Grade Eight

As students enter eighth grade they have written and interpreted expressions, solved equations and inequalities, explored quantitative relationships between dependent and independent variables, and solved problems involving area, surface area, and volume. Students have also begun to develop an understanding of statistical thinking (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

WHAT STUDENTS LEARN IN GRADE EIGHT

[Note: Sidebar]

Grade Eight Critical Areas of Instruction

In grade eight, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence and understanding and applying the Pythagorean Theorem.

Students also work towards fluency with solving simple sets of two equations with two unknowns by inspection.

Grade Eight Standards for Mathematical Content

The Standards for Mathematical Content emphasize key content, skills, and practices at each grade level and support three major principles:

- **Focus:** Instruction is focused on grade level standards.
- **Coherence:** Instruction should be attentive to learning across grades and should link major topics within grades.
- **Rigor:** Instruction should develop conceptual understanding, procedural skill and fluency, and application.

Grade level examples of focus, coherence, and rigor will be indicated throughout the chapter.

22 Not all of the content in a given grade is emphasized equally in the standards. Cluster

23 headings can be viewed as the most effective way to communicate the **focus and**

24 **coherence** of the standards. Some clusters of standards require a greater instructional

25 emphasis than the others based on the depth of the ideas, the time that they take to

26 master, and/or their importance to future mathematics or the later demands of college

27 and career readiness.

28

29 The following Grade 8 Cluster-Level Emphases chart highlights the content emphases

30 in the standards at the cluster level for this grade. The bulk of instructional time should

31 be given to "Major" clusters and the standards within them. However, standards in the

32 "Supporting/Additional" clusters should not be neglected. To do so will result in gaps in

33 students' learning, including skills and understandings they may need in later grades.

34 Students need opportunities to reinforce topics in major clusters at a grade by utilizing

35 topics in the supporting and additional clusters. Instruction should include problems and

36 activities that support natural connections between the clusters.

37

38 Teachers and administrators alike should note that the standards are not topics to be

39 checked off a list during isolated units of instruction, but rather content to be developed

40 throughout the school year through rich instructional experiences and presented in a

41 coherent manner (Adapted from the Partnership for Assessment of Readiness for

42 College and Careers [PARCC] 2012).

43

44 [**Note**: The Emphases chart should be a graphic inserted in the grade level section. The

45 explanation "key" needs to accompany it.]

46

47   **Grade 8 Cluster-Level Emphases**

48 **The Number System**

49 •   [a/s]: Know that there are numbers that are not rational, and approximate them by rational
50   numbers[1]. **(8.NS.1-2)**

---

[1] Work with the number system in this grade is intimately related to work with radicals, and both of these may be connected to the Pythagorean Theorem as well as to volume problems, e.g., in which a cube has known volume but unknown edge lengths.

The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. The *Mathematics Framework* has not been edited for publication.

51 **Expressions and Equations**

52 - [m]: Work with radicals and integer exponents. **(8.EE.1-4▲)**

53 - [m]: Understand the connection between proportional relationships, lines, and linear
54   equations. **(8.EE.5-6▲)**

55 - [m]: Analyze and solve linear equations and pairs of simultaneous linear equations.
56   **(8.EE.7-8▲)**

57 **Functions**

58 - [m]: Define, evaluate, and compare functions. **(8.F.1-3▲)**

59 - [a/s]: Use functions to model relationships between quantities.[2] **(8.F.4-5)**

60 **Geometry**

61 - [m]: Understand congruence and similarity using physical models, transparencies, or
62   geometry software. **(8.G.1-5▲)**

63 - [m]: Understand and apply the Pythagorean Theorem. **(8.G.6-8▲)**

64 - [a/s]: Solve real-world and mathematical problems involving volume of cylinders, cones,
65   and spheres. **(8.G.9)**

66 **Statistics and Probability**

67 - [a/s]: Investigate patterns of association in bivariate data.[3] **(8.SP.1-4)**

68

Explanations of Major, Additional and Supporting Cluster-Level Emphases
Major[4] **[m]** (▲) clusters – areas of intensive focus where students need fluent understanding and application of the core concepts. These clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness.
Additional [a] clusters – expose students to other subjects; may not connect tightly or explicitly to the major work of the grade
Supporting [s] clusters – rethinking and linking; areas where some material is being covered, but in a way that applies core understanding; designed to support and strengthen areas of major emphasis.   *A Note of Caution: Neglecting material will leave gaps in students' skills and understanding and will leave students unprepared for the challenges of a later grade.

69 (Adapted from Smarter Balanced Assessment Consortia [Smarter Balanced], DRAFT

70 Content Specifications 2012).

71

---

[2] The work in this cluster involves functions for modeling linear relationships and a rate of change/initial value, which supports work with proportional relationships and setting up linear equations.

[3] Looking for patterns in scatterplots and using linear models to describe data is directly connected to the work in the Expressions and Equations clusters. Together, these represent a connection to the Standard for Mathematical Practice 4, Model with mathematics.

[4] The ▲ symbol will indicate standards in a Major Cluster in the narrative.

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72 **Connecting Mathematical Practices and Content**

73 The Standards for Mathematical Practice (MP) are developed throughout each grade

74 and, together with the content standards, prescribe that students experience

75 mathematics as a rigorous, coherent, useful, and logical subject that makes use of their

76 ability to make sense of mathematics. The MP standards represent a picture of what it

77 looks like for students to understand and do mathematics in the classroom and should

78 be integrated into every mathematics lesson for all students.

79

80 Although the description of the MP standards remains the same at all grades, the way

81 these standards look as students engage with and master new and more advanced

82 mathematical ideas does change. Below are some examples of how the MP standards

83 may be integrated into tasks appropriate for grade eight students. (Refer to pages 9–12

84 in the Overview of the Standards Chapters for a complete description of the MP

85 standards.)

86

87   **Standards for Mathematical Practice (MP)**

88   **Explanations and Examples for Grade Eight**

Standards for Mathematical Practice	Explanation and Examples
MP.1 Make sense of problems and persevere in solving them.	In grade eight, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"
MP.2 Reason abstractly and quantitatively.	Students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students *contextualize* to understand the meaning of the number or variable as related to the problem and *decontextualize* to manipulate symbolic representations by applying properties of operations.
MP. 3 Construct viable arguments and critique the reasoning of others.	Students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (e.g., box plots, dot plots, histograms). They further refine their mathematical

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	communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" and "Does that always work?" They explain their thinking to others and respond to others' thinking.
MP.4 Model with mathematics.	Eighth grade students model real-world problem situations symbolically, graphically, in tables, and contextually. Working with the new concept of a *function*, students learn that relationships between variable quantities in the real world often satisfy a dependent relationship, in that one quantity determines the value of another. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use scatterplots to represent data and describe associations between variables. They should be able to use any of these representations as appropriate to a problem context. Students should be encouraged to answer questions, such as "What are some ways to represent the quantities?" or "How might it help to create a table, chart, graph,…?"
MP.5 Use appropriate tools strategically.	Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade eight may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal that intersects parallel lines. Teachers might ask, "What approach are you considering trying first?" or "Why was it helpful to use…?"
MP.6 Attend to precision.	In grade eight, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Teachers might ask "What mathematical language, definitions, properties…can you use to explain…?
MP.7 Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In grade eight, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
MP.8 Look for and express regularity in repeated reasoning.	In grade eight, students use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. Through multiple opportunities to model linear relationships, they notice that the slope of the graph of the linear relationship and the rate of change of the associated function are the same. As students repeatedly check whether points are on the line of slope 3 through the point (1, 2), they might abstract the equation of the line in the form $(y - 2)/(x - 1) = 3$. Students divide to find

	decimal equivalents of rational numbers (e.g. $^2/_3 = 0.\bar{6}$) and generalize their observations. They use iterative processes to determine more precise rational approximations for irrational numbers. Students should be encouraged to answer questions, such as "How would we prove that…?" or "How is this situation like and different from other situations using this operations?"

89 (Adapted from Arizona Department of Education [Arizona] 2012 and North Carolina

90 Department of Public Instruction [N. Carolina] 2013)

91

92 **Standards-based Learning at Grade Eight**

93 The following narrative is organized by the domains in the Standards for Mathematical

94 Content and highlights some necessary foundational skills from previous grades and

95 provides exemplars to explain the content standards, highlight connections to the

96 various Standards for Mathematical Practice (**MP**), and demonstrate the importance of

97 developing conceptual understanding, procedural skill and fluency, and application. A

98 triangle symbol (**▲**) indicates standards in the major clusters (refer to the Grade 8

99 Cluster-Level Emphases chart on page 2).

100

101 **Domain: The Number System**

102 In seventh grade, adding, subtracting, multiplying, and dividing rational numbers was

103 the culmination of numerical work with the four basic operations. The number system

104 continues to develop in grade eight, expanding to the real numbers by introducing

105 irrational numbers, and develops further in high school, expanding to become the

106 complex numbers with the introduction of imaginary numbers (Adapted from PARCC

107 2012).

The Number System	8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.
1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g. π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

108

109 In eighth grade, students learn that not all numbers can be expressed in the form a/b

110 where a and b are positive or negative whole numbers with $b \neq 0$. Such numbers are

111 called *irrational*, and students explore cases of both rational and irrational numbers and

112 their decimal expansions to begin to understand the distinction. That rational numbers

113 have eventually repeating decimal expansions is a direct result of how one uses long

114 division to produce a decimal expansion.

115

Why rational numbers have terminating or repeating decimal expansions.

In each step of the standard algorithm to divide a by b, one determines a partial quotient and a remainder; the requirement being that each remainder is smaller than the divisor (b). In simpler examples students will have noticed or been led to notice that once a remainder is repeated, from that point onward the decimal repeats, as with $\frac{1}{6} = 0.16\mathbf{666} \ldots = 0.1\overline{6}$ or $\frac{3}{11} = 0.27\mathbf{2727} \ldots = 0.\overline{27}$. If one were to imagine using long division to convert the fraction $\frac{3}{13}$ to a decimal without going through the tedium of actually producing the decimal, one can reason that the possible remainders are 1 through 12. Consequently, by the thirteenth remainder we MUST obtain a remainder that has occurred already and therefore a decimal that is repeating.

116

117 The full reasoning for why the converse is true, that eventually repeating decimals

118 represent numbers that are rational, is beyond the scope of this grade. But students can

119 use algebraic reasoning to show that eventually repeating decimals represent rational

120 numbers in certain simple cases. **(8.NS.1)**

121

Example: Convert the repeating decimal $0.\overline{18}$ into a fraction of the form a/b.

Solution: One method for converting such a decimal into a fraction is to set $N = 0.\overline{18} = 0.18181818 \ldots$. If this is the case, then $100N = 18.\overline{18}$. Subtracting $100N$ and N, we obtain $99N$. But this means that $99N = 18.\overline{18} - .\overline{18} = 18$. Solving for N, we see that $N = 18/99 = 2/11$.

122

123 Since every decimal is of one of the two forms eventually repeating or non-repeating,

124 this leaves irrational numbers as precisely those numbers whose decimal expansions

125 do not have a repeating pattern. Students understand this informally in grade eight, and

126 they experience approximating irrational numbers by rational numbers in simple cases.

127

Example: Finding better and better approximations of $\sqrt{2}$.

The following reasoning can be used to approximate simple irrational square roots.

- Since $1^2 < 2 < 2^2$, this means that $\sqrt{2}$ must be between 1 and 2.
- By guessing and checking, since $1.4^2 = 1.96$, and $1.5^2 = 2.25$, we know that $\sqrt{2}$ is between 1.4 and 1.5.
- By some more guessing and checking, and using a calculator, we see that since $1.41^2 = 1.9881$ and $1.42^2 = 2.0164$, we know that $\sqrt{2}$ is between 1.41 and 1.42.

Continuing in this manner yields better and better approximations of $\sqrt{2}$. Allowing students to investigate this process with their calculators, they can get some experience with the idea that the decimal expansion of $\sqrt{2}$ never repeats. In addition, students should graph their successive approximations on number lines to reinforce their understanding of the number line as a tool for representing the real numbers.

128

129   Ultimately, students should come to an informal understanding that the set of real

130   numbers is comprised of rational numbers and irrational numbers. They will continue to

131   work with irrational numbers and their rational approximations when solving equations

132   such as $x^2 = 18$ and in problems involving the Pythagorean Theorem. In the

133   Expressions and Equations domain that follows, students learn to use radicals to

134   represent such numbers. (Adapted from California Department of Education *A Look at*

135   *Grades Seven and Eight in California Public Schools: Transitioning to the Common*

136   *Core State Standards in English Language Arts and Mathematics* [CDE Transition

137   Document] 2012, Arizona 2012, and N. Carolina 2013).

138

139   [Note: Sidebar]

Focus, Coherence, and Rigor:

The standards in the grade eight domain "The Number System" support major work at the grade with the Pythagorean Theorem (**8.G.6-8▲**) and connection to volume problems **(8.G.9)**, e.g., in which a cube has known volume but unknown edge lengths.

140

141

142   **Domain: Expressions and Equations**

143

144   In grade seven students formulated expressions and equations in one variable, and

145   they used these equations to solve problems and fluently solved equations of the form

146 $px + q = r, p(x + q) = r$. In grade eight, students apply their previous understandings of

147 ratio and proportional reasoning to the study of linear equations and pairs of

148 simultaneous linear equations, which is a critical area of instruction at the grade.

149

Expressions and Equations	8.EE

Work with radicals and integer exponents.
1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

150

151 Grade eight students add the following properties of integer exponents to their

152 repertoire of rules for transforming expressions, and they use these properties to

153 generate equivalent expressions.

154

Properties of Integer Exponents
For any nonzero numbers a and b and integers n and m.
1. $a^n a^m = a^{n+m}$
2. $(a^n)^m = a^{nm}$
3. $a^n b^n = (ab)^n$
4. $a^0 = 1$
5. $a^{-n} = 1/a^n$

155

156 Since students have been focusing on place value relationships in the base-ten number

157 system since elementary school, working with powers of 10 is a natural place to start

158 investigating the patterns that give rise to these properties. However, general bases

159 should be explored, and will foreshadow the study of exponential functions in higher

160 mathematics courses.

161

	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Example: Students can reason about patterns to explore the properties of exponents. Students fill in the blanks in the table shown and discuss with a neighbor any patterns they find:							
Expanded	$2 \times 2 \times 2$	2×2	2	?	?	?	?
Evaluate	8	4	2	?	?	?	?

Students can reason about why the value of 2^0 should be 1, based on patterns they may see, such as that in the bottom row of the table, each value is 1/2 of the value to the left of it. Students should explore similar examples with other bases to arrive at the general understanding that $a^n = a \times a \times \cdots \times a$ (n factors), $a^0 = 1$, and $a^{-n} = 1/a^n$.

162

163 Generally, Mathematical Practice standard 3 (**MP.3**) calls for students to construct

164 mathematical arguments; thus reasoning should be emphasized when it comes to

165 learning the properties of exponents. For example, students can reason that $5^3 \times 5^2 =$

166 $(5 \times 5 \times 5) \times (5 \times 5) = 5^5$. Students generalize the properties of exponents after

167 many experiences working with them before they use them fluently.

168

169 Students do not learn the properties of rational exponents until high school. However, in

170 grade eight they start to work systematically with the square root and cube root

171 symbols, writing, for example, $\sqrt{64} = 8$ and $\sqrt[3]{5^3} = 5$. Since $\sqrt{p}$ is defined to only mean

172 the positive solution to the equation $x^2 = p$ (when it exists), it is not correct to say that

173 $\sqrt{64} = \pm 8$. However, a correct solution to $x^2 = 64$ would be $x = \pm\sqrt{64} = \pm 8$. Students in

174 grade eight are not in a position to prove that these are the only solutions, but rather

175 use informal methods such as guess and check to verify them (Progressions 6-8 EE

176 2011).

177

178 Students recognize perfect squares and cubes, understanding that non-perfect squares

179 and non-perfect cubes are irrational. Students should generalize from many

180 experiences that: (**MP.2, MP.5, MP.6, and MP.7**)

181   • Squaring a square root of a number returns the number back (e.g., $\left(\sqrt{5}\right)^2 = 5$)

182   • Taking the square root of the square of a number *sometimes* returns the number

183   back (e.g., $\sqrt{7^2} = \sqrt{49} = 7$, while $\sqrt{(-3)^2} = \sqrt{9} = 3 \neq -3$)

184   • Cubing a number and taking the cube root can be considered inverse operations.

185

186 Students expand their exponent work as they perform operations with numbers

187 expressed in scientific notation, including problems where both decimal and scientific

188 notation are used. Students use scientific notation to express very large or very small

189 numbers. Students compare and interpret scientific notation quantities in the context of

190 the situation, recognizing that the powers of ten indicated in quantities expressed in

191 scientific notation follow the rules of exponents shown above. (Adapted from CDE

192 Transition Document 2012, Arizona 2012, and N. Carolina 2013)

193

Example: Ants and Elephants. An ant has a mass of approximately 4×10^{-3} grams and an elephant has a mass of approximately 8 metric tons. How many ants does it take to have the same mass as an elephant?

(Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg.)

Solution: To compare the masses of an ant and an elephant, we convert the mass of an elephant into grams:

$$8 \text{ metric tons } \times \frac{1000 \text{ kg}}{1 \text{ metric ton}} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 8 \times 10^3 \times 10^3 \text{grams} = 8 \times 10^6 \text{grams}.$$

If we let N represent the number of ants that have the same mass as an elephant, then $(4 \times 10^{-3})N$ is their total mass in grams. This should equal 8×10^6 grams. This gives us a simple equation:

$$(4 \times 10^{-3})N = 8 \times 10^6 \text{ which means that } N = \frac{8 \times 10^6}{4 \times 10^{-3}} = 2 \times 10^{6-(-3)} = 2 \times 10^9$$

Thus, 2×10^9 ants would have the same mass as an elephant.

(Adapted from Illustrative Mathematics, 8.EE Ant and Elephant.)

194

195   [Note: Sidebar]

Focus, Coherence, and Rigor:

As students work with scientific notation, they learn to choose units of appropriate size for measurement of very large or very small quantities. (**MP.2, MP.5, MP.6**)

196

Expressions and Equations	8.EE

Understand the connections between proportional relationships, lines, and linear equations.
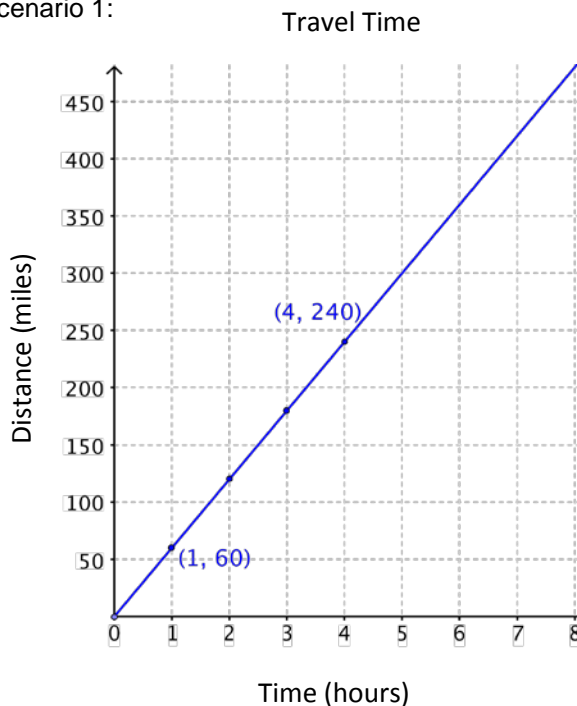5.   Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two

> different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation and determine which of the two moving objects has greater speed.*
> 6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y = mx* for a line through the origin and the equation *y = mx + b* for a line intercepting the vertical axis at *b*.

197

198 Students build on their work with unit rates from sixth grade and proportional

199 relationships in seventh grade to compare graphs, tables, and equations of proportional

200 relationships (**8.EE.5▲**). Students identify the unit rate (or slope) to compare two

201 proportional relationships represented in different ways (e.g., as graph of the line

202 through the origin, a table exhibiting a constant rate of change, or an equation of the

203 form $y = kx$). Students interpret the unit rate in a proportional relationship (e.g., r miles

204 per hour) as the slope of the graph. They understand that the slope of a line represents

205 a constant rate of change.

206

Example: Compare the scenarios below to determine which represents a greater speed. Include a description of each scenario that discusses unit rates in your explanation.	
Scenario 1: **Travel Time** Time (hours)	Scenario 2: The equation for the distance y in miles as a function of the time x in hours is: $$y = 55x.$$
	Solution: "The unit rate in Scenario 1 can be read from the graph; it is 60 miles per hour. In Scenario 2, I can see that this looks like an equation $y = kx$, and in that type of equation the unit rate is the constant k. Therefore the speed in Scenario 2 is 55 miles per hour. So the person traveling in Scenario 1 is going faster."

207 (Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina 2013)

208

209   Following is an example of connecting the Standards for Mathematical Content with the

210   Standards for Mathematical Practice.

211

Connecting to the Standards for Mathematical Practice—Grade Eight

Standard(s) Addressed	Example(s) and Explanations
8.EE.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in two different ways.	**Task:** Below is a table that shows the costs of various amounts of almonds.

Almonds (pounds)	3	5	8	10	15
Cost (dollars)	15.00	25.00	40.00	50.00	75.00

1.   Graph the cost versus the number of pounds of almonds. The number of pounds of almonds should be on the horizontal axis and the cost of the almonds on the vertical axis.
2.   Use the graph to find the cost of 1 pound of almonds. Explain how you got your answer.
3.   The table shows that 5 pounds of almonds costs $25.00. Use your graph to find out how much 6 pounds of almonds costs.
4.   Suppose that walnuts cost $3.50 per pound. Draw a line on your graph that might represent the cost of different numbers of pounds of walnuts.
5.   Which is cheaper? Almonds or walnuts? How do you know?

Solution:
1.   A graph is shown.
2.   To find the cost of 1 pound of almonds, one would locate the point that has first coordinate 1; this is the point (1, 5). This shows that the *unit cost* is $5 per pound.
3.   Students can do this by simply locating 6 pounds on the horizontal axis and finding the point on the graph associated with this number of pounds. However, the teacher can also urge students to notice that one can move along the graph by moving to the right 1 unit and noticing that we move 5 units up to the next point on the graph. This idea is the genesis of *slope* of a line and should be explored.
4.   Ideally, students draw a line that passes through (0,0) and the approximate point (1, 3.50). Proportional thinkers might notice that 2 pounds of walnuts cost $7, so they can plot a point with whole number coordinates.
5.   Walnuts are cheaper. Students can explore several different ways to see this, including the unit cost, the steepness of the line, by comparing common quantities of nuts, etc.

Classroom Connections
The concept of slope can be approached in its simplest form with directly proportional quantities. In this case, when two quantities x and y are directly proportional, they are related by an equation $y = kx$, equivalently, $\frac{y}{x} = k$, where k is a constant known as the constant of proportionality. In the case of almonds above, the k in an equation would represent the unit cost of almonds. Students should have several experiences with graphing and exploring directly proportional relationships to build a foundation for understanding more general linear equations of the form $y = mx + b$.

Connecting to the Standards for Mathematical Practice
(MP.1) Students are encouraged to attack the entire problem and make sense in each step required.
(MP.4) Students are modeling a very simple real-life cost situation.

212

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213   [Note: Sidebar]

> **Focus, Coherence, and Rigor:**
>
> The connection between the unit rate in a proportional relationship and the slope of its graph depends on a connection with the geometry of similar triangles. (See Standards **8.G.4-5▲**.) The fact that a line has a well-defined slope—that the ratio between the rise and run for any two points on the line is always the same—depends on similar triangles (Adapted from Progressions 6-8 EE 2011).

214

215   Standard **(8.EE.6▲)** represents a convergence of several ideas in this and

216   previous grade levels. Students have graphed proportional relationships and

217   found the slope of the resulting line, interpreting it as the unit rate **(8.EE.5▲)**. It is

218   here that the language of "rise over run" comes into use. In the Functions

219   domain, students will see that any linear equation $y = mx + b$ determines a

220   function whose graph is a straight line (a *linear function*), and they verify that the

221   slope of the line is equal to m (**8.F.3**). In standard **(8.EE.6▲)**, students go further

222   and explain why the slope m is the same through any two points on a line. They

223   justify this fact using similar triangles, which are studied in standards **(8.G.4-5▲)**.

224

225

226

227

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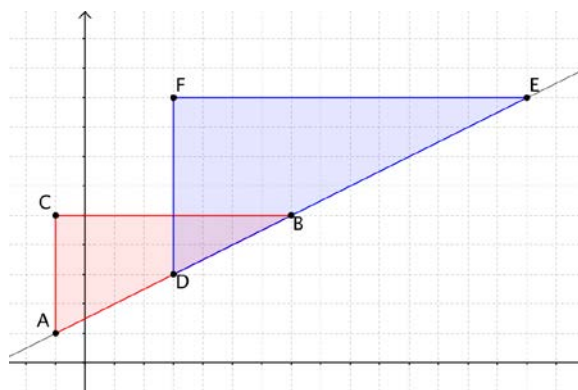
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239

Example of Reasoning (8.EE.6 ▲). Showing that the slope is the same between two points on a line.	
In seventh grade, students made scale drawings of figures and observed the proportional relationships between side lengths of such figures (**7.G.1▲**). In grade eight, students generalize this idea and study *dilations* of plane figures, and they define figures as being *similar* in terms of dilations (see standard **8.G.4▲**). It is discovered that similar figures share a proportional relationship between side lengths just like scale drawings did: there is a *scale factor* $k > 0$ such that corresponding side lengths of similar figures are related by the equation $s_1 = k \cdot s_2$. Furthermore, the ratio of two sides in one shape is equal to the ratio of the corresponding two sides in the other shape. Finally, in standard (**8.G.5▲**), students informally argue that triangles that have two corresponding angles of the same measure must be similar, and this is the final piece of the puzzle for the first result in standard (**8.EE.6▲**).	**Example:** "Explain why the slope between points A and B and points D and E are the same." **Solution:** "Angles $\angle A$ and $\angle D$ are equal since they are corresponding angles formed by the transversal crossing the vertical lines through points A and D. Since $\angle C$ and $\angle F$ are both right angles, the triangles are similar. This means the ratios $\frac{AC}{BC}$ and $\frac{DF}{EF}$ are equal. But when you find the 'rise over the run,' these are exactly the ratios that you find, and so the slope is the same between these two sets of points."

240

241   In grade eight students build on previous work with proportional relationships,

242   unit rates, and graphing to connect these ideas and understand that the points

243   (x, y) on a non-vertical line are the solutions of the equation $y = mx + b$, where

244   m is the slope of the line, as well as the unit rate of a proportional relationship in
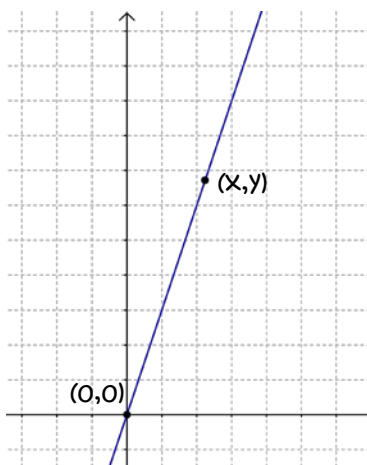
245   the case $b = 0$.

246

247

248

Example of Reasoning (8.EE.6). Deriving the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

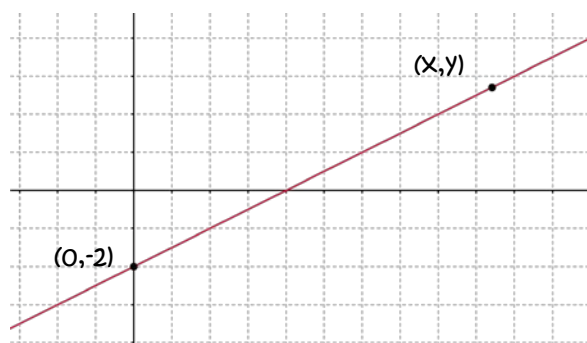|	
Example: "Explain how to derive the equation $y = 3x$ for the line of slope $m = 3$ shown below." **Solution: "**I know that the slope is the same between any two points on a line. So I'll choose the origin $(0,0)$ and a generic point on the line, calling it (x, y). By choosing a generic point like this I know that any point on the line will fit the equation I come up with. The slope between these two points is found by $$3 = \frac{\text{rise}}{\text{run}} = \frac{y - 0}{x - 0} = \frac{y}{x}$$ This equation can be rearranged to $y = 3x$."	**Example:** "Explain how to derive the equation $y = \frac{1}{2}x - 2$ for the line of slope $m = \frac{1}{2}$ with intercept $b = -2$ shown below." **Solution:** "I know the slope is $\frac{1}{2}$ so I'll calculate the slope using the point $(0, -2)$ and the generic point (x, y). The slope between these two points is found by $$\frac{1}{2} = \frac{\text{rise}}{\text{run}} = \frac{y - (-2)}{x - 0} = \frac{y + 2}{x}$$ This can be rearranged to $y + 2 = \frac{1}{2}x$, which is the same as $y = \frac{1}{2}x - 2$."

249   (Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina

250   2013)

251

Expressions and Equations	8.EE

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equation of the form *x = a*, *a = a*, or *a = b* results (where *a* and *b* are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. The *Mathematics Framework* has not been edited for publication.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

252

253   Students have worked informally with one-variable linear equations as early as

254   kindergarten. This important line of development culminates in grade eight as

255   much of students' work involves analyzing and solving linear equations and pairs

256   of simultaneous linear equations.

257

258   Grade eight students solve linear equations in one variable, including cases with

259   one solution, infinitely many solutions, and no solutions (**8.EE.7▲**). Students

260   show examples of each of these cases by successively transforming an equation

261   into simpler forms ($x = a$, $a = a$, and $a = b$, where a and b represent different

262   numbers). Solving some linear equations will require students to expand

263   expressions using the distributive property and to collect like terms.

264

Solutions to One Variable Equations

- When an equation has only one solution, there is only one value of the variable that makes the equation true, e.g., with $12 - 4y = 16$.

- When an equation has infinitely many solutions, the equation is true for all real numbers, and is sometimes referred to as an *identity*, e.g., with $7x + 14 = 7(x + 2)$. Solving this equation using familiar steps might yield $14 = 14$, a statement that is true no matter the value of x. Students should be encouraged to think about why this means that any real number solves the equation, and relate it to back to the original equation (e.g., the equation is just showing the distributive property).

- When an equation has no solutions, we sometimes say the equation is *inconsistent*, e.g. with $5x - 2 = 5(x + 1)$. Attempting to solve this equation might yield $-2 = 5$, a false statement no matter the value of x. Students should be encouraged to reason why there are no solutions to the equation, for example, by observing that the original equation is equivalent to $5x - 2 = 5x + 5$, and reasoning that it is never the case that $N - 2 = N + 5$ no matter what N is.

265

266   Grade eight students also analyze and solve pairs of simultaneous linear

267   equations. Solving pairs of simultaneous linear equations builds on the skills and

268   understandings students used to solve linear equations with one variable (**8.EE.8**

269   **a-c▲**), and systems of linear equations can also have one solution, infinitely

270   many solutions, or no solutions. Students will discover these cases as they graph

271   systems of linear equations and solve them algebraically. Grade eight students

272   learn that for a system of linear equations:

273   •   If the graphs of the lines meet at one point (the lines intersect), then there is

274   one solution, the ordered pair of the point of intersection representing the

275   solution of the system.

276   •   If the graphs of the lines do not meet (the lines are parallel), the system has

277   no solutions, and the slopes of these lines are the same.

278   •   If the graphs of the lines are *coincident* (the graphs are exactly the same line)

279   then the system has infinitely many solutions, the solutions being the set of all

280   ordered pairs on the line.

281

Example: Introducing Systems of Linear Equations.

To introduce the concept of a system of linear equations, the teacher might ask students to get into small groups and think about how they would start a business selling coffee at school during lunch. Then, each group would create a budget that details the cost of the items they would have to purchase each month (students could use the internet to acquire pricing or use their best estimate), as well as a monthly total. Each group would also establish a price for a cup of their coffee. Students can also discuss a model (equation) for the profit their business will make in a month. The teacher might ask students thoughtful questions, such as:

1. What are some variable quantities in our situation? (Important are cups of coffee sold, monthly profit.)

2. What is the profit at the beginning of the month? (A negative number corresponding to the monthly total of items purchased)

3. How many cups of coffee will you need to sell to make a profit?

Instruct students to make a table that shows profit vs. cups of coffee sold, for multiples of 10 cups of coffee up to 200. Instruct students to create a graph from the data in their table. The teacher can demonstrate the graphs of the lines $y = 0$ and $y = 50$, then ask students to draw the same lines on their graph. Ask students the meaning of those lines. (Solution: the point when the business is no longer losing money, the point when the business is making money). The teacher

---

> can demonstrate the points of intersection. Discuss the importance of those two coordinates. Finally, ask two students from different groups (pre-select groups whose graphs will intersect) to graph their data on the same axis for the whole class to see. Discuss the significance of the point of intersection of the two lines, including the concept that the number of cups sold and the profit will be the same at that point. As a class, students write equations for both lines and demonstrate by substitution that the coordinates of the intersection point are solutions to both equations.

282

283 By making connections between algebraic and graphical solutions and the

284 context of the system of linear equations, students are able to make sense of

285 their solutions.

286

287 Students solve real-world and mathematical problems leading to two linear

288 equations in two variables. Below is an example of how reasoning about real-

289 world situations can be used to introduce and make sense out of solving systems

290 of equations by elimination. The technique of elimination can be used in general
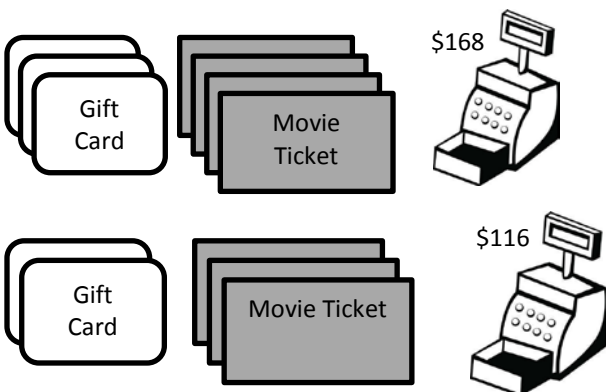
291 cases to solve systems of equations.

292

Example: Solving a System of Equations by Elimination.

Suppose you knew that 3 gift cards and 4 movie tickets altogether cost $188, while 2 gift cards and 3 movie tickets altogether cost $116.

1. Explain how to use this information to find the cost of one gift card and one movie ticket.
2. Next, explain how you could find the cost of 1 movie ticket.
3. Explain how you would find the cost of 1 gift card.

Solution:

1. If I let g represent the cost of a gift card and t represent the cost of a movie ticket, then I know that $3g + 4t = 168$ and $2g + 3t = 116$. I can represent this in a diagram:

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I can see that if I subtract the 2 gift cards and 3 movie tickets from the 3 gift cards and 4 movie tickets, I get $168 – $116 = $52. This means the cost of 1 of each item together is $52. I can represent this by

$$3g + 4t = 168$$
$$\underline{2g + 3t = 116}$$
$$g + t = 52$$

2. Now I can see that 2 of each item would cost $104. If I subtract this result from the second equation above, I am left with 1 movie ticket and it costs $12.

$$2g + 3t = 116$$
$$\underline{2g + 2t = 104}$$
$$1t = 12$$

3. Now it is easy to see that if 1 gift card and 1 ticket together cost $52, then 1 gift card alone would cost $52 – $12 = $40.

293

294

295   **Domain: Functions**

296

297   In grade seven, students learned to determine if two quantities represented a

298   proportional relationship. Proportional reasoning is a transitional topic, coming

299   between arithmetic and algebra. Underlying the progression from proportional

300   reasoning through algebra and beyond is the idea of a *function*— a rule that

301   assigns to each input exactly one output. In grade eight a critical area of

302   instruction is the concept of a function. Students are introduced to functions, and

303   they learn proportional relationships are part of a broader group of linear

304   functions.

305

Functions   8.F
Define, evaluate, and compare functions.
1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[5]
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line;

[5] Function notation is not required in Grade 8.
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> give examples of functions that are not linear. *For example, the function A = s² giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

306

307 In grade eight, students understand two main points in regards to functions

308 (**8.F.1▲**):

- a *function* is a rule that assigns to each input exactly one output, and

- the *graph* of a function is the set of ordered pairs consisting of an input and the corresponding output.

312 In general, students understand that functions describe situations in which one

313 quantity determines another. The main work in grade eight concerns linear

314 functions, though students are exposed to non-linear functions to contrast them

315 with linear functions. Thus, students may view a linear equation like $y = -.75x +$

316 12 as a rule that defines a quantity y whenever the quantity x is given. In this

317 case, the function may describe the amount of money remaining after x turns

318 when a student who starts with $12 plays a game that costs $.75 per turn. Or,

319 students may view the formula for the area of a circle, $A = \pi r^2$ as a (non-linear)

320 function in the sense that the area of a circle is dependent on its radius. Student

321 work with functions at grade eight remains informal, but sets the stage for more

322 formal work in the higher mathematics courses.

323

Example: Introduction to Functions.

To introduce the concept of a function, a teacher might have students contrast two workers' wages at two different jobs, one with an hourly wage and the other based on a combination of hourly wage and tips. Students read through scenarios and make a table for each of the two workers, listing hours worked and money earned during 20 different shifts varying from 3 to 8 hours in length. Students answer questions about the data, including the level of predictability of the wage of each worker, based on the number of hours worked. Students graph the data and observe the patterns of the graph. Next, the teacher could introduce the concept of a function and relate the tables and graphs from the activity to the idea of a function, emphasizing that an input value completely determines an output value. Students could then be challenged to find other quantities that are functions and to create and discuss corresponding tables and/or graphs.

324

325   Students are able to connect foundational understandings about functions to their

326   work with proportional relationships. The same kinds of tables and graphs

327   students used in seventh grade to recognize and represent proportional

328   relationships between quantities are used in eighth grade when students

329   compare the properties of two functions that are represented in different ways

330   (e.g., numerically in tables, visually in graphs). Students also compare the

331   properties of two functions that are represented algebraically or verbally

332   (**8.F.2▲**).

333

Example: Functions Represented Differently. Which function has a greater rate of change?		
Function 1: The function represented by the graph shown. *Function 2*: The function whose input x and output y are related by the equation $y = 4x + 7.$		**Solution:** The graph of the function shows that when $x = 0$ the value of the function is $y = 7$, while when $x = 2$ the value of the function is $y = 13$. This means that Function 1 increases by 6 units when x increases by 2 units. Function 2 also has an output of $y = 7$ when $x = 0$, but when $x = 2$ the value of Function 2 is $y = 15$. This means that Function 2 increases by 8 units when x increases by 2 units. Therefore, Function 2 has a greater rate of change.

334

335   Students' understanding of the equation $y = mx + b$ deepens as they learn that

336   the equation defines a linear function whose graph is a straight line (**8.F.3▲**), a

337   concept closely related to standard (**8.EE.6▲)**. To avoid the mistaken impression

338   that all functional relationships are linear, students also work with nonlinear

339   functions and provide examples of nonlinear functions, recognizing that the graph

340   of a nonlinear function is not a straight line.

341

Functions	8.F
Use functions to model relationships between quantities.	
4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description or from two (*x, y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a	

> table of values.
> 5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

342

343 In grade eight students learn to use functions to represent relationships between

344 quantities. This work is also closely tied to Mathematical Practice (**MP.4 *Model***

345 ***with Mathematics***). There are many situations in real-world problems that can

346 be modeled with linear functions, including instances of constant payment plans

347 (e.g. phone plans), costs associated with running a business, and relationships

348 between certain associated bivariate data (see standard **8.SP.3**). Students also

349 recognize that linear functions in which $b = 0$ are proportional relationships,

350 something they have studied since grade six.

351

352 Standard **8.F.4** refers to students finding the *initial value* of a linear function.

353 What is intended by this standard is that if f represents a linear function with a

354 domain $[a, b]$, that is, the input values for f are between the values a and b, then

355 the initial value for f would be $f(a)$. The term "initial value" takes its name from

356 an interpretation of the independent variable as representing time, although the

357 term can apply to any function. Note that formal introduction of the term *domain*

358 does not occur until the higher mathematics courses, but teachers may desire to

359 include this language if it clarifies these ideas for students. The example below

360 will illustrate this definition.

361

Example: Modeling With a Linear Function. A car rental company charges $45 per day for the car as well as a one-time $25 fee for the car's GPS navigation system. Write an equation for the cost in dollars, c, as a function of the number of days the car is rented, d. What is the initial value for this function?

Solution: There are several aids that may help students determine an equation for the cost:
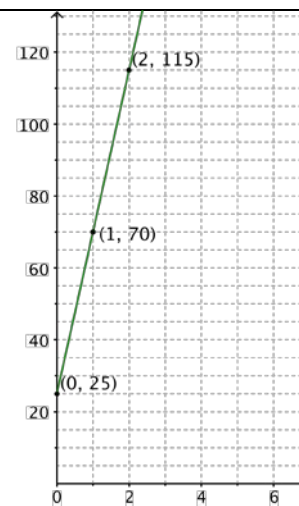
- A verbal description: "Each day adds $45 to the cost, but there is the one-time $25 GPS fee. This means that the cost should be $25 plus $45 times whatever number of days you rent the car. This gives me $c = 25 + 45d$. Since the least number of days you could rent the car is 1 day, the initial value is $25 + 45 = 70$, which means it costs $70 to rent the car for 1 day."

- A table: "I made a table to give me a feel for how much the rental might cost after d days.

d (days)	1	2	3	4	d
c (dollars)	70	115	160	205	25
	$= 25 + (1)45$	$= 25 + (2)45$	$= 25 + (3)45$	$= 25 + (4)45$	$+ (d)45$

The table helped me see that the cost in dollars is given by $c = 25 + (d)45$. Since it only

makes sense to rent the car for 1 day or more, the initial value is when $d = 1$, which is $c = 70$,

or $70."

- A graph: "I made a rough graph and saw that the relationship between the cost and the days rented appeared to be linear. So I found the slope of the line to be 45 (which is the cost per day) and the y intercept to be 25. This means the equation is $c = 45d + 25$. What's important to see is that even though the y intercept of the graph is 25, that isn't the initial value since that would be when someone rents the car for 1 day. Since the point on the graph is $(1,70)$, the initial value is $70."

362

363 Students analyze graphs and then describe qualitatively the functional

364 relationship between two quantities (e.g., where the function is increasing or

365 decreasing, linear or nonlinear). They are able to sketch graphs that illustrate the

366 qualitative features of functions that are described verbally (**8.F.5**).

367

368   [Note: Sidebar]

Focus, Coherence, and Rigor:

Work in the cluster "Use functions to model relationships between quantities" involves functions

for modeling linear relationships and computing a rate of change or initial value, which supports

major work at grade eight with proportional relationships and setting up linear equations. (**8.EE.5-**

8▲)

369

370   **Domain: Geometry**

371

372 In grade seven, students solved problems involving scale drawings and informal

373 geometric constructions, and they worked with two- and three-dimensional

374   shapes to solve problems involving area, surface area, and volume. In grade,

375   eight students complete their work on volume by solving problems involving

376   cones, cylinders, and spheres. The also analyze two- and three-dimensional

377   space and figures using distance, angle, similarity, and congruence and through

378   understanding and applying the Pythagorean Theorem, which is a critical area of

379   instruction at the grade.

380

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.
1. Verify experimentally the properties of rotations, reflections, and translations:
     a. Lines are taken to lines, and line segments to line segments of the same length.
     b. Angles are taken to angles of the same measure.
     c. Parallel lines are taken to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

381

382   A major shift in the traditional curriculum occurs in this domain in grade eight with

383   the introduction of basic transformational geometry. In particular, the notion of

384   *congruence* is defined differently than it has been defined in the past. Whereas

385   previously two shapes were taken to be congruent if they had the "same size and

386   same shape," this imprecise notion is exchanged for a more precise one, that a

387   two-dimensional figure is congruent to another if the second can be obtained

388   from the first by a sequence of rotations, reflections, and translations. Given that

389   this is the case, students need ample opportunities to explore these three

390   geometric transformations and their properties. The work in the grade eight

391   Geometry domain is designed to provide a seamless transition to the Geometry

392   conceptual category in the higher mathematics courses, which begins with

393   transformational geometry from a more advanced perspective.

394

395   With the aid of physical models, transparencies, and geometry software, eighth

396   grade students gain an understanding of transformations and their relationship to

397   congruence of shapes. Through experimentation, students verify the properties

398   of rotations, reflections, and translations, including discovering that these

399   transformations change the position of a geometric figure but not its shape or

400   size **(8.G.1a-c▲)**. Finally, students come to understand that congruent shapes

401   are precisely those that can be "mapped" one onto the other by using rotations,

402   reflections or translations.

403

Characteristics of Rotations, Reflections, Translations.[6] Students come to understand that the following transformations result in shapes that are *congruent* to one another.
Students understand a *rotation* as the spinning of a figure around a fixed point known as the center of rotation. Rotations are usually performed counterclockwise according to a certain angle of rotation, unless otherwise specified.
Students understand a *reflection* as the flipping of an object over a line, known as the line of reflection.
Students understand a *translation* as the shifting of an object in one direction a fixed distance, so that any point lying on the shape moves the same distance in the same direction.

404

405   Students also study dilations in standard **(8.G.3▲)**. A *dilation* with scale factor

406    $k > 0$ can be thought of as a stretching (if $k > 1$) or shrinking (if $k < 1$) of an

407   object. In a dilation, a point is specified from which the distance to the points of a

408   figure are multiplied to obtain new points, and hence a new figure.

409

Examples of Geometric Transformations. (Note that the original figure is called the *preimage*, while the new figure is called the *image*.)

---

[6] An example of an interactive online tool that shows transformation is "Interactive Transmographer," which allows students to work with rotation, reflection, and translation. Available at http://www.shodor.org/interactivate/activities/Transmographer/ (Shodor 2013).

Rotation: A figure can be rotated up to 360° about the center of rotation.

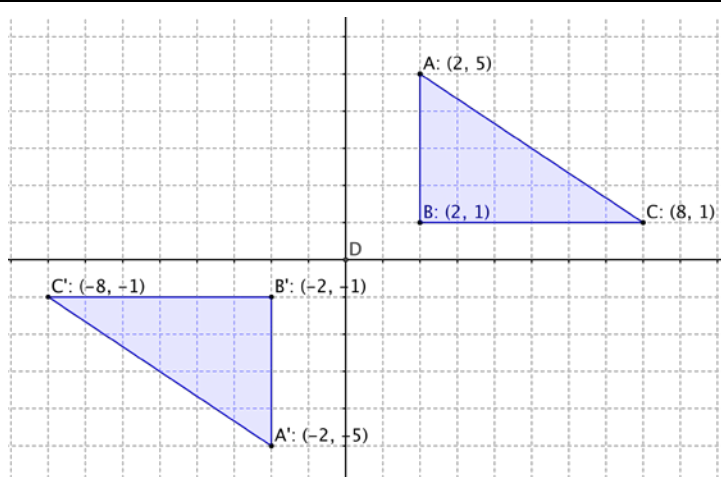Consider when $\triangle ABC$ is rotated 180° *clockwise* about the origin. The coordinates of $\triangle ABC$ are $A(2,5)$, $B(2,1)$, and $C(8,1)$. When rotated 180°, the image triangle $\triangle A'B'C'$ has coordinates $A'(-2,-5)$, $B'(-2,-1)$, $C'(-8,-1)$. Each coordinate is the opposite of its preimage point's coordinate.

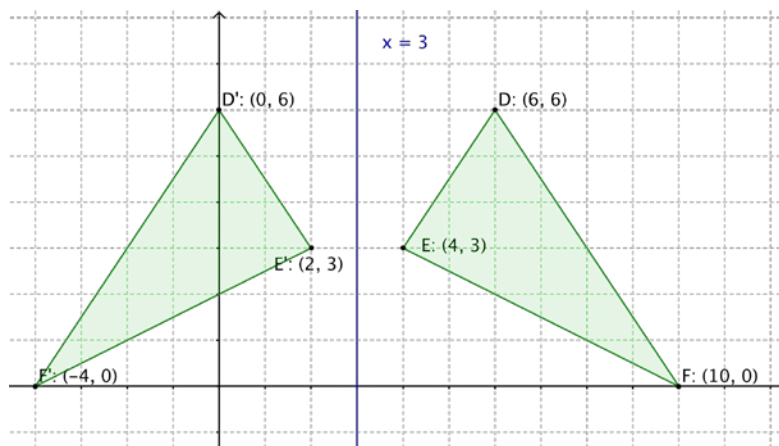**Reflection:** In the picture shown, $\triangle DEF$ has been reflected across the line $x = 3$. Notice the change in the orientation of the points, in 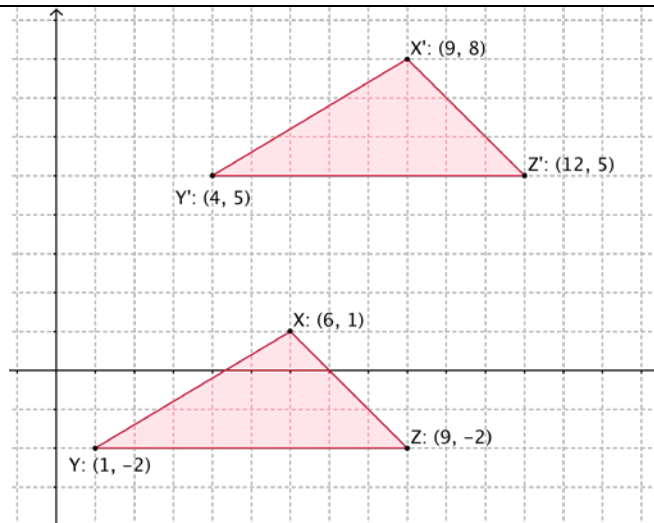the sense that the clockwise order of the preimage D-E-F is reversed in the image to D'-F'-E'. Notice also that each point on the image is at a same distance from the line of reflection as its corresponding point on the preimage.

Translation: Here, $\triangle XYZ$ has been translated 3 units to the right and 7 units up. Orientation is preserved. It is not too hard to see that under this transformation, a point (x, y) yields the image point $(x + 3, y + 7)$.

Dilation: In the picture, $\triangle UVW$ has been dilated from the origin $P:(0,0)$ by a factor of $k = 3$. The picture shows that the segments $OU, OV,$ and OW have all been multiplied by the factor $k = 3$, which results in new triangle, $\triangle U'V'W'$. By definition, $\triangle UVW$ and $\triangle U'V'W'$ are *similar* triangles. Students should experiment and find that the ratios of correspondi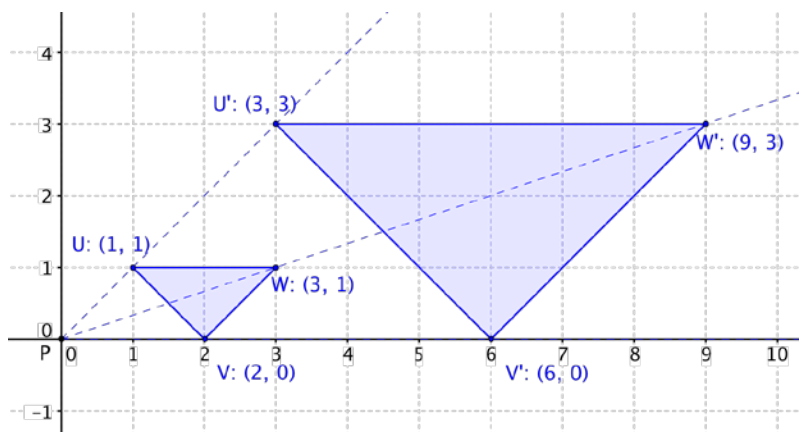ng side lengths satisfy $\frac{U'V'}{UV} = \frac{V'W'}{VW} = \frac{U'W'}{UW} = 3$, which corresponds to k. Students can apply the Pythagorean Theorem (**8.G.7-8**) to find the side lengths and justify this result: e.g., by finding the lengths of VW and $V'W'$:

$$\overline{VW} = \sqrt{1^2 + 1^2} = \sqrt{2} \text{ and } \overline{V'W'} = \sqrt{3^2 + 3^2} = \sqrt{18}$$

(Students can check informally that $\sqrt{18} = 3\sqrt{2}$ as formal work with radicals has not yet begun.)

410
411

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

412

413   Analogous to the new definition of congruence, the definition of *similar shapes*

414   has been refined to be more precise. Whereas previously shapes were said to

415   be similar if they had the "same shape but not necessarily the same size," now,

416   two shapes are called *similar* if the second can be obtained from the first by a

417   sequence of rotations, reflections, translations, and dilations **(8.G.4▲)**. Through

418   investigating dilations and using reasoning such as in the previous example,

419   students find that:

420   1. When two shapes are similar, the length of a segment AB in the first shape is

421   multiplied by the scale factor k to give the length of the corresponding

422   segment $A'B'$ in the second shape: $\overline{A'B'} = k \cdot \overline{AB}$.

423   2. Since the previous fact is true for all sides of a dilated shape, the ratio of the

424   lengths any two corresponding sides of the first and second shape is equal to

425   k.

426   3. It is also true that the ratio of any two side lengths from the first shape is the

427   same as the ratio of the corresponding side lengths from the second shape,

428   e.g., $\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{A'B'}}{\overline{B'C'}}$. (Students can justify this algebraically since fact 2 yields that

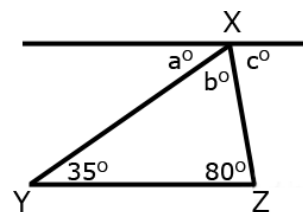429 $\frac{\overline{AB}}{\overline{A'B'}} = \frac{\overline{BC}}{\overline{B'C'}}$.)

430

431   Students use informal arguments to establish facts (**8.G.5▲**) about the angle

432   sum and exterior angles of triangles (e.g., consecutive exterior angles are

433   supplementary), the angles created when parallel lines are cut by a transversal

434   (e.g., corresponding angles are congruent), and the angle–angle criterion for

435   similarity of triangles (if two angles of a triangle are congruent to two angles of

436   another triangle, the two triangles are similar). The angle-angle criterion for

437   triangle similarity, when coupled with the previous three properties of similar

438   shapes, allows students to justify the fact that the slope of a line is the same

439   between any two points on the line (see discussion of standard **8.EE.6▲**).

440

Example: The sum of the measures of the angles of a triangle is 180°.
In the figure shown, the line through point X is parallel to segment YZ. I know that $a = 35$ because it is the measure of an angle that is alternating with angle $\angle Y$. For a similar reason, $c = 80$. Because lines have an angle measure of 180°, we know that $b = 180 - (35 + 80) = 65°$. So the sum of the measures of the angles in this triangle is 180°.

441

Geometry   8.G
Understand and apply the Pythagorean Theorem.
6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-

> world and mathematical problems in two and three dimensions.
> 8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

442

443 The Pythagorean Theorem is useful in practical problems, relates to grade-level

444 work in irrational numbers, and plays an important role mathematically in

445 coordinate geometry in higher mathematics. In grade eight, students explain a

446 proof of the Pythagorean Theorem (**8.G.6▲**). There are many varied and

447 interesting proofs of the Pythagorean Theorem.[7] In grade eight students apply

448 the theorem to determine unknown side lengths in right triangles (**8.G.7▲**) and to

449 find the distance between two points in a coordinate system (**8.G.8▲**). Work with

450 the Pythagorean Theorem will support students' work in high school-level

451 coordinate geometry.

452

453   [Note: Sidebar]

Focus, Coherence, and Rigor:

Understanding, modeling and applying (**MP.4**) the Pythagorean Theorem and its converse requires that student look for and make use of structure (**MP.7**) and express repeated reasoning (**MP.8**). Students also construct and critique arguments as they explain a proof of the theorem and its converse to others (**MP.3**). (Adapted from The Charles A. Dana Center Mathematics Common Core Toolbox 2012).

454

<table>
<tr><td>Geometry</td><td>8.G</td></tr>
</table>

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

455

456 In grade seven students learned about the area of a circle. In eighth grade

457 students learn the formulas for calculating the volumes of cones, cylinders, and

458 spheres and use the formulas to solve real-world and mathematical problems

459 (**8.G.9**). When students learn to solve problems involving volumes of cones,

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[7] For example, geometric "proof without words" of the Pythagorean Theorem available at http://illuminations.nctm.org/activitydetail.aspx?id=30 (NCTM Illuminations 2013).

The *Mathematics Framework* was adopted by the California State Board of Education on November 6, 2013. The *Mathematics Framework* has not been edited for publication.

460 cylinders and spheres — together with their previous grade seven work in angle

461 measure, area, surface area and volume — they will have acquired a well-

462 developed set of geometric measurement skills. These skills, along with

463 proportional reasoning and multistep numerical problem solving, can be

464 combined and used in flexible ways as part of modeling during high school and in

465 college and careers (Adapted from PARCC 2012).

466

467

468   **Domain: Statistics and Probability**

469 Building on work in earlier grades with univariate measurement data and

470 analyzing data on line plots and histograms, eighth-grade students begin to work

471 with bivariate measurement data and use scatter plots to represent and analyze

472 the data.

473 Bivariate measurement data consist of data that represent two measurements.

474 Scatter plots can show the relationship between the two measured variables.

475 Collecting and analyzing bivariate measurement data help students to answer

476 questions such as how does more time spent on homework affect test grades

477 and what is the relationship between years of education and annual income.

478

Statistics and Probability	8.SP
Investigate patterns of association in bivariate data.	
1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	
4. Understand that patters of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class*	

> *on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

479
480 Students construct and interpret scatter plots to investigate patterns of
481 association between two quantities (**8.SP.1**). Students build on their previous
482 knowledge of scatter plots to examine relationships between variables. They
483 analyze scatter plots to determine positive and negative associations, the degree
484 of association, and type of association. Students examine outliers to determine if
485 data points are valid or represent a recording or measurement error.
486

Example: Creating Scatter Plots.

Customer satisfaction is vital to the success of fast food restaurants, and speed of service is a key component of that satisfaction. In order to determine the best staffing level, the owners of a local fast food restaurant have collected the data below showing the number of staff members and the average time for filling an order. Describe the association between the number of staff and the average time for filling an order, and make a recommendation as to how many staff should be hired.

Number of staff	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

487
488 Students can use tools such as those at the National Center for Educational
489 Statistics (NCES) to create a graph or generate data sets, available
490 at http://nces.ed.gov/nceskids/createagraph/default.aspx (NCES Kids Zone
491 2013).
492
493 Students know that straight lines are widely used to model relationships between
494 two quantitative variables (**8.SP.2**). For scatter plots that appear to show a linear
495 association, students informally fit a line (e.g., by drawing a line on the coordinate
496 plane between data points) and informally assess the fit by judging the closeness
497 of the data points to the straight line.
498

Example: Informally Determining a Line of Best Fit.

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles

---

traveled and how many gallons of gas are used. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles Traveled	0	75	120	160	250	300
Gallons Used	0	2.3	4.5	5.7	9.7	10.7

499

500   Students solve problems in the context of bivariate measurement data by using

501   the equation of a linear model (**8.SP.3**). They interpret the slope and the y-

502   intercept. For example in a linear model for a biology experiment, students

503   interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each

504   day is associated with an additional 1.5 cm in the height of the plant.
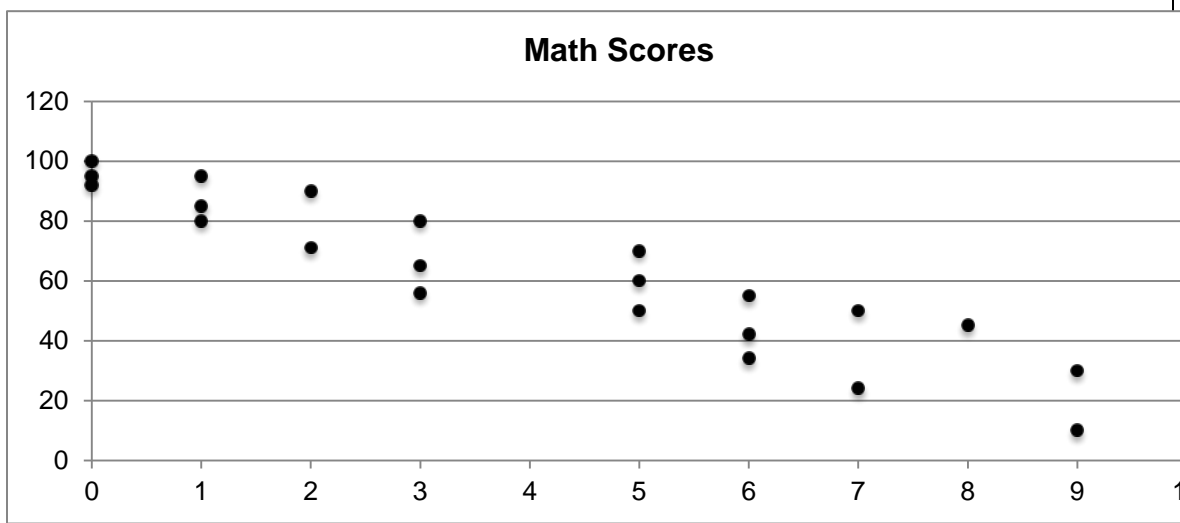
505

506

Example: Finding a Linear Model for a Data Set.

Given data from students' math scores and absences, make a scatter plot. Informally fit a line to the graph, and determine an approximate linear function that models the data. What would you expect to be the score of a student with 4 absences?

Solution:

Absences	3	5	1	1	3	6	5	3	0	7	8
Math Scores	65	50	95	85	80	34	70	56	100	24	45
Absences	2	9	0	6	6	2	0	5	7	9	1
Math Scores	71	30	95	55	42	90	92	60	50	10	80

Students would most likely make a scatter plot using simple data software, and find a graph that looks like the following:

Math Scores

> Students can use graphing software to find a line of best fit. Such a line might be $y = -8x + 95$. They interpret this equation as defining a function the gives the approximate score of a student based on the number of absences they have. We would expect a student with 4 absences to have a score of approximately $y = -8(4) + 95 = 63$.

507 (Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina
508 2013)

509

510   [Note: Sidebar]

> **Focus, Coherence, and Rigor:**
>
> In eight grade students apply their experience with coordinate geometry and linear functions to plot bivariate data as points on a plane and to make use of the equation of a line in analyzing the relationship between two paired variables. Students develop mathematical practices as they build statistical models to explore the relationship between two variables (**MP.4**) and look for and make use of structure to describe possible association in bivariate data (**MP.7**) (Adapted from Progressions 6-8 SP 2011).

511

512 Students learn to see patterns of association in bivariate categorical data in a
513 two-way table (**8.SP.4**). They construct and interpret a two-way table that
514 summarizes data on two categorical variables collected from the same subjects.
515 The two-way table displays frequencies and relative frequencies. Students use
516 relative frequencies calculated from rows or columns to describe a possible
517 association between the two variables. For example, students collect data from
518 their classmates about whether they have a curfew and whether they do chores
519 at home. The two-way table allows students to easily see if students who have a
520 curfew also tend to do chores at home.

521

> **Example: Two-Way Tables for Categorical Data.**
>
> The table illustrates the results when 100 students were asked the survey questions: (1) Do you have a curfew? (2) Do you have assigned chores?
>
> | Chores | | Curfew | |
> | --- | --- | --- | --- |
> | | | Yes | No |
> | | Yes | 40 | 10 |

Students can examine the survey results to determine if there is evidence that those who have a curfew also tend to have chores.	No	10	40

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores since it appears that most students with chores have a curfew and most students without chores do not have a curfew.

522

523   [Note: Sidebar]

Focus, Coherence, and Rigor:

Work in the Statistics and Probability cluster "Investigate patterns of association in bivariate data" involves looking for patterns in scatterplots and using linear models to describe data. This is directly connected to major work in the Expressions and Equations clusters (**8.EE.1-8▲**) and also provides opportunities for students to model with mathematics (**MP.4**).

524 (Adapted from CDE Transition Document 2012, Arizona 2012, and N. Carolina

525   2013)

526 A detailed discussion of statistics and probability is provided online at Draft 6–8
527 Progression on Statistics and Probability (Progressions 6-8 SP 2011).

528

529 **Essential Learning for the Next Grade**

530 In middle school, multiplication and division develop into powerful forms of ratio

531 and proportional reasoning. The properties of operations take on prominence as

532 arithmetic matures into algebra. The theme of quantitative relationships also

533 becomes explicit in grades six through eight, developing into the formal notion of

534 a function by grade eight. Meanwhile, the foundations for later courses in

535 deductive geometry are laid in the middle grades. The gradual development of

536 data representations in kindergarten through grade five leads to statistics in

537 middle school: the study of shape, center, and spread of data distributions;

538 possible associations between two variables; and the use of sampling in making

539 statistical decisions.

540

541   In higher mathematics courses, algebra, functions, geometry, and statistics

542   develop with an emphasis on modeling. Students continue to take a thinking

543   approach to algebra, learning to see and make use of structure in algebraic

544   expressions of growing complexity (Adapted from PARCC 2012).

545

546   To be prepared for the higher mathematics courses students should be able to

547   demonstrate they have acquired certain mathematical concepts and procedural

548   skills by the end of grade eight. Prior to grade eight, some standards identify

549   fluency expectations at the grade level. In eighth grade linear algebra is an

550   instructional focus and although the grade eight standards do not specifically

551   identify fluency expectations, eighth grade students who can fluently solve linear

552   equations (**8.EE.7▲**) and pairs of simultaneous linear equations (**8.EE.8▲)** will

553   be better prepared to complete courses in higher mathematics. These fluencies

554   and the conceptual understandings that support them are foundational for work in

555   higher mathematics. Students have been working informally with one-variable

556   linear equations since as early as kindergarten. This important line of

557   development culminates in grade eight with the solution of general one-variable

558   linear equations, including cases with infinitely many solution or no solutions as

559   well as cases requiring algebraic manipulation using properties of operations.

560

561   Of particular importance for students to attain in grade eight are skills and

562   understandings to work with radical and integer exponents **(8.EE.1-4▲);**

563   understand connections between proportional relationships, lines, and linear

564   equations **(8.EE.5-6▲);** analyze and solve linear equations and pairs of

565   simultaneous linear equations **(8.EE.7-8▲);** and define, evaluate, and compare

566   functions **(8.F.1-3▲)**. In addition, the skills and understandings to use functions

567   to model relationships between quantities (**8.F.4-5**) will better prepare students to

568   use mathematics to model real-world problems in the higher grades.

569

570   **Guidance on Course Placement and Sequences**

571   The standards support a progression of learning. Many culminating standards

572   that remain important far beyond the particular grade level appear in the middle

573   grades. As stated in the Common Core standards:

574   "…some of the highest priority content for college and career readiness comes

575   from grades 6–8. This body of material includes powerfully useful proficiencies

576   such as applying ratio reasoning in real-world and mathematical problems,

577   computing fluently with positive and negative fractions and decimals, and solving

578   real-world and mathematical problems involving angle measure, area, surface

579   area, and volume…"

580   (CCSSI 2010, Note on courses and transitions).

581

582   The Common Core standards for grades 6 – 8 are comprehensive, rigorous, and

583   non-redundant. Acceleration will require compaction not the former strategy of

584   deletion. Therefore, careful consideration needs to be made before placing a

585   student into higher mathematics coursework in middle grades. Acceleration may

586   get students to advanced coursework but might create gaps in students'

587   mathematical background. Careful consideration and systematic collection of

588   multiple measures of individual student performance on both the content and

589   practice standards will be required. For additional information and guidance on

590   course placement, see "Appendix A: Course Placement and Sequences" in this

591   framework.

592

593

594 # Grade 8 Overview
595

596

597 ## The Number System

598 - Know that there are numbers that are not rational, and
599   approximate them by rational numbers.

600

601 ## Expressions and Equations

602 - Work with radicals and integer exponents.

603 - Understand the connection between proportional
604   relationships, lines, and linear equations.

605 - Analyze and solve linear equations and pairs of
606   simultaneous linear equations.

607

608 ## Functions

609 - Define, evaluate, and compare functions.

610 - Use functions to model relationships between quantities.

611

612 ## Geometry

613 - Understand congruence and similarity using physical models, transparencies, or
614   geometry software.

615 - Understand and apply the Pythagorean Theorem.

616 - Solve real-world and mathematical problems involving volume of cylinders, cones,
617   and spheres.

618

619 ## Statistics and Probability

620 - Investigate patterns of association in bivariate data.

621

Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

622   Grade 8

The Number System	8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π_2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations	8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.

   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

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Functions	8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry	8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:
    a. Lines are taken to lines, and line segments to line segments of the same length.
    b. Angles are taken to angles of the same measure.
    c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

623
624   [1]Function notation is not required in Grade 8.

Statistics and Probability	8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

625
626