Mississippi College-and Career-Readiness Standards for Mathematics 2014

Scaffolding Document

INTRODUCTION

Mission Statement

The Mississippi Department of Education is dedicated to student success, including the improvement of student achievement in mathematics, in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a technological society. The *2014 Mississippi College- and Career-Readiness Standards for Mathematics* ("The Standards") provide a consistent, clear understanding of what students are expected to know and be able to do by the end of each grade level and course. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success in college and careers and to compete in the global economy.

Purpose

The primary purpose of the *2014 Mississippi College- and Career-Readiness Standards for Mathematics Scaffolding Document* is to provide teachers with a deeper understanding of the Standards as they plan for classroom instruction. Based on the *2014 Mississippi College- and Career-Readiness Standards*, this document provides a close analysis of the requirements for student mastery. Because of the rigor and depth of the Standards, scaffolding instruction to meet the needs of all learners is essential to individual success. The Scaffolding Document will aid teachers' understanding of how to teach the Standards through a natural progression of student mastery.

Organization

The *2014 Mississippi College- and Career-Readiness Standards for Mathematics Scaffolding Document* is divided by grade level/course. Within each grade level/course, the document is separated into domains for Grades K-8 and conceptual categories for Grades 9-12. The content of this document is centered on the mathematical domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking** (Grades 1-5); **Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations - Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions and Equations, Geometry,**

Statistics and Probability (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics and Probability**.

Each standard is then broken down into three categories: "What a Student Should Know" (Prerequisite Knowledge), "What a Student Should Understand" (Conceptual Understanding), and "What a Student Should Be Able to Do" (Evidence of Knowledge). The Prerequisite Knowledge column lists the skills that students should have mastered in previous grades/courses in order to work towards mastery of the grade-specific standard. In other words, this column details what a student needs to KNOW before mastering the grade-specific standard. The Conceptual Understanding column explains the deeper understanding of concepts, not actions or skills, that are required for mastery of the grade-specific standard. In summary this column explains what a student needs to UNDERSTAND before mastering the grade-specific standard. The last column, Evidence of Knowledge, explains what student mastery looks like, including what work a student may produce to exhibit mastery of the grade-specific standard. Conclusively, this column describes what a student needs to DO to show mastery of the grade-specific standard.

OVERVIEW OF THE COMMON CORE STATE STANDARDS FOR MATHEMATICS

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge. Aiming for clarity and specificity, these Standards endeavor to follow a design that not only stresses conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the laws of arithmetic to structure those ideas.

Understanding Mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific expectations but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary for college and/or careers. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with reading disabilities the use of Braille, screen reader technology, or other assistive devices should be made available. In addition, while writing, these students should have access to a scribe, computer, or speech-to-text technology in their classroom. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of College- and Career- Readiness for all students.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the 2014 Mississippi College- and Career- Readiness Standards for Mathematics

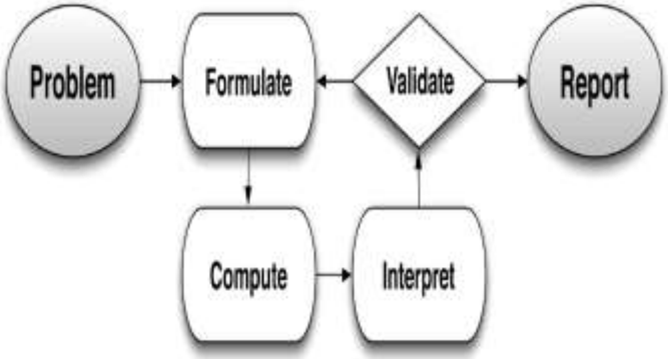
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to the Standards in mathematics instruction.

The Standards are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Modeling (High School Courses only)

Modeling standards are noted throughout the high school courses with an asterisk (*). *Modeling* links classroom mathematics and statistics to everyday life, work, and decision-making. *Modeling* is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.



Making mathematical models is a Standard for Mathematical Practice, and specific *Modeling* standards appear throughout the high school standards. The basic modeling cycle above involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

PARCC Assessment Emphases

Each tested grade level or course describes cluster emphases for content standards. These are provided because curriculum, instruction, and assessment in each course must reflect the focus and emphasis of the grade level/course standards. To make relative emphases in the Standards more transparent and useful, cluster headings are designated as **Major**, **Additional** and **Supporting**. Key: ■ Major Clusters; ■ Supporting Clusters; and ■ Additional Clusters.

Some clusters that are not major emphases in themselves are designed to *support* and strengthen areas of major emphasis, while other clusters that may not connect tightly or explicitly to the major work of the grade would fairly be called *additional*. To say that some things have greater emphasis is not to say that anything in the Standards can be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

All standards figure in a mathematical education and therefore will be *eligible* for inclusion on the PARCC end-of-year assessments. The assessments will mirror the message that is communicated here: Major clusters will be a majority of the assessment, Supporting clusters will be assessed through their success at supporting the Major clusters and Additional clusters will be assessed as well. The assessments will strongly focus where the Standards strongly focus.

Finally, the following are some recommendations for using the cluster-level emphases:

Do …

- Use the guidance to inform instructional decisions regarding time and other resources spent on clusters of varying degrees of emphasis.
- Allow the focus on the major work of the grade to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials taking the cluster-level emphases into account. The major work of the grade must be presented with the highest possible quality; the supporting work of the grade should indeed support the major focus, not detract from it.
- Set priorities for other implementation efforts taking the emphases into account, such as staff development; new curriculum development; or revision of existing formative or summative testing at the district or school level.

Don't …

- Neglect any material in the standards. (Instead, use the information provided to connect Supporting Clusters to the other work of the grade.)

- Sort clusters from Major to Supporting, and then teach them in that order. To do so would strip the coherence of the mathematical ideas and miss the opportunity to enhance the major work of the grade with the supporting clusters.

- Use the cluster headings as a replacement for the standards. All features of the standards matter — from the practices to surrounding text to the particular wording of individual content standards. Guidance is given at the cluster level as a way to talk about the content with the necessary specificity yet without going so far into detail as to compromise the coherence of the standards.