useful for understanding the notion of completing the square, as described below.

Creating Equations A-CED

Create equations that describe numbers or relationships. [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only.]

1. Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **CA**★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* ★

An equation is a statement of equality between two expressions. The values that make the equation true are the solutions to the equation. An identity, in contrast, is true for all values of the variables; rewriting an expression in an equivalent form often creates identities. The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. In this set of standards, students create equations to solve problems, they correctly graph the equations on coordinate axes, and they interpret solutions in a modeling context. The example below requires students to understand the multiple variables that appear in a given equation and to reason with them.

Example: The height h of a ball at time t seconds thrown vertically upward at a speed of v ft/sec is given by the equation $h = 6 + vt - 16t^2$ Write an equation whose solution is a. The time it takes a ball thrown at a speed of 88 ft/sec to rise 20 feet. b. The speed with which the ball must be thrown to rise 20 feet in 2 seconds.	**Solution and Comments:** a. We want $h = 20$, and we are told that $v = 88$, so the equation is $20 = 6 + 88t - 16t^2$. b. We want $h = 20$, and we are told that $t = 2$, so the equation is $20 = 6 + 2v - 16 \cdot 2^2$. While this is a straightforward example, students must be able to flexibly see some of the variables in the equation as constants when others are given values. It also does not explicitly state $v = 88$ or $t = 2$, so that students must understand the meaning of the variables in order to proceed with the problem (MP.1).

One change in the CA CCSSM is the creation of equations involving absolute values (A-CED.1). The basic absolute value function has at least two very useful definitions, a descriptive, verbal definition, and a formula definition. A common definition of the absolute value of x is:

$|x| =$ the distance from the number x to 0 (on a number line).

An understanding of the number line easily yields that, for example, $|0| = 0$, $|7| = 7$, and $|-3.9| = 3.9$. However, an equally valid "formula" definition of absolute value reads:

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

In other words, $|x|$ is simply x whenever x is 0 or positive, but $|x|$ is the opposite of x whenever x is negative. Either definition can be extended to an understanding of the expression $|x - a|$ as the distance between x and a on a number line, an interpretation that has many uses. For a simple application of this idea, suppose a certain bolt is to be mass-produced in a factory with the specification that its width should be 5mm with an error no larger than 0.01mm. If w represents the width of a given bolt produced on the production line, then we want w to satisfy the inequality $|w - 5| \leq 0.01$, i.e., the difference between the actual width w and the target width should be less than or equal to 0.01 (MP.4, MP.6). Students should become comfortable with the basic properties of absolute values (e.g. $|x| + a \neq |x + a|$ in general etc.) and with solving absolute value equations and interpreting the solution.

In higher math courses intervals on the number line are often denoted by an inequality of the form $|x - a| \leq d$ for a positive number d. For example $|x - 2| \leq \frac{1}{2}$ represents the closed interval $1\frac{1}{2} \leq x \leq 2\frac{1}{2}$. This can be seen by interpreting $|x - 2| \leq \frac{1}{2}$ as "the distance from x to 2 is less than or equal to $\frac{1}{2}$" and deciding which numbers fit this description. On the other hand, in the case that $x - 2 < 0$, we would have $|x - 2| = -(x - 2) \leq \frac{1}{2}$, so that $x \geq 1\frac{1}{2}$. In the case that $x - 2 \geq 0$, we have $|x - 2| = x - 2 \leq \frac{1}{2}$ which means that $x \leq 2\frac{1}{2}$. Since we are looking for all x that satisfy both inequalities, the interval is $1\frac{1}{2} \leq x \leq 2\frac{1}{2}$. This shows how the formula definition can be used to find this interval.

Reasoning with Equations and Inequalities — A-REI

Understand solving equations as a process of reasoning and explain the reasoning. [Master linear; learn as general principle.]

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable. [Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions.]

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

3.1 Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context. CA

4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions. In Algebra I, students solve linear equations and inequalities in one variable, including ones with absolute values and equations with coefficients represented by letters (A-REI.3, A-REI.3.1). In addition, this is students' first exposure to quadratic equations, and they learn various techniques for solving them and the relationship between those techniques (A-REI.4.a-b). When solving equations, students make use of the symmetric and transitive properties, and certain properties of equality with regards to operations (e.g. "equals added to equals is equal"). Standard A-REI.1 requires that in any situation, students can solve an equation *and explain the steps* as resulting from previous true equations and using the aforementioned properties (MP.3). In this way, the idea of *proof*, while not explicitly named, is given a prominent role in the solving of equations, and the reasoning and justification process is not simply relegated to a future mathematics course.

On Solving Equations: A written sequence of steps is code for a narrative line of reasoning that would use words like "if", "then", "for all" and "there exists." In the process of learning to solve equations, students should learn certain "if-then" moves: e.g. "if $x = y$ then $x + c = y + c$ for any c." The danger in learning algebra is that students emerge with nothing but the moves, which may make it difficult to detect incorrect or made-up moves later on. Thus the first requirement in this domain (REI) is that students understand that solving equations is a process of reasoning (A.REI.1).	**Fragments of Reasoning** $x^2 = 4$ $x^2 - 4 = 0$ $(x-2)(x+2) = 0$ $x = 2, -2$ This sequence of equations is short-hand for a line of reasoning: "*If x is a number whose square is 4, then $x^2 - 4 = 0$, by properties of equality. Since $x^2 - 4 = (x-2)(x+2)$ for all numbers, it follows that $(x-2)(x+2) = 0$. So either $(x-2) = 0$, in which case $x = 2$, or $(x+2) = 0$, in which case $x = -2$.*"

Students in Algebra I extend their work with exponents to working with quadratic functions and equations that have real roots. To extend their understanding of these quadratic expressions and the functions they define, students investigate properties of quadratics and their graphs in the Functions domain.

Example: When solving quadratic equations of the form $(x-p)^2 = q$ in standard A-REI.4.a, students rely on the understanding that they can take square roots of both sides of the equation to obtain,

$$\sqrt{(x-p)^2} = \sqrt{q}. \qquad (1)$$

In the case that $\sqrt{q}$ is a real number, we can solve this equation for x. A common mistake is to quickly introduce the symbol ± here, without understanding where it comes from. Doing so without care often leads to students thinking that $\sqrt{9} = \pm 3$, for example.

Note that the quantity $\sqrt{a^2}$ is simply a when $a \geq 0$ (as in $\sqrt{5^2} = \sqrt{25} = 5$), while $\sqrt{a^2}$ is equal to $-a$ (the opposite of a) when $a < 0$ (as in $\sqrt{(-4)^2} = \sqrt{16} = 4$). But this means that $\sqrt{a^2} = |a|$. Applying this to equation (1) yields $|x-p| = \sqrt{q}$. Solving this simple absolute value equation yields that $x - p = \sqrt{q}$ or $-(x-p) = \sqrt{q}$. This results in the two solutions $p + \sqrt{q}, p - \sqrt{q}$.

Students also transform quadratic equations into the form $ax^2 + bx + c = 0$, for $a \neq 0$, the *standard form* of a quadratic equation. In some cases, the quadratic expression factors nicely and students can apply the zero product property of the real numbers to solve the resulting equation. The *zero product property* states that for two real numbers m and n, $m \cdot n = 0$ if and only if either $m = 0$ or $n = 0$. Hence, when a quadratic polynomial can be rewritten as $a(x-r)(x-s) = 0$, the solutions can be found by setting each of the linear factors equal to 0 separately, and obtaining the solution set $\{r, s\}$. In other cases, a means for solving a quadratic equation arises by *completing the square*. Assuming for simplicity that $a = 1$ in the standard equation above, and that the equation has been rewritten as $x^2 + bx = -c$, we can "complete the square" by adding the square of half the coefficient of the x-term to each side of the equation:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2. \qquad (2)$$

The result of this simple step is that the quadratic on the left side of the equation is a perfect square,

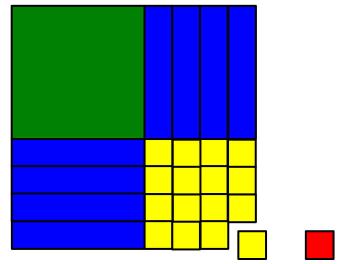
$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2.$$

Thus, we have now converted equation (2) into an equation of the form $(x - p)^2 = q$, namely,

$$\left(x + \frac{b}{2}\right)^2 = -c + \frac{b^2}{4},$$

which can be solved by the method described above, as long as the term on the right is positive. The case when $a \neq 1$ can be handled similarly and ultimately results in the familiar quadratic formula. Tile representations of quadratics illustrate that the process of completing the square has a geometric interpretation that explains the origin of the name. Students should be encouraged to explore these representations in order to make sense out of the process of completing the square (MP.1, MP.5). Completing the square is an example of a theme that reoccurs throughout algebra: finding ways of transforming equations into certain standard forms that have the same solutions.

Completing the Square: The method of completing the square is a useful skill in Algebra. It is generally used to change a quadratic in standard form, $ax^2 + bx + c$, into one in vertex-form, $a(x - h)^2 + k$. The vertex form can help determine several properties of quadratic functions.	subtract 1 to the quadratic expression: $y = x^2 + 8x + 15 + 1 - 1 = x^2 + 8x + 16 - 1$. Factoring gives us $y = (x + 4)^2 - 1$. In the picture, note that the tiles used to represent $x^2 + 8x + 15$ have been rearranged to try to form a square, and that a positive unit tile and a "negative"

Completing the square also has applications in Geometry (G-GPE.1) and later higher mathematics courses. **Example:** To complete the square for the quadratic $y = x^2 + 8x + 15$, we take half the coefficient of the x-term and square it to yield 16. We realize that we need only to add 1 and	unit tile are added into the picture to "complete the square." 

The same solution techniques used to solve equations can be used to rearrange formulas to highlight specific quantities and explore relationships between the variables involved. For example, the formula for the area of a trapezoid, $A = \left(\frac{b_1+b_2}{2}\right)h$, can be solved for *h* using the same deductive process (MP.7, MP.8). As will be discussed later, functional relationships can often be explored more deeply by rearranging equations that define such relationships; thus, the ability to work with equations with letters as coefficients is an important skill.

Reasoning with Equations and Inequalities **A-REI**

Solve systems of equations. [Linear-linear and linear-quadratic.]

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system. The process of adding one equation to another is understood as, "if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum (or difference) of the left sides of the two equations is equal to the sum (or difference) of the right sides." The reversibility of these steps justifies that we achieve an equivalent system of equations by doing this. This crucial point should be consistently noted when reasoning about solving systems of equations.

Example: *Solving simple systems of equations.* To get started with understanding how to solve systems of equations by linear combinations, students can be encouraged to interpret the system in terms of real-world quantities, at least in some cases. For instance, suppose one wanted to solve the system

$$3x + y = 40$$
$$4x + 2y = 58$$

Now consider the following scenario:

- Suppose 3 CD's and a magazine cost $40, while 4 CD's and 2 magazines cost $58.

And the questions

- What happens to the price when you add 1 CD and 1 magazine to your purchase?
- What is the price if you decided to buy only 2 CD's and no magazine?

Answering these questions amounts to realizing that since $(3x + y) + (x + y) = 40 + 18$, we must have that $x + y = 18$. Therefore, $(3x + y) + (-1)(x + y) = 40 + (-1)18$, which implies that $2x = 22$, or 1 CD costs $11.

The value of y can now be found using either of the original equations: $y = 7$.

When solving systems of equations, students also make frequent use substitution, e.g., when solving the system $2x - 9y = 5$ and $y = \frac{1}{3}x + 1$ by substituting the expression $\frac{1}{3}x + 1$ for y in the first equation, to obtain $2x - 9\left(\frac{1}{3}x + 1\right) = 5$. Students also solve such systems approximately, by using graphs and tables of values (A-REI.5-7).

Reasoning with Equations and Inequalities **A-REI**

Represent and solve equations and inequalities graphically. [Linear and exponential; learn as general principle.]

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

One of the most important goals of instruction in mathematics is to illuminate connections between various mathematical concepts. In particular, in standards A-REI.10-12, students learn the relationship between the algebraic representation of an equation and its graph plotted in the coordinate plane, and understand geometric interpretations of solutions to equations and inequalities. In Algebra I students work only with linear, exponential, quadratic, step, piecewise, and absolute value functions. As students become more comfortable with function notation, e.g. writing $f(x) = 3x - 2$ and $g(x) = x^2 - x + 2$, they begin to see solving the equation $3x - 2 = x^2 - x + 2$ as solving the equation $f(x) = g(x)$; that is, they are finding those x-values where two functions take on the same output value. Moreover, they graph the two equations, and see that the x-coordinate(s) of the point(s) of intersection of the graphs of $y = f(x)$ and $y = g(x)$ are the solutions to the original equation (shown in Figure 3).

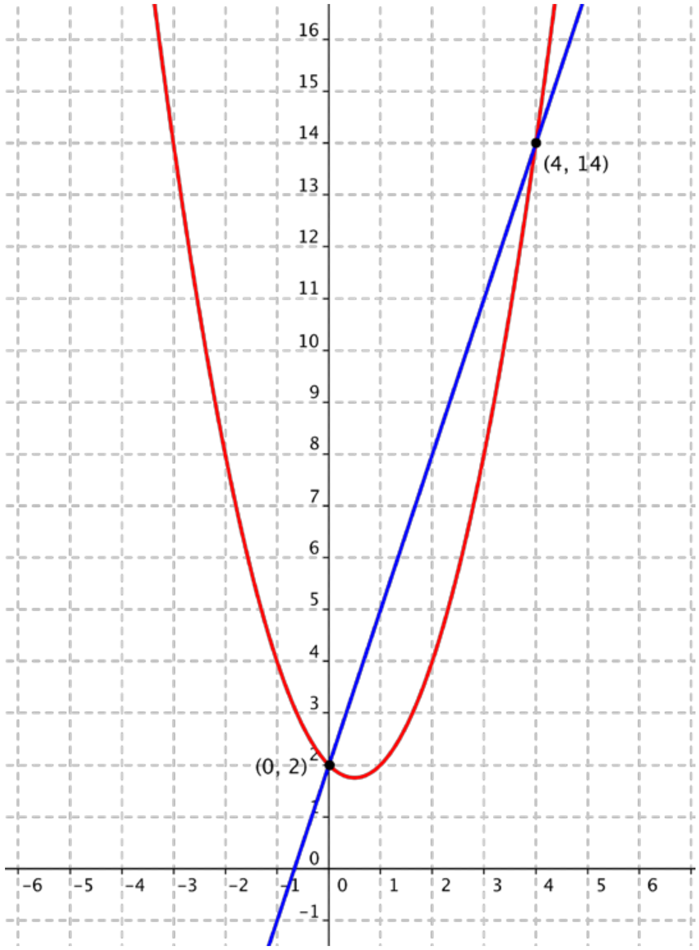


Figure 3: Graphs of $y = 3x + 2$ and $y = x^2 - x + 2$.

Students also create tables of values for functions to approximate or find exact solutions to equations such as that above. For example, they may use spreadsheet software to construct a table such as the one below.

x	f(x)=3x+2	g(x)=x^2-x+2
-6	-16	44
-5	-13	32
-4	-10	22
-3	-7	14
-2	-4	8
-1	-1	4
0	2	2
1	5	2
2	8	4
3	11	8
4	14	14
5	17	22
6	20	32
7	23	44
8	26	58
9	29	74

Figure 4: Table of values for $\boldsymbol{f(x) = 3x + 2}$ and $\boldsymbol{g(x) = x^2 - x + 2}$.

While a table like this one is in general insufficient proof that all the solutions to a given equation have been found, students can reason in certain situations why they have found all solutions (MP.3, MP.6). In this example, since the original equation is of degree two, we know that there are at most two solutions, so that the solution set is $\{0,4\}$.

Conceptual Category: Statistics and Probability

In Algebra I, students build on their understanding of key ideas for describing distributions—shape, center, and spread—presented in the standards for grades six through eight. This enhanced understanding allows them to give more precise answers to deeper questions, often involving comparisons of data sets.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

Summarize, represent, and interpret data on two categorical and quantitative variables. [Linear focus, discuss general principle.]

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
    a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* ★
    b. Informally assess the fit of a function by plotting and analyzing residuals. ★
    c. Fit a linear function for a scatter plot that suggests a linear association. ★

Interpret linear models.

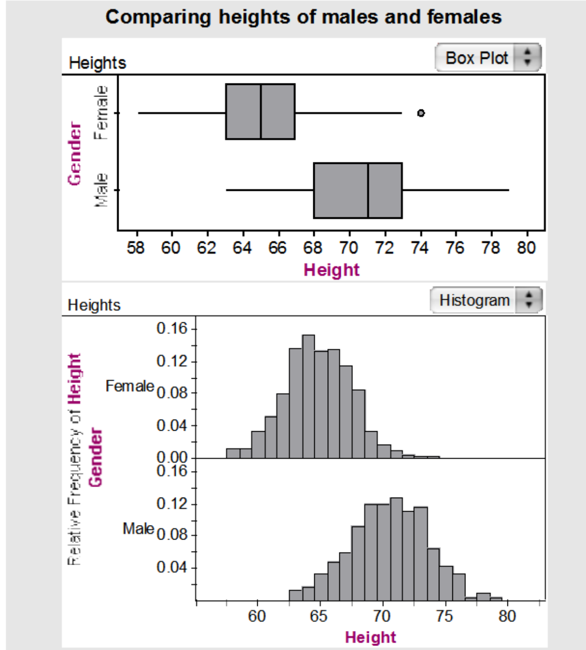
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation. ★

Standards S.ID.1-6 can be considered supporting standards with regards to Standards S.ID.7-9, in the sense that they extend concepts students began learning in grades six through eight. In general, students use shape and the question(s) to be answered to decide on the median or mean as the more appropriate measure of center and to justify their choice through statistical reasoning. Students may use parallel box-plots or histograms to compare differences in the shape, center and spread of comparable data sets (S-ID.1, 2).

Example. The graphs below show two ways of comparing height data for males and females in the 20-29 age group. Both involve plotting the data or data summaries (box plots or histograms) on the same scale, resulting in what are called parallel (or questions about it just from knowledge of these three facts (shape, center, and spread). For either group, about 68% of the data values will be within one standard deviation of the mean (S.ID.2, S.ID.3). Students also observe that the two side-by-side) box plots and parallel histograms (S-ID.1). The parallel box plots show an obvious difference in the medians and the interquartile ranges (IQRs) for the two groups; the medians for males and females are, respectively, 71 inches and 65 inches, while the IQRs are 4 inches and 5 inches. Thus, male heights center at a higher value but are slightly more variable.

The parallel histograms show the distributions of heights to be mound shaped and fairly symmetrical (approximately normal) in shape. Therefore, the data can be succinctly described using the mean and standard deviation. Heights for males and females have means of 70.4 inches and 64.7 inches, respectively, and standard deviations of 3.0 inches and 2.6 inches. Students should be able to sketch each distribution and answer

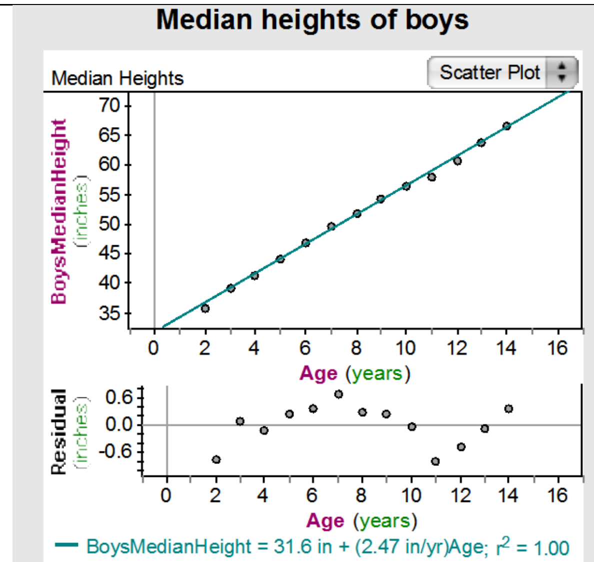
measures of center, median and mean, tend to be close to each other for symmetric distributions.

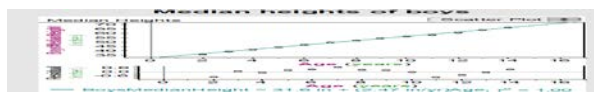


Heights of U.S. males and females in the 20–29 age group. Source: *U.S. Census Bureau, Statistical Abstract of the United States: 2009*, Table 201.

As with univariate data analysis, students now take a deeper look at bivariate data, using their knowledge of proportions to describe categorical associations and using their knowledge of functions to fit models to quantitative data. (S-ID-5, 6) Students have seen scatter plots in the grade eight standards and now extend that knowledge to fit mathematical models that capture key elements of the relationship between two variables and to explain what the model tells us about the relationship. Students must learn to take a careful look at scatter plots, as sometimes the “obvious” pattern does not tell the whole story and may be misleading. A line of best fit may appear to fit data almost perfectly, while an examination of the *residuals*, the collection of differences between corresponding coordinates on a least squares line and the actual data value for a variable may reveal more about the behavior of the data.

Example. Students must learn to take a careful look at scatter plots, as sometimes the "obvious" pattern does not tell the whole story, and can even be misleading. The graphs show the median heights of growing boys through the ages 2 to 14. The line (least squares regression line) with slope 2.47 inches per year of growth looks to be a perfect fit (S-ID.6c). But, the *residuals*, the differences between the corresponding coordinates on the least squares line and the actual data values for each age, reveal additional information. A plot of the residuals shows that growth at a constant rate does not proceed at a constant rate over those years (S.ID.6b).





Finally, students extend their work from topics covered in the grade eight standards and other topics in Algebra I to interpret the parameters of a linear model in the context of data that it represents. They compute *correlation coefficients* using technology and interpret the value of the coefficient (MP.4, MP.5). Students see situations where correlation and causation are mistakenly interchanged, and they are careful to closely examine the story that data and computed statistics are trying to tell. (S-ID.7-9).

Algebra I Overview

Number and Quantity

The Real Number System

- Extend the properties of exponents to rational exponents.
- Use properties of rational and irrational numbers.

Quantities.

- Reason quantitatively and use units to solve problems.

Algebra

Seeing Structure in Expressions

- Interpret the structure of expressions.
- Write expressions in equivalent forms to solve problems.

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials.

Creating Equations

- Create equations that describe numbers or relationships.

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning.
- Solve equations and inequalities in one variable.
- Solve systems of equations.
- Represent and solve equations and inequalities graphically.

Functions

Interpreting Functions

- Understand the concept of a function and use function notation.
- Interpret functions that arise in applications in terms of the context.
- Analyze functions using different representations.

Building Functions

- Build a function that models a relationship between two quantities.
- Build new functions from existing functions.

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Statistics and Probability

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable.
- Summarize, represent, and interpret data on two categorical and quantitative variables.
- Interpret linear models.

★ Indicates a modeling standard linking mathematics to everyday life, work, and decision-making

(+) Indicates additional mathematics to prepare students for advanced courses.

Algebra I

Number and Quantity

The Real Number System N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N-Q

Reason quantitatively and use units to solve problems. [Foundation for work with expressions, equations and functions.]

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★
2. Define appropriate quantities for the purpose of descriptive modeling. ★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Algebra

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions. [Linear, exponential, and quadratic.]

1. Interpret expressions that represent a quantity in terms of its context. ★
   a. Interpret parts of an expression, such as terms, factors, and coefficients. ★
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.* ★
2. Use the structure of an expression to identify ways to rewrite it.

Write expressions in equivalent forms to solve problems. [Quadratic and exponential.]

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
   a. Factor a quadratic expression to reveal the zeros of the function it defines. ★
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. ★
   c. Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*★

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials. [Linear and quadratic.]

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials**.**

Creating Equations A-CED

Create equations that describe numbers or relationships. [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only.]

1. Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* **CA**★
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.* ★

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning. [Master linear; learn as general principle.]

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable. [Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions.]

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

3.1 Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context. CA

4. Solve quadratic equations in one variable.
   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations. [Linear-linear and linear-quadratic.]

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Represent and solve equations and inequalities graphically. [Linear and exponential; learn as general principle.]

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions

Interpreting Functions F-IF

Understand the concept of a function and use function notation. [Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences]

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context. [Linear, exponential, and quadratic.]

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★

Analyze functions using different representations. [Linear, exponential, quadratic, absolute value, step, piecewise-defined.]

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ★
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. ★

  e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. ★

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
  b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities. [For F.BF.1, 2, linear, exponential, and quadratic.]

2. Write a function that describes a relationship between two quantities. ★
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context. ★
  b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ★
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

Build new functions from existing functions. [Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only.]

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
  a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
  a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ★
  b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ★
  c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ★
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context. ★ [Linear and exponential of form $f(x)=b^x+k$.]

6. Apply quadratic functions to physical problems, such as the motion of an object under the force of gravity. CA ★

Statistics and Probability

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

Summarize, represent, and interpret data on two categorical and quantitative variables. [Linear focus, discuss general principle.]

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* ★
   b. Informally assess the fit of a function by plotting and analyzing residuals. ★
   c. Fit a linear function for a scatter plot that suggests a linear association. ★

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
8. Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
9. Distinguish between correlation and causation. ★