Introduction

"The highest form of pure thought is mathematics."

Plato (427 – 347 BCE)

All students need a high-quality mathematics program designed to prepare them to graduate from high school ready for college, a career, and civic life. In support of this goal, California adopted the California Common Core State Standards for Mathematics (CA CCSSM) in August 2010, replacing the 1997 statewide mathematics academic standards. As part of the modification of the CA CCSSM in January 2013, the California State Board of Education also approved the organization of the higher mathematics standards into model courses.

The development of standards began as a voluntary, state-led effort, coordinated by the Council of Chief State School Officers (CCSSO) and the National Governors Association Center for Best Practices (NGA), committed to developing a set of standards that would help prepare students for success in career and college. The standards developed by this Common Core State Standards Initiative are based on evidence of the skills and knowledge needed for college and career readiness and an expectation that students be able to both know and do mathematics by solving a range of problems and engaging in key mathematical practices (Achieve 2013).

Why is Mathematics Important?

Mathematics impacts everyday life, future careers, and good citizenship. A solid foundation in mathematics prepares students for future occupations including the fields of business, medicine, science, engineering, and technology. Students' understanding of probability and the ability to quantify and analyze information enables them to interpret economic data, participate in political discussions, and make wiser personal financial decisions. Mathematical modeling is a tool for solving everyday problems, making informed decisions, improving life skills (i.e., logical thinking, reasoning, and problem solving), planning, designing, predicting, and developing financial literacy.

Success in mathematics education provides students with college and career options and increases prospects for future income. Knowledge and understanding of high school mathematics correlates to access to college, graduation from college, and earnings in the top quartile of income from employment. The value of such preparation promises to be even greater in the future. The National Science Board indicates that the growth of jobs in the mathematics-intensive science and engineering workforce is outpacing overall job growth by 3:1 (National Mathematics Advisory Panel 2008).

Mathematics Achievement

Student achievement in mathematics is important to prepare students for college and future careers. Nationally students' achievement in mathematics has not kept pace with their international peers. Achievement gaps still exist in our country, college remediation rates are too high, and some students are unprepared to perform and thrive in the workforce.

California's student achievement data reflects similar challenges for some students. The 2011 National Assessment of Educational Progress (NAEP) results indicate that California's fourth and eighth grade students continue to make incremental gains in their mathematics scores; however, too many students also continue to place at the "Basic" achievement level, which denotes partial mastery of fundamental skills (CDE 2011).

Standards Implementation

The CA CCSSM resemble the standards of the highest-achieving nations and reflect the importance of focus, coherence, and rigor. California's implementation of the CA CCSSM demonstrates a commitment to providing a world-class education for all students that supports narrowing the achievement gap, life-long learning, and the skills and knowledge necessary to fully participate in the 21st century global economy. The CA CCSSM will build on California's standards-based educational system in which standards, curriculum, instruction, assessment, and accountability are aligned to support student attainment of the standards. Teachers and local school officials, in collaboration with families and community partners, will use standards to help students achieve academic success (CDE 2012).

California Common Core State Standards for Mathematics (CA CCSSM)

Toward Greater Focus, Coherence and Rigor

For over a decade, research conducted on mathematics education in high-performing countries has pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent to improve mathematics

achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep” (CCSSI 2012).

The development of these standards were informed by international benchmarking and began with research-based learning progressions detailing what is known about how students’ mathematical knowledge, skills, and understanding develop over time. The progression from kindergarten standards to standards for higher mathematics exemplifies the three principles of focus, coherence, and rigor that are the basis for the CA CCSSM. The standards stress conceptual understanding, procedural skill and fluency, and application, to ensure students are learning and absorbing the critical information they need to succeed at higher levels and can apply their learning in increasingly complex situations.

The CA CCSSM include two types of standards: Eight Standards for Mathematical Practice (the same at each grade level) and Standards for Mathematical Content (different at each grade level). These standards address both “habits of mind” that students should develop to foster mathematical understanding and expertise, and also skills and knowledge – what students need to know and be able to do. The standards also call for mathematical practices and mathematical content to be connected as students engage in mathematics. The Standards for Mathematical Practice are defined in the Overview chapter beginning on page 10. The Standards for Mathematical Content and the Standards for Mathematics Practice are listed at the end of each grade level (K-8) and higher mathematics course.

Table 1: Guiding Principles for Mathematics Programs in California
Guiding Principle 1: Learning Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.
Guiding Principle 2: Teaching An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.
Guiding Principle 3: Technology Technology is an essential tool that should be used strategically in mathematics education.
Guiding Principle 4: Equity All students should have a high quality mathematics program that prepares them for college and career.
Guiding Principle 5: Assessment Assessment of student learning in mathematics should take many forms to inform instruction and learning.

Guiding Principles for Mathematics Programs in California

The following five Guiding Principles are philosophical statements that underlie the Standards for Mathematical Practice, Standards for Mathematical Content, and other resources in this framework. They should guide the construction and evaluation of mathematics programs in the schools and the broader community. The Standards for Mathematical Practice are interwoven throughout the Guiding Principles.

Guiding Principle 1: Learning

Mathematical ideas should be explored in ways that stimulate curiosity, create enjoyment of mathematics, and develop depth of understanding.

Students need to understand mathematics deeply and use it effectively. The Standards for Mathematical Practice (MP) describe ways in which students increasingly engage with the subject matter as they grow in mathematical maturity and expertise through the elementary, middle, and high school years.

To achieve mathematical understanding, instruction and learning must balance mathematical procedures and conceptual understanding. Students should be actively engaged in doing meaningful mathematics, discussing mathematical ideas, and applying mathematics in interesting, thought-provoking situations. Student understanding is further developed through ongoing reflection about cognitively demanding and worthwhile tasks.

Tasks should be designed to challenge students in multiple ways. Short- and long-term investigations that connect procedures and skills with conceptual understanding are integral components of an effective mathematics program. Activities should build upon students' curiosity and prior knowledge and enable them to solve progressively deeper, broader, and more sophisticated problems. (See MP.1: *Make sense of problems and persevere in solving them.*) Mathematical tasks reflecting sound and significant mathematics should generate active classroom talk, promote the development of conjectures, and lead to an understanding of the necessity for mathematical reasoning. (See MP. 2: *Reason abstractly and quantitatively.*)

Guiding Principle 2: Teaching

An effective mathematics program is based on a carefully designed set of content standards that are clear and specific, focused, and articulated over time as a coherent sequence.

The sequence of topics and instruction should be based on what is known about how students' mathematical knowledge, skill, and understanding develop over time. What and how students are taught should reflect not only the topics within mathematics but also the key ideas that determine how knowledge is organized and generated within mathematics. (See MP.7: *Look for and make use of structure.*) Students should be asked to apply their learning and to show their mathematical thinking and understanding. This high-quality instruction requires teachers who have a deep knowledge of mathematics as a discipline.

Mathematical problem solving is the hallmark of an effective mathematics program. Skill in mathematical problem solving requires practice with a variety of mathematical problems as well as a firm grasp of mathematical techniques and their underlying principles. Armed with this deeper knowledge, students can use mathematics in a flexible way to attack various problems and devise different ways of solving any particular problem. (See MP.8: *Look for and express regularity in repeated reasoning.*) Mathematical problem solving calls for reflective thinking, persistence, learning from the ideas of others, and going back over one's own work with a critical eye. Students should be able to construct viable arguments and critique the reasoning of others. They should analyze situations and justify their conclusions, communicate their conclusions to others, and respond to the arguments of others. (See MP.3: *Construct viable arguments*

and critique the reasoning of others.) Students at all grades should be able to listen to or read the arguments of others, decide whether they make sense, and ask questions to clarify or improve the arguments.

Mathematical problem solving provides students with experiences to develop other mathematical practices. Success in solving mathematical problems helps to create an abiding interest in mathematics. Students learn to model with mathematics and to apply the mathematics that they know to solve problems arising in everyday life, society, and the workplace. (See MP.4: *Model with mathematics.*)

For a mathematics program to be effective, it must also be taught by knowledgeable teachers. According to Liping Ma, “The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics” (Ma 2010). Research on the relationship between teachers’ mathematical knowledge and students’ achievement confirms the importance of teachers’ content knowledge (National Mathematics Advisory Panel 2008). The message from the research is clear: having knowledgeable teachers really does matter; teacher expertise in a subject drives student achievement. “Improving teachers’ content subject matter knowledge and improving students’ mathematics education are thus interwoven and interdependent processes that must occur simultaneously” (Ma 2010). (See the “Instructional Strategies” and “Supporting High Quality Common Core Mathematics Instruction” chapters for more information.

Guiding Principle 3: Technology

Technology is an essential tool that should be used strategically in mathematics education.

Technology enhances the mathematics curriculum in many ways. Tools such as measuring instruments, manipulatives (such as base-ten blocks and fraction pieces), scientific and graphing calculators, and computers with appropriate software, if properly used, contribute to a rich learning environment for investigating, exploring, developing, and applying mathematical concepts. Appropriate use of calculators is essential; calculators should not be used as a replacement for basic understanding and skills. Elementary students should learn how to perform the basic arithmetic operations independent of the use of a calculator (NCES 1998). The use of a graphing calculator can help middle and secondary students visualize properties of functions and their graphs. Graphing calculators should be used to enhance student understanding and skills rather than replace them.

Teachers and students should consider the available tools when presenting or solving a problem. Students should be familiar with tools appropriate for their grade level to be able to make sound decisions about which of these tools would be helpful. (See MP.5: *Use appropriate tools strategically.)*

Technology enables students to communicate ideas within the classroom or to search for information in external databases such as the Internet, an important supplement to a

school's internal library resources. Technology can be especially helpful in assisting students with special needs in regular and special classrooms, at home, and in the community.

Technology changes the mathematics to be learned, as well as when and how it is learned. For example, currently available technology provides a dynamic approach to such mathematical concepts as functions, rates of change, geometry, and averages that was not possible in the past. Some mathematics becomes more important because technology requires it, some becomes less important because technology replaces it, and some becomes possible because technology allows it. (See the "Technology in the Teaching of Mathematics" chapter for additional information.)

Guiding Principle 4: Equity

All students should have a high quality mathematics program that prepares them for college and a career.

All California students should have a high quality mathematics program that meets the goals and expectations of these standards and addresses students' individual interests and talents. The standards provide clear signposts along the way to the goal of college and career readiness for all students. The standards provide for a broad range of students, from those requiring a significant amount of extra support in mathematics to others needing minimal support or enrichment opportunities. To promote achievement of these standards teachers should plan for, instruct, model, and support classroom talk, reflection, use of multiple problem solving strategies, and a positive disposition

toward mathematics. They should have high expectations for all students. At every level of the education system, teachers should act on the belief that every child can and should learn challenging mathematics. Teachers and guidance personnel should advise students and parents about why it is important to take advanced courses in mathematics and how this will prepare students for success in college and the workplace.

All students should have the benefit of quality instructional materials, good libraries, and adequate technology. All students must have the opportunity to learn and meet the same high standards. In order to meet the needs of the greatest range of students, mathematics programs should provide the necessary intervention and support for those students who are below or above grade-level expectations. Practice and enrichment should extend beyond the classroom. Tutorial sessions, mathematics clubs, competitions, and apprenticeships are examples of mathematics activities that promote learning.

Because mathematics is the cornerstone of many disciplines, a comprehensive curriculum should include applications to everyday life and modeling activities that demonstrate the connections among disciplines. Schools should also provide opportunities for communicating with experts in applied fields to enhance students' knowledge of these connections. (See Standard for Mathematical Practice 4: *Model with mathematics.*)

An important part of preparing students for college and careers is to ensure that they have the necessary mathematics and problem-solving skills to make sound financial decisions that they will face in the world every day, such as setting up a bank account; learning about saving and interest; understanding student loans; reading credit and debit statements; selecting the best buy when shopping; and choosing the most cost effective cell phone plan based on monthly usage. (See the "Universal Access" chapter and the "Financial Literacy and Mathematics Education" and "Mathematical Modeling" appendices for additional information.)

Guiding Principle 5: Assessment

Assessment of student learning in mathematics should take many forms to inform instruction and learning.

A comprehensive assessment program is an integral component of an instructional program. It provides students with frequent feedback on their performance, teachers with diagnostic tools for gauging students' depth of understanding of mathematical concepts and skills, parents with information about their children's performance in the context of program goals, and administrators with a means for measuring student achievement.

Assessments take a variety of forms, require varying amounts of time, and address different aspects of student learning. Gaps in knowledge and errors in reasoning can be identified when students "think aloud" or talk through their reasoning. By observing and questioning students as they work, teachers can gain insight into students' abilities to

apply appropriate mathematical concepts and skills, make conjectures, and draw conclusions. Homework, mathematics journals, portfolios, oral presentations, and group projects offer additional means for capturing students' thinking, knowledge of mathematics, facility with the language of mathematics, and ability to communicate what they know to others. Tests and quizzes assess knowledge of mathematical concepts, operations, and skills and their efficient application to problem solving; they can also pinpoint areas in need of more practice or teaching. Taken together, the results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.

Assessment should also be a major component of the learning process. As students help identify goals for lessons or investigations, they gain greater awareness of what they need to learn and how they will demonstrate that learning. Engaging students in this kind of goal-setting can help them reflect on their own work, understand the standards to which they are held accountable, and take ownership of their learning. (Adapted from Massachusetts Department of Elementary and Secondary Education 2011) (See the "Assessment" chapter for additional information.)

Supporting 21st Century Learning

California is part of a growing national movement to teach students the critical thinking and problem-solving skills they need for college, career, and civic life. The Partnership for 21st Century Skills (P21) developed a framework for 21st century learning that

consists of student outcomes and support systems. The student outcomes are organized into 1) core subjects and 21st century interdisciplinary themes, which include global awareness; financial, economic, business, and entrepreneurial literacy; civic literacy; health literacy; and environmental literacy, 2) life and career skills, which include flexibility and adaptability, initiative and self-direction, social and cross-cultural skills, productivity and accountability, and leadership and responsibility, 3) learning and innovation skills, often referred to as the "4Cs": creativity and innovation, critical thinking and problem solving, communication, and collaboration, and 4) information, media and technology skills, which include information literacy, media literacy, and information, communications and technology literacy. Support systems include standards and assessments, curriculum and instruction, professional development, and learning environments.

California educators need to intentionally include the 4Cs in mathematics instruction. A fundamental goal is to promote higher order mathematical thinking skills and interdisciplinary approaches that integrate the use of supportive technologies, inquiry, and problem-based learning to provide contexts for pupils to apply learning in relevant, real-world scenarios and that prepare all pupils for college, career, and citizenship in the 21st century. Mathematics instruction is instrumental to mastering P21 interdisciplinary themes, particularly financial, economic, business, and entrepreneurial literacy. Resources connecting the Partnership for 21st Century Skills with the Common Core State Standards can be found at www.p21.org.

Purpose of the Framework

The purpose of this framework is to guide the curriculum development and instruction that teachers provide in their efforts to ensure that all students meet or exceed the CA CCSSM. The framework provides a context for implementing the standards in the form of guidelines for educators and developers of instructional materials. Building on the standards, the framework addresses how all students in California public schools can best meet the standards.

Implementation of the CA CCSSM will take time and effort, but it also provides a new opportunity to ensure that California' students are held to the same high expectations in mathematics as their national and global peers. Educators and administrators, as well as parents, are challenged to become familiar with the standards and to support raising the bar for student achievement through rigorous curriculum and instruction that develops students' conceptual understanding, procedural skill and fluency, and application of mathematics to solve problems.