**Criteria for Evaluating Mathematics Instructional Materials**

**for Kindergarten through Grade Eight**

**Adopted by the State Board of Education on January 16, 2013**

Instructional materials that are adopted by the state help teachers to present and students to learn the content set forth in the *Common Core State Standards for Mathematics with California Additions* (Standards)(this refers to the content standards and the standards for mathematical practice),as revised pursuant to California *Education Code* Section 60605.11*,* (added by Senate Bill 1200, Statutes of 2012). To accomplish this purpose, this document establishes criteria for evaluating instructional materials for the eight-year adoption cycle beginning with the primary adoption in 2013-14. These criteria serve as evaluation guidelines for the statewide adoption of mathematics instructional materials for kindergarten through grade eight, as called for in *Education Code* Section 60207.

The Standardsrequire focus, coherence, and rigor, with content and mathematical practice standards intertwined throughout. The standards are organized by grade-level in kindergarten through grade eight and by conceptual categories for higher mathematics. For this adoption, the standards for higher mathematics are organized into model courses and are assigned to a first course in a traditional or an integrated sequence of courses. There are number of supportive and advisory documents that are available for publishers and producers of instructional materials that define the depth of instruction necessary to support the focus, coherence, and rigor of the standards. These documents include the *Progressions Documents for Common Core Math Standards* (<http://ime.math.arizona.edu/progressions/>), the *PARCC Model Content Frameworks (*<http://www.parcconline.org>*)*, *Smarter Balanced test specifications (*http://www.[smarterbalanced](http://www.smarterbalanced.org/).org)*, The Illustrative Mathematics Project*, (<http://illustrativemathematics.org/>), and draft chapters of *California Mathematics Curriculum Framework*. Overall, the Standards do not dictate a singular approach to instructional resources—to the contrary, they provide opportunities to raise student achievement through innovations.

It is the intent of the State Board of Education that these criteria be seen as neutral on the format of instructional materials in terms of digital, interactive online, and other types of curriculum materials.

I. Focus, Coherence, and Rigor in the Common Core State Standards for

Mathematics

With the advent of the Common Core, a decade’s worth of recommendations for greater focus and coherence finally have a chance to bear fruit. Focus and coherence are the two major evidence-based design principles of the Standards. These principles are meant to fuel greater achievement in a rigorous curriculum, in which students acquire conceptual understanding, procedural skill and fluency, and the ability to apply mathematics to solve problems. Thus, the implications of the standards for mathematics education could be summarized briefly as follows:

**Focus**: Place strong emphasis where the Standards focus

**Coherence**: Think across grades, and link to major topics in each grade

**Rigor**: In major topics, pursue with equal intensity:

 conceptual understanding,

 procedural skill and fluency, and

 applications

**Focus**

Focus requires that we significantly narrow the scope of content in each grade so that students more deeply experience that which remains.

The overwhelming focus of the Standards in early grades is arithmetic, along with the components of measurement that support it. That includes the **concepts** underlying arithmetic, the **skills** of arithmetic computation, and the ability to **apply** arithmetic to solve problems and put arithmetic to engaging uses. Arithmetic in the K–5 standards is an important life skill, as well as a thinking subject and a rehearsal for algebra in the middle grades.

Focus remains important through the middle and high school grades in order to prepare students for college and careers; surveys suggest that postsecondary instructors value greater mastery of prerequisites over shallow exposure to a wide array of topics with dubious relevance to postsecondary work.

Both of the assessment consortia have made the focus, coherence, and rigor of the Standards central to their assessment designs.[[1]](#footnote-1) Choosing materials that also embody the Standards will be essential for giving teachers and students the tools they need to build a strong mathematical foundation and succeed on standards-aligned assessments.

**Coherence**

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.[[2]](#footnote-2) The standards define progressions of learning that leverage these principles as they build knowledge over the grades.[[3]](#footnote-3)

When people talk about coherence, they often talk about making connections between topics. The most important connections are vertical: the links from one grade to the next that allow students to progress in their mathematical education. That is why it is critical to think across grades and examine the progressions in the standards to see how major content develops over time.

Connections at a single grade level can be used to improve focus, by tightly linking secondary topics to the major work of the grade. For example, in grade 3, bar graphs are not “just another topic to cover.” Rather, the standard about bar graphs asks students to use information presented in bar graphs to solve word problems using the four operations of arithmetic. Instead of allowing bar graphs to detract from the focus on arithmetic, the standards are showing how bar graphs can be positioned in support of the major work of the grade. In this way coherence can support focus.

Materials cannot match the contours of the Standards by approaching each individual content standard as a separate event. Nor can materials align to the Standards by approaching each individual **grade** as a separate event: “The standards were not so much assembled out of topics as woven out of progressions. Maintaining these progressions in the implementation of the standards will be important for helping all students learn mathematics at a higher level. . . For example, the properties of operations, learned first for simple whole numbers, then in later grades extended to fractions, play a central role in understanding operations with negative numbers, expressions with letters, and later still the study of polynomials. As the application of the properties is extended over the grades, an understanding of how the properties of operations work together should deepen and develop into one of the most fundamental insights into algebra. The natural distribution of prior knowledge in classrooms should not prompt abandoning instruction in grade-level content, but should prompt explicit attention to connecting grade-level content to content from prior learning. To do this, instruction should reflect the progressions on which the CCSSM [*Common Core State Standards for Mathematics*] are built.”[[4]](#footnote-4)

**Rigor**

To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications. The word “understand” is used in the Standards to set explicit expectations for conceptual understanding, the word “fluently” is used to set explicit expectations for fluency, and the phrase “real-world problems” and the star symbol () are used to set expectations and flag opportunities for applications and modeling (which is a standard for mathematical practice as well as a content category in high school). Real-world problems and standards that support modeling are also opportunities to provide activities related to careers and the work-world.

To date, curricula have not always been balanced in their approach to these three aspects of rigor. Some curricula stress fluency in computation, without acknowledging the role of conceptual understanding in attaining fluency. Some stress conceptual understanding, without acknowledging that fluency requires separate classroom work of a different nature. Some stress pure mathematics, without acknowledging first of all that applications can be highly motivating for students, and moreover, that a mathematical education should make students fit for more than just their next mathematics course. At another extreme, some curricula focus on applications, without acknowledging that math doesn’t teach itself.

The Standards do not take sides in these ways, but rather they set high expectations for all three components of rigor in the major work of each grade. Of course, that makes it necessary that we first follow through on the focus in the Standards—otherwise we are asking teachers and students to do more with less.

II. Criteria for Materials and Tools Aligned to the Standards

**Three Types of Programs**

Three types of programs will be considered for adoption: basic grade-level for kindergarten through grade eight, Algebra I, and Integrated Mathematics I (hereafter referred to as Mathematics I). All three types of programs must stand alone and will be reviewed separately. Publishers may submit programs for one grade or any combination of grades. In addition, publishers may include intervention and acceleration components to support students.

*Basic Grade-Level Program*

The basic grade-level program is the comprehensive curriculum in mathematics for students in kindergarten through grade eight. It provides the foundation for instruction and is intended to ensure that all students master the *Common Core State Standards for Mathematics with California Additions*.

*Common Core Algebra I and Common Core Mathematics I*

When students have mastered the content described in the *Common Core State Standards for Mathematics with California Additions* for kindergarten through grade eight, they will be ready to complete Common Core Algebra I or Common Core Mathematics I. The course content will be consistent with its high school counterpart and will articulate with the subsequent courses in the sequence.

**Criteria for Materials and Tools Aligned to the Standards**

The criteria for the evaluation of mathematics instructional resources for kindergarten through grade eight are organized into six categories:

**1. Mathematics Content/Alignment with the Standards**. Content as specified in the *Common Core State Standards for Mathematics with California Additions*, including the Standards for Mathematical Practices, and sequence and organization of the mathematics program that provide structure for what students should learn at each grade level.

**2**. **Program Organization.** Instructional materials support instruction and learning of the standards and include such features as lists of the standards, chapter overviews, and glossaries.

**3. Assessment**. Strategies presented in the instructional materials for measuring what students know and are able to do.

**4**. **Universal Access**. Access to the standards-based curriculum for all students, including English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.

**5**. **Instructional Planning**. Information and materials that contain a clear road map for teachers to follow when planning instruction.

**6**. **Teacher Support**. Materials designed to help teachers provide effective standards-based mathematics instruction.

Materials that fail to meet the criteria category 1 for Mathematics Content/Alignment with the Standards will not be considered suitable for adoption. The criteria for category 1 must be met in the core materials or via the primary means of instruction, rather than in ancillary components. In addition, programs must have strengths in each of categories 2 through 6 to be suitable for adoption.

**Category 1: Mathematics Content/Alignment with the Standards**

Mathematics materials should support teaching to the *Common Core State Standards for Mathematics with California Additions*. Instructional materials suitable for adoption must satisfy the following criteria:

1. **The mathematics content is correct, factually accurate, and written with precision. Mathematical terms are defined and used appropriately. Where the standards provide a definition, materials use that as their primary definition to develop student understanding.**

2. **The materials in basic instructional programs support comprehensive teaching of the *Common Core State Standards for Mathematics with California Additions* and include the standards for mathematical practice at each grade level or course.** The standards for mathematical practice must be taught in the context of the content standards at each grade level or course. The principles of instruction must reflect current and confirmed research. The materials must be aligned to and support the design of the *Common Core State Standards for Mathematics with California Additions* and address the grade-level content standards and standards for mathematical practice in their entirety.

3. **In any single grade in the kindergarten through grade eight sequence, students and teachers using the materials as designed spend the large majority of their time on the major work of each grade.** The major work (major clusters) of each grade is identified in the Content Emphases by Cluster documents for K–8[[5]](#footnote-5). In addition, major work should especially predominate in the first half of the year (e.g., in grade 3 this is necessary so that students have sufficient time to build understanding and fluency with multiplication). Note that an important **subset** of the major work in grades K–8 is the progression that leads toward Algebra I and Mathematics I (see Table 1, next page). Materials give especially careful treatment to these clusters and their interconnections. Digital or online materials that allow navigation or have no fixed pacing plan are explicitly designed to ensure that students’ time on task meets this criterion.

Table 1. Progress to Algebra in Grades K–8

**K 1 2 3 4 5 6 7 8**

Represent &

Know number names and the count sequence

Count to tell the number of objects

Compare numbers

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from

Work with numbers 11-19 to gain foundations for place value

Represent and solve problems involving addition and subtraction

Understand and apply properties of operations and the relationship between addition and subtraction

Add and subtract within 20

Work with addition and subtraction equations

Extend the counting sequence

Understand place value

Use place value understanding and properties of operations to add and subtract

Measure lengths indirectly and by iterating length units

Represent and solve problems involving addition and subtraction

Add and subtract within 20

Understand place value

Use place value understanding and properties of operations to add and subtract

Measure and estimate lengths in standard units

Relate addition and subtraction to length

solve problems involving multiplication and division

Understand properties of multiplication and the relationship between multiplication and division

Multiply & divide within 100

Solve problems involving the four operations, and identify & explain patterns in arithmetic

Develop understanding of fractions as numbers

Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects

Geometric measurement: understand concepts of area and

relate area to multiplication and to addition

Use the four operations with whole numbers to solve problems

Generalize place value understanding for multi-digit whole numbers

Use place value understanding and properties of operations to perform multi- digit arithmetic

Extend understanding of fraction equivalence and ordering

Build fractions from unit fractions by applying and extending previous

understandings of

operations

Understand decimal notation for fractions, and compare decimal fractions

Understand the place value system

Perform operations with multi-digit whole numbers and decimals to hundredths

Use equivalent fractions as a strategy to add and subtract fractions

Apply and extend previous understandings of multiplication and division to

multiply and

divide fractions

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition

Graph points in the coordinate plane to solve real-world and mathematical problems\*

Apply and extend previous understandings of multiplication and division to divide fractions by fractions

Apply and extend previous understandings of numbers to the system of rational numbers

Understand ratio concepts and use ratio reasoning to solve problems

Apply and extend previous understandings of arithmetic to algebraic expressions

Reason about and solve one-variable equations and inequalities

Represent and analyze quantitative relationships between dependent and independent variables

Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers

Analyze proportional relationships and use them to solve real-world and mathematical problems

Use properties of operations to generate equivalent expressions

Solve real-life and mathematical problems using numerical and algebraic expressions and equations

Work with radical and integer exponents

Understand the connections between proportional relationships, lines, and linear equations

Analyze and solve linear equations and pairs of simultaneous linear equations

Define, evaluate, and compare functions

Use functions to model relationships between quantities\*

\*Indicates a cluster that is well thought of as part of a student’s progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the two asterisked exceptions, the clusters listed here are a subset of those designated as Major in both of the assessment consortia’s draft documents.

4. **Focus:** In aligned materials there are no chapter tests, unit tests, or other assessment components that make students or teachers responsible for any topics before the grade in which they are introduced in the Standards. (One way to meet this criterion is for materials to omit these topics entirely prior to the indicated grades.) If the materials address topics outside of the *Common Core State Standards for Mathematics with California Additions,* the publisher will provide a mathematical and pedagogical justification.

5. **Focus and Coherence through Supporting Work: Supporting clusters do not detract from focus, but rather enhance focus and coherence simultaneously by engaging students in the major clusters of the grade.** For example, materials for K–5 generally treat data displays as an occasion for solving grade-level word problems using the four operations.[[6]](#footnote-6)

6. **Rigor and Balance: Materials and tools reflect the balances in the Standards and help students meet the Standards’ rigorous expectations, by all of the following:**

a. **Developing students’ conceptual understanding of key mathematical concepts, where called for in specific content standards or cluster headings, including connecting conceptual understanding to procedural skills.** Materials amply feature high-quality conceptual problems and questions that can serve as fertile conversation- starters in a classroom if students are unable to answer them. In addition, group discussion suggestions include facilitation strategies and protocols. In the materials, conceptual understanding is not a generalized imperative applied with a broad brush, but is attended to most thoroughly in those places in the content standards where explicit expectations are set for understanding or interpreting. (Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.)

b. **Giving attention throughout the year to individual standards that set an expectation of**

**fluency.** The Standards are explicit where fluency is expected. In grades K–6 materials should help students make steady progress throughout the year toward fluent (accurate and reasonably fast) computation, including knowing single-digit products and sums from memory (see, e.g., 2.OA.2 and 3.OA.7). The word “fluently” in particular as used in the Standards refers to fluency with a written or mental method, not a method using manipulatives or concrete representations. Progress toward these goals is interwoven with developing conceptual understanding of the operations in question.[[7]](#footnote-7)

Manipulatives and concrete representations such as diagrams that enhance conceptual understanding are closely connected to the written and symbolic methods to which they refer (see, e.g., 1.NBT). As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable—e.g., the sum 698 + 240 or the system *x* + *y* = 1, 2*x* + 2*y* = 3—as well as an ample number of generic cases so that students can learn and practice efficient algorithms (e.g., the sum 8767 + 2286). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.[[8]](#footnote-8) Materials do not make fluency a generalized imperative to be applied with a broad brush, but attend most thoroughly to those places in the content standards where explicit expectations are set for fluency. In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8).

c. **Allowing teachers and students using the materials as designed to spend sufficient time working with engaging applications, without losing focus on the major work of each grade.** Materials in grades K–8 include an ample number of single-step and multi- step contextual problems that develop the mathematics of the grade, afford opportunities for practice, and engage students in problem solving. Materials for grades 6–8 also include problems in which students must make their own assumptions or simplifications in order to model a situation mathematically. Applications take the form of problems to be worked on individually as well as classroom activities centered on application scenarios. Materials attend thoroughly to those places in the content standards where expectations for multi-step and real-world problems are explicit. Applications in the materials draw only on content knowledge and skills specified in the content standards, with particular stress on applying major work, and a preference for the more fundamental techniques from additional and supporting work. Modeling builds slowly across K–8, and applications are relatively simple in early grades. Problems and activities are grade-level appropriate, with a sensible tradeoff between the sophistication of the problem and the difficulty or newness of the content knowledge the student is expected to bring to bear.[[9]](#footnote-9)

**Additional aspects of the Rigor and Balance Criterion**:

(1) *The three aspects of rigor are not always separate in materials.* (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)

(2) *Nor are the three aspects of rigor always together in materials.* (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free unless explicitly taught.)

(3) Digital and online materials with no fixed lesson flow or pacing plan are not designed for superficial browsing but rather instantiate the Rigor and Balance criterion and promote depth and mastery.

7. **Consistent Progressions: Materials are consistent with the progressions in the Standards, by (all of the following):**

a. **Basing content progressions on the grade-by-grade progressions in the Standards.**

Progressions in materials match closely with those in the Standards. This does not require the table of contents in a book to be a replica of the content standards; but the match between the Standards and what students are to learn should be close in each grade. Discrepancies are clearly aimed at helping students meet the Standards as written, rather than effectively rewriting the standards. Comprehensive materials do not introduce gaps in learning by omitting content that is specified in the Standards.

The basic model for grade-to-grade progression involves students making tangible progress during each given grade, as opposed to substantially reviewing then marginally extending from previous grades. Remediation may be necessary, particularly during transition years, and resources for remediation may be provided, but review is clearly identified as such to the teacher, and teachers and students can see what their specific responsibility is for the current year.

Digital and online materials that allow students and/or teachers to navigate content across grade levels promote the Standards’ coherence by tracking the structure and progressions in the Standards. For example, such materials might link problems and concepts so that teachers and students can browse a progression.

b. **Giving all students extensive work with grade-level problems.** Differentiation is sometimes necessary, but materials often manage unfinished learning from earlier grades inside grade-level work, rather than setting aside grade-level work to reteach earlier content. Unfinished learning from earlier grades is normal and prevalent; it should not be ignored nor used as an excuse for cancelling grade-level work and retreating to below-grade work. (For example, the development of fluency with division using the standard algorithm in grade 6 is the occasion to surface and deal with unfinished learning about place value; this is more productive than setting aside division and backing up.) Likewise, students who are “ready for more” can be provided with problems that take grade-level work in deeper directions, not just exposed to later grades’ topics.

c. **Relating grade-level concepts explicitly to prior knowledge from earlier grades.** The materials are designed so that prior knowledge becomes reorganized and extended to accommodate the new knowledge. Grade-level problems in the materials often involve application of knowledge learned in earlier grades. Although students may well have learned this earlier content, they have not learned how it extends to new mathematical situations and applications. They learn basic ideas of place value, for example, and then extend them across the decimal point to tenths and beyond. They learn properties of operations with whole numbers, and then extend them to fractions, variables, and expressions. The materials make these extensions of prior knowledge explicit. Note that cluster headings in the Standards sometimes signal key moments where reorganizing and extending previous knowledge is important in order to accommodate new knowledge (e.g., see the cluster headings that use the phrase “Apply and extend previous understanding”).

8. **Coherent Connections: Materials foster coherence through connections at a single grade,**

**where appropriate and where required by the Standards, by (all of the following):**

a. **Including learning objectives that are visibly shaped by CCSSM cluster headings, with meaningful consequences for the associated problems and activities.** While some clusters are simply the sum of their individual standards (e.g., Grade 8, Expressions and Equations, Cluster C: Analyze and solve linear equations and pairs of simultaneous linear equations.), many are not (e.g., Grade 8, Expressions and Equations, Cluster B: Understand the connection between proportional relationships, lines, and linear equations.). In the latter cases, cluster headings function like topic sentences in a paragraph in that they state the point of, and lend additional meaning to, the individual content standards that follow. Cluster headings can also signal multi-grade progressions, by using phrases such as “Apply and extend previous understandings of [X] to do [Y].” Hence an important criterion for coherence is that some or many of the learning objectives in the materials are visibly shaped by CCSSM cluster headings, with meaningful consequences for the associated problems and activities. Materials do not simply treat the Standards as a sum of individual content standards and individual practice standards.

b. **Including problems and activities that serve to connect two or more clusters in a domain, or two or more domains in a grade, in cases where these connections are natural and important**. If instruction only operates at the individual standard level, or even at the individual cluster level, then some important connections will be missed. For example, robust work in 4.NBT should sometimes or often synthesize across the clusters listed in that domain; robust work in grade 4 should sometimes or often involve students applying their developing computation NBT skills in the context of solving word problems detailed in OA. Materials do not invent connections not explicit in the standards without first attending thoroughly to the connections that are required explicitly in the Standards (e.g., 3.MD.7 connects area to multiplication, to addition, and to properties of operations; A-REI.11 connects functions to equations in a graphical context; proportion connects to percentage, similar triangles, and unit rates.) Not everything in the standards is naturally well connected or needs to be connected (e.g., Order of Operations has essentially nothing to do with the properties of operations, and connecting these two things in a lesson or unit title is actively misleading). Instead, connections in materials are mathematically natural and important (e.g., base-ten computation in the context of word problems with the four operations), reflecting plausible direct implications of what is written in the Standards without creating additional requirements. Instructional materials include problems and activities that connect to real-world and career settings, where appropriate.

9. **Practice-to-Content Connections: Materials meaningfully connect content standards and practice standards.** “Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.” (CCSSM, p. 8.) Over the course of any given year of instruction, each mathematical practice standard is meaningfully present in the form of activities or problems that stimulate students to develop the habits of mind described in the practice standards. These practices are well-grounded in the content standards. Materials are accompanied by an analysis, aimed at evaluators, of how the authors have approached each practice standard in relation to content within each applicable grade or grade band. Materials do not treat the practice standards as static across grades or grade bands, but instead tailor the connections to the content of the grade and to grade-level-appropriate student thinking. Materials also include teacher-directed materials that explain the role of the practice standards in the classroom and in students’ mathematical development.

10. **Focus and Coherence via Practice Standards: Materials promote focus and coherence by connecting practice standards with content that is emphasized in the Standards**. Content and practice standards are not connected mechanistically or randomly, but instead support focus and coherence. Examples: Materials connect looking for and making use of structure (MP.7) with structural themes emphasized in the Standards such as properties of operations, place value decompositions of numbers, numerators and denominators of fractions, numerical and algebraic expressions, etc.; materials connect looking for and expressing regularity in repeated reasoning (MP.8) with major topics by using regularity in repetitive reasoning as a *tool* with which to explore major topics. (In K–5, materials might use regularity in repetitive reasoning to shed light on, e.g., the 10  10 addition table, the 10  10 multiplication table, the properties of operations, the relationship between addition and subtraction or multiplication and division, and the place value system; in 6–8, materials might use regularity in repetitive reasoning to shed light on proportional relationships and linear functions; in high school, materials might use regularity in repetitive reasoning to shed light on formal algebra as well as functions, particularly recursive definitions of functions.)

11. **Careful Attention to Each Practice Standard: Materials attend to the full meaning of each practice standard.** For example, MP.1 does not say, “Solve problems.” Or “Make sense of problems.” Or “Make sense of problems and solve them.” It says “Make sense of problems and persevere in solving them.” Thus, students using the materials as designed build their perseverance in grade-level-appropriate ways by occasionally solving problems that require them to persevere to a solution beyond the point when they would like to give up. MP.5 does not say, “Use tools.” Or “Use appropriate tools.” It says “Use appropriate tools strategically.” Thus, materials include problems that reward students’ strategic decisions about how to use tools, or about whether to use them at all. MP.8 does not say, “Extend patterns.” Or “Engage in repetitive reasoning.” It says “Look for and express regularity in repeated reasoning.” Thus, it is not enough for students to extend patterns or perform repeated calculations. Those repeated calculations must lead to an insight (e.g., “When I add a multiple of 3 to another multiple of 3, then I get a multiple of 3.”). The analysis for evaluators explains how the full meaning of each practice standard has been attended to in the materials.

12. **Emphasis on Mathematical Reasoning: Materials support the Standards’ emphasis on mathematical reasoning, by all of the following:**

a. **Prompting students to construct viable arguments and critique the arguments of others concerning key grade-level mathematics that is detailed in the content standards (cf. MP.3).** Materials provide sufficient opportunities for students to reason mathematically in independent thinking and express reasoning through classroom discussion and written work. Reasoning is not confined to optional or avoidable sections of the materials but is inevitable when using the materials as designed. Materials do not approach reasoning as a generalized imperative, but instead create opportunities for students to reason *about* key mathematics detailed in the content standards for the grade. Materials thus attend first and most thoroughly to those places in the content standards setting explicit expectations for explaining, justifying, showing, or proving. Students are asked to critique given arguments, e.g., by explaining under what conditions, if any, a mathematical statement is valid. Materials develop students’ capacity for mathematical reasoning in a grade-level appropriate way, with a reasonable progression of sophistication from early grades up through high school.[[10]](#footnote-10) Teachers and students using the materials as designed spend classroom time communicating reasoning (by constructing viable arguments and explanations and critiquing those of others’ concerning key grade-level mathematics) — recognizing that learning mathematics also involves time spent working on applications and practicing procedures. Materials provide examples of student explanations and arguments (e.g., fictitious student characters might be portrayed).

b. **Engaging students in problem solving as a form of argument**. Materials attend thoroughly to those places in the content standards that explicitly set expectations for multi-step problems; multi-step problems are not scarce in the materials. Some or many of these problems require students to devise a strategy autonomously. Sometimes the goal is the final answer alone (cf. MP.1); sometimes the goal is to show work and lay out the solution as a sequence of well justified steps. In the latter case, the solution to a problem takes the form of a cogent argument that can be verified and critiqued, instead of a jumble of disconnected steps with a scribbled answer indicated by drawing a circle around it (cf. MP.6). Problems and activities of this nature are grade-level appropriate, with a reasonable progression of sophistication from early grades up through high school.

c. **Explicitly attending to the specialized language of mathematics.** Mathematical reasoning involves specialized language. Therefore, materials and tools address the development of mathematical and academic language associated with the standards. The language of argument, problem solving and mathematical explanations are taught rather than assumed. Correspondences between language and multiple mathematical representations including diagrams, tables, graphs, and symbolic expressions are identified in material designed for language development. Note that variety in formats and types of representations—graphs, drawings, images, and tables in addition to text—can relieve some of the language demands that English language learners face when they have to show understanding in math.

d. **Materials help English learners access challenging mathematics, learn content, and develop grade-level language.** For example, materials might include annotations to help with comprehension of words, sentences and paragraphs, and give examples of the use of words in other situations. Modifications to language do not sacrifice the mathematics, nor do they put off necessary language development.

**Category 2: Program Organization**

The organization and features of the instructional materials support instruction and learning of the Standards. Teacher and student materials include such features as lists of the standards, chapter overviews, and glossaries. Instructional materials must have strengths in these areas to be considered suitable for adoption.

1. A list of *Common Core State Standards for Mathematics with California Additions* is included in the teacher's guide together with page number citations or other references that demonstrate alignment with the content standards and standards for mathematical practice*.* All standards must be listed in their entirety with their cluster heading included.

2. Materials drawn from other subject-matter areas are consistent with the currently adopted California standards at the appropriate grade level, including the *California Career Technical Education Model Curriculum Standards* where applicable.

3. Intervention components, if included, are designed to support students’ progress in mathematics and develop fluency. Intervention materials should provide targeted instruction on standards from previous grade levels and develop student learning of the standards for mathematical practice.

4. Middle school acceleration components, if included, are designed to support students’ progress beyond grade-level standards in mathematics. Acceleration materials should provide instruction targeted toward readiness for higher mathematics at the middle school level.

5. Teacher and student materials contain an overview of the chapters, clearly identify the mathematical concepts, and include tables of contents, indexes, and glossaries that contain important mathematical terms.

6. Support materials are an integral part of the instructional program and are clearly aligned with the *Common Core State Standards for Mathematics with California Additions.*

7. The grade-level content standards and the standards for mathematical practice demonstrating alignment to student lessons shall be explicitly stated in the student editions.

**Category 3: Assessment**

Instructional materials should contain strategies and tools for continually measuring student achievement. Formative assessment is a systematic process to continuously gather evidence and provide feedback about learning while instruction is under way. Formative assessments can take multiple forms and occur over varied durations of time. They are to be used to gather information about student learning and to address student misunderstandings. Formative assessments are to provide guidance for the teacher in determining whether the student needs additional materials or resources to achieve grade-level standards and conceptual understanding. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

1. Not every form of assessment is appropriate for every student or every topic area, so a variety of assessment types need to be provided for formative assessment. Some of these could include (but is not limited to) graphic organizers, student observation, student interviews, journals and learning logs, exit ticket activities, mathematics portfolios, self- and peer-evaluations, short tests and quizzes, and performance tasks.

2. Summative assessment is the assessment of learning at a particular time point and is meant to summarize a learner's skills and knowledge at a given point of time. Summative assessments frequently come in the form of chapter or unit tests, weekly quizzes, end-of-term tests, or diagnostic tests.

3. All assessments should have content validity and measure individual student progress both at regular intervals and at strategic points of instruction. The assessments should be designed to:

* Monitor student progress toward meeting the content and mathematical practice standards.
* Assess all three aspects of rigor: conceptual understanding, procedural skill and fluency, and applications.
* Provide summative evaluations of individual student achievement.
* Provide multiple methods of assessing what students know and are able to do, such as selected response, constructed response, real-world problems, performance tasks, and open-ended questions.
* Assist the teacher in keeping parents and students informed about student progress.

4. Intervention aspects of mathematics programs should include initial assessments to identify areas of strengths and weaknesses, formative assessments to demonstrate student progress toward meeting grade-level standards, and a summative assessment to determine student preparedness for grade-level work.

5. Suggestions on how to use assessment data to guide decisions about instructional practices and how to modify instruction so that all students are consistently progressing toward meeting or exceeding the standards should be included.

6. Assessments that ask for variety in what students produce, answers and solutions, arguments and explanations, diagrams, mathematical models.

7. Assessment tools for grades six through eight help to determine student readiness for Common Core Algebra I and Common Core Mathematics I.

8. Middle school acceleration aspects of mathematics programs include an initial assessment to identify areas of strengths and weaknesses, formative assessments to demonstrate student progress toward exceeding grade-level standards, and a summative assessment to determine student preparedness for above grade-level work.

**Category 4: Universal Access**

Students with special needs must be provided access to the same standards-based curriculum that is provided to all students, including both the content standards and the standards for mathematical practice*.* Instructional materials should provide access to the standards-based curriculum for all students, including English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

1. Comprehensive guidance and differentiation strategies, based on current and confirmed research, to adapt the curriculum to meet students' identified special needs and to provide effective, efficient instruction for all students. Strategies may include:

* Working with students’ misconceptions to strengthen their conceptual understanding.
* Intervention strategies that describe specific ways to address the learning needs of students using rich problems that engage them in the mathematics reviewed and stress conceptual development of topics rather than focusing only on procedural skills.
* Suggestions for reinforcing or expanding the curriculum.
* Additional instructional time and additional practice, including specialized teaching methods or materials and accommodations for students with special needs.
* Help for students who are below grade level, including more explicit explanations with ample and different opportunities for review and practice of both content and mathematical practices standards, or other assistance that will help to accelerate student performance to grade level.
* Technology may be a used to aid in the implementation of these strategies.

2. Strategies for English learners that are consistent with the English Language Development Standards adopted under *Education Code* Section 60811. Materials incorporate strategies for English learners in both lessons and teacher’s editions, as appropriate, at every grade level and course level.

3. Materials incorporate instructional strategies to address the needs of students with disabilities in both lessons and teacher’s editions, as appropriate, at every grade level and course level, pursuant to *Education Code* section 60204(b)(2).

4. Teacher and student editions include thoughtful and well-conceived alternatives for advanced students and that allow students to accelerate beyond their grade-level content (acceleration) or to study the content in the *Common Core State Standards for Mathematics with California Additions* in greater depth or complexity (enrichment).

5. Materials should help students understand and use appropriate academic language and participate in discussions about mathematical concepts and reasoning. Materials should include content that is relevant to English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.

6. Materials help English learners access challenging mathematics, learn content, and develop grade-level language. For example, materials might include annotations to help with comprehension of words, sentences and paragraphs, and give examples of the use of words in other situations. Modifications to language do not sacrifice the mathematics, nor do they put off necessary language development.

7. Materials are consistent with the strategies found in Response to Intervention and Instruction (<http://www.cde.ca.gov/ci/cr/ri/>).

1. The visual design of the materials does not distract from the mathematics, but instead serves to support students in engaging thoughtfully with the subject.

**Category 5: Instructional Planning**

Instructional materials must contain a clear road map for teachers to follow when planning instruction. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

1. A teacher's edition with ample and useful annotations and suggestions on how to present the content in the student edition and in the ancillary materials, including modifications for English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.

2. A list of program lessons in the teacher's edition, cross-referencing the standards covered and providing an estimated instructional time for each lesson, chapter, and unit.

3. Unit and lesson plans, including suggestions for organizing resources in the classroom and ideas for pacing lessons.

4. A curriculum guide for the academic instructional year.

5. All components of the program are user friendly and, in the case of electronic materials, platform neutral.

6. Answer keys for all workbooks and other related student activities.

7. Concrete models, including manipulatives, support instruction of the *Common Core State Standards for Mathematics with California Additions* and include clear instructions for teachers and students*.*

8. A teacher’s edition that explains the role of the specific grade-level mathematics in the context of the overall mathematics curriculum for kindergarten through grade twelve.

9. Technical support and suggestions for appropriate use of audiovisual, multimedia, and information technology resources.

10. Homework activities, if included, that extend and reinforce classroom instruction and provide additional practice of mathematical content, practices, and applications that have been taught.

11. Strategies for informing parents or guardians about the mathematics program and suggestions for how they can help support student progress and achievement.

**Category 6: Teacher Support**

Instructional materials should be designed to help teachers provide mathematics instruction that ensures opportunities for all students to learn the essential skills and knowledge specified for in the *Common Core State Standards for Mathematics with California Additions*. Instructional materials in mathematics must have strengths in these areas to be considered suitable for adoption:

1. Clear, grade-appropriate explanations of mathematics concepts that teachers can easily adapt for instruction of all students, including English learners, advanced learners, students below grade level in mathematical skills, and students with disabilities.

2. Strategies to identify, address, and correct common student errors and misconceptions.

3. Suggestions for accelerating or decelerating the rate at which new material is introduced to students.

4. Different kinds of lessons and multiple ways in which to explain concepts, offering teachers choice and flexibility.

5. Materials designed to help teachers identify the reason(s) that students may find a particular type of problem(s) more challenging than another (e.g., identify skills not mastered) and point to specific remedies.

6. Learning objectives that are explicitly and clearly associated with instruction and assessment.

7. A teacher’s edition that contains full, adult-level explanations and examples of the more advanced mathematics concepts in the lessons so that teachers can improve their own knowledge of the subject, as necessary.

8. Explanations of the instructional approaches of the programs and identification of the research-based strategies.

9. Explanations of the mathematically appropriate use of manipulatives or other visual and concrete representations.

1. See the Smarter/Balanced content specification and item development specifications, and the PARCC Model Content Framework and item development ITN. Complete information about the consortia can be found at http://www.smarterbalanced.org and <http://www.parcconline.org.> [↑](#footnote-ref-1)
2. For some remarks by Phil Daro on this theme, see the excerpt at [http://vimeo.com/achievethecore/darofocus,](http://vimeo.com/achievethecore/darofocus) and/or the full

   video available at [http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/.](http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/) [↑](#footnote-ref-2)
3. For more information on progressions in the Standards, see [http://ime.math.arizona.edu/progressions.](http://ime.math.arizona.edu/progressions) [↑](#footnote-ref-3)
4. See “Appendix: The Structure of the Standards” in *K–8 Publishers’ Criteria for the Common Core State Standards for Mathematics*, p. 21 (<http://www.corestandards.org/assets/Math_Publishers_Criteria_K-8_Summer%202012_FINAL.pdf>) [↑](#footnote-ref-4)
5. For cluster-level emphases at grades K–8, see [http://www.achievethecore.org/downloads/Math%20Shifts%20and%20Major%20Work%20of%20Grade.pdf.](http://www.achievethecore.org/downloads/Math%20Shifts%20and%20Major%20Work%20of%20Grade.pdf) [↑](#footnote-ref-5)
6. For more information about this example, see Table 1 in the *Progression* for K–3 Categorical Data and 2–5 Measurement Data, [http://commoncoretools.files.wordpress.com/2011/06/ccss\_progression\_md\_k5\_2011\_06\_20.pdf.](http://commoncoretools.files.wordpress.com/2011/06/ccss_progression_md_k5_2011_06_20.pdf) More generally, the *PARCC Model Content Frameworks* give examples in each grade of how to improve focus and coherence by linking supporting topics to the major work. [↑](#footnote-ref-6)
7. For more about how students develop fluency in tandem with understanding, see the *Progressions* for Operations and Algebraic Thinking, <http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf>and for Number and Operations in Base Ten,

   [http://commoncoretools.files.wordpress.com/2011/04/ccss\_progression\_nbt\_2011\_04\_073.pdf.](http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf) [↑](#footnote-ref-7)
8. Non-mathematical approaches (such as the “butterfly method” of adding fractions) compromise focus and coherence and displace mathematics in the curriculum (cf. 5.NF.1). For additional background on this point, see the remarks by Phil Daro excerpted at <http://vimeo.com/achievethecore/darofocus>and/or the full video, available at [http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/.](http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/) [↑](#footnote-ref-8)
9. Cf. *Common Core State Standards for Mathematics* (CCSSM), p. 84 at <http://www.corestandards.org/the-standards>. Also note that modeling is a mathematical practice in every grade, but in high school it is also a content category (CCSSM, pp. 72, 73); therefore, modeling is generally enhanced in high school materials, with more elements of the modeling cycle (CCSSM, p. 72). [↑](#footnote-ref-9)
10. As students progress through the grades, their production and comprehension of mathematical arguments evolves from informal and concrete toward more formal and abstract. In early grades students employ imprecise expressions which with practice over time become more precise and viable arguments in later grades. Indeed, the use of imprecise language is part of the process in learning how to make more precise arguments in mathematics. Ultimately, conversation about arguments helps students transform assumptions into explicit and precise claims. [↑](#footnote-ref-10)